

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-a+b-x+c-x²-^p-d+e-x+f-x²-^q

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3.92	$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3} dx$	516
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3.103	$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx$	585
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3.114	$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}} dx$	658
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3.117	$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$	678
3.118	$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}} dx$	685
3.119	$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}} dx$	695
3.120	$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}} dx$	702

3.121	$\int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx$	706
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [123]. This is test number [36].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (123)	% 0. (0)
Mathematica	% 100. (123)	% 0. (0)
Maple	% 98.37 (121)	% 1.63 (2)
Maxima	% 54.47 (67)	% 45.53 (56)
Fricas	% 89.43 (110)	% 10.57 (13)
Sympy	% 34.96 (43)	% 65.04 (80)
Giac	% 70.73 (87)	% 29.27 (36)

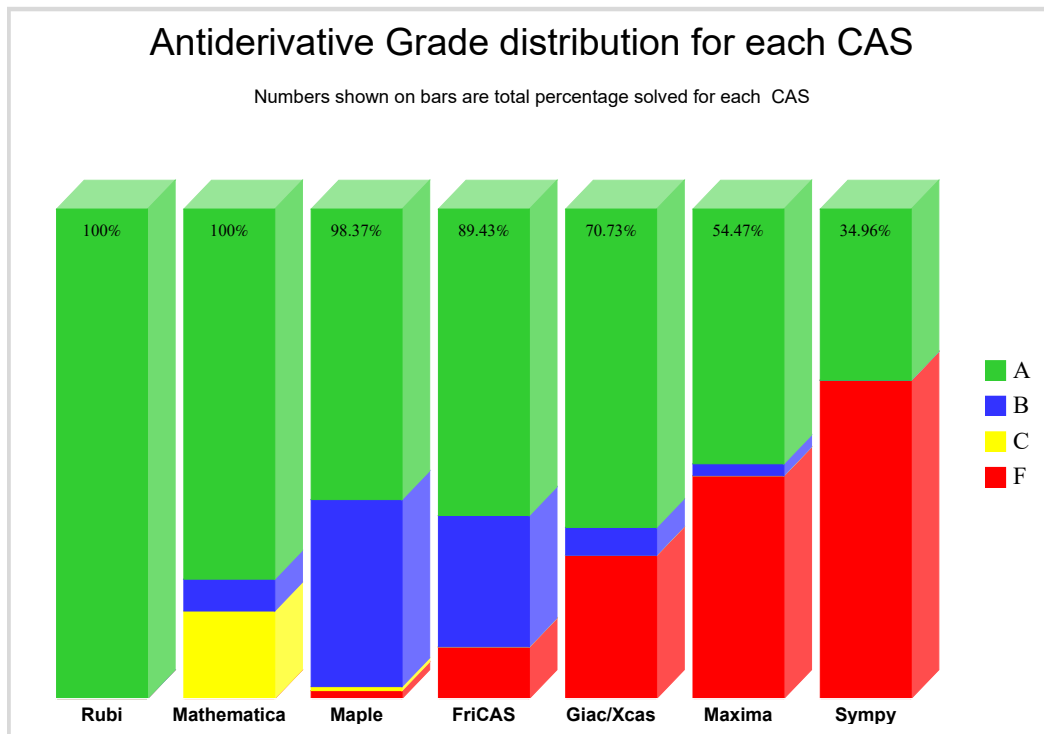
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

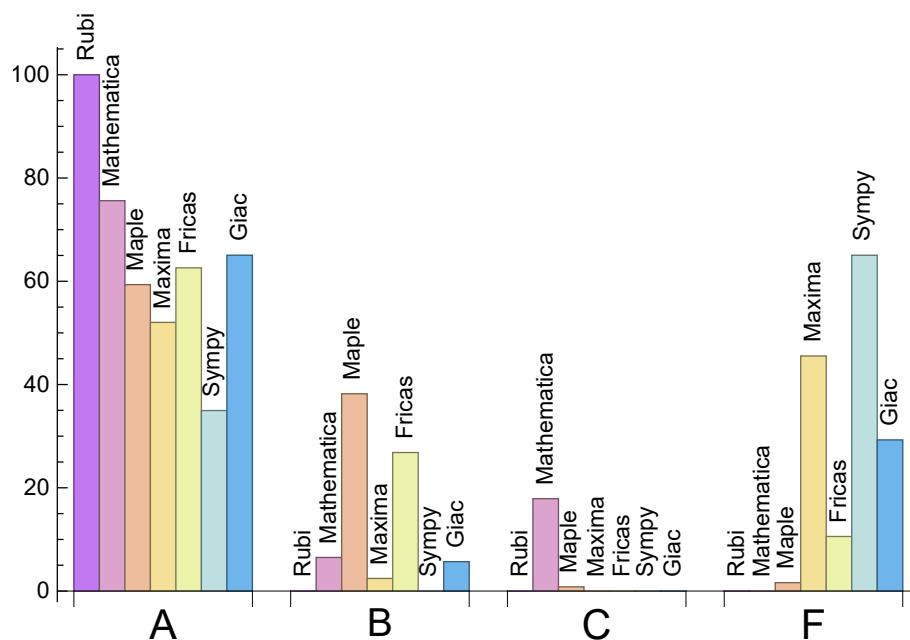
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	75.61	6.5	17.89	0.
Maple	59.35	38.21	0.81	1.63
Maxima	52.03	2.44	0.	45.53
Fricas	62.6	26.83	0.	10.57
Sympy	34.96	0.	0.	65.04
Giac	65.04	5.69	0.	29.27

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.71	205.62	1.	128.	1.
Mathematica	0.94	322.93	1.34	95.	1.
Maple	0.16	6251.53	19.31	136.	0.82
Maxima	1.39	121.69	1.22	97.	1.12
Fricas	3.72	2411.01	11.67	395.5	4.02
Sympy	0.22	76.74	0.99	73.	0.97
Giac	1.37	202.79	1.32	97.	1.05

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {6, 106, 107, 108, 113, 122, 123}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

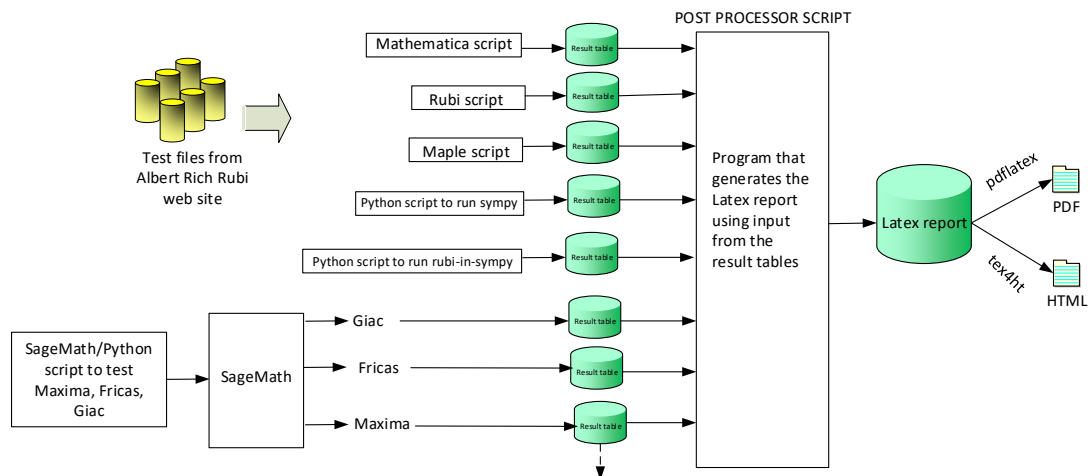
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 107, 108 }

C grade: { 8, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 121 }

F grade: { }

2.1.3 Maple

A grade: { 1, 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 96, 111, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 121 }

C grade: { 11 }

F grade: { 9, 10 }

2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96 }

B grade: { 93, 94, 95 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.5 FriCAS

A grade: { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade: { 2, 3, 4, 5, 7, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 95, 97, 98, 99, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade: { }

F grade: { 6, 9, 10, 13, 102, 103, 106, 107, 108, 113, 117, 122, 123 }

2.1.6 Sympy

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

2.1.7 Giac

A grade: { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119 }

B grade: { 3, 8, 12, 14, 104, 120, 121 }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 10, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 108, 112, 113, 117, 122, 123 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	87	136	0	494	0	113
normalized size	1	1.	0.85	1.33	0.	4.84	0.	1.11
time (sec)	N/A	0.094	0.191	0.052	0.	1.157	0.	1.302

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	178	491	0	2226	0	0
normalized size	1	1.	2.17	5.99	0.	27.15	0.	0.
time (sec)	N/A	0.111	0.388	0.388	0.	2.14	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	161	307	0	1755	0	620
normalized size	1	1.	2.44	4.65	0.	26.59	0.	9.39
time (sec)	N/A	0.079	0.226	0.327	0.	1.458	0.	14.668

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	296	829	0	3297	0	0
normalized size	1	1.	2.29	6.43	0.	25.56	0.	0.
time (sec)	N/A	0.169	0.91	0.301	0.	3.384	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	1748	1884	0	8058	0	0
normalized size	1	1.	7.8	8.41	0.	35.97	0.	0.
time (sec)	N/A	0.427	6.376	0.242	0.	19.058	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	328	328	3386	3695	0	0	0	0
normalized size	1	1.	10.32	11.27	0.	0.	0.	0.
time (sec)	N/A	0.97	6.58	0.259	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	490	1377	0	4059	0	0
normalized size	1	1.	3.02	8.5	0.	25.06	0.	0.
time (sec)	N/A	0.306	2.127	0.352	0.	8.127	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	84	27	0	123	0	70
normalized size	1	1.	3.	0.96	0.	4.39	0.	2.5
time (sec)	N/A	0.017	0.068	0.048	0.	1.034	0.	1.142

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.138	2.905	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	172	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	0.238	3.482	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	27	16	0	55	0	66
normalized size	1	1.	0.56	0.33	0.	1.15	0.	1.38
time (sec)	N/A	0.015	0.018	0.168	0.	0.866	0.	1.182

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	84	0	235	0	193
normalized size	1	1.	0.94	1.2	0.	3.36	0.	2.76
time (sec)	N/A	0.051	0.083	0.059	0.	0.808	0.	1.168

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1077	1077	600	661	0	0	0	0
normalized size	1	1.	0.56	0.61	0.	0.	0.	0.
time (sec)	N/A	3.099	1.528	0.618	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	159	341	0	441	0	231
normalized size	1	1.	1.62	3.48	0.	4.5	0.	2.36
time (sec)	N/A	0.21	0.401	0.112	0.	0.975	0.	1.177

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	73	176	65	73
normalized size	1	1.	1.	0.81	1.07	2.59	0.96	1.07
time (sec)	N/A	0.049	0.003	0.044	0.972	0.847	0.084	1.154

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	59	131	53	59
normalized size	1	1.	1.	0.8	1.05	2.34	0.95	1.05
time (sec)	N/A	0.038	0.002	0.044	0.983	0.717	0.084	1.175

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	46	96	41	46
normalized size	1	1.	1.	0.8	1.05	2.18	0.93	1.05
time (sec)	N/A	0.029	0.001	0.042	0.961	0.712	0.079	1.185

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	59	26	32
normalized size	1	1.	1.	0.83	1.07	1.97	0.87	1.07
time (sec)	N/A	0.016	0.001	0.043	0.993	0.711	0.063	1.174

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	45	119	49	45
normalized size	1	1.	1.	0.81	1.07	2.83	1.17	1.07
time (sec)	N/A	0.04	0.015	0.049	1.484	0.834	0.154	1.168

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	49	146	42	49
normalized size	1	1.	1.	0.79	1.14	3.4	0.98	1.14
time (sec)	N/A	0.027	0.015	0.046	1.466	0.851	0.213	1.223

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	76	242	63	62
normalized size	1	1.	0.83	0.73	1.19	3.78	0.98	0.97
time (sec)	N/A	0.037	0.025	0.044	1.472	0.801	0.281	1.251

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	86	221	76	86
normalized size	1	1.	1.	0.81	1.08	2.76	0.95	1.08
time (sec)	N/A	0.06	0.003	0.043	0.957	0.622	0.133	1.199

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	55	73	167	63	73
normalized size	1	1.	1.	0.83	1.11	2.53	0.95	1.11
time (sec)	N/A	0.048	0.002	0.042	0.964	0.734	0.137	1.16

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	122	51	59
normalized size	1	1.	1.	0.83	1.09	2.26	0.94	1.09
time (sec)	N/A	0.042	0.002	0.044	0.967	0.904	0.108	1.195

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	46	95	41	46
normalized size	1	1.	1.	0.76	1.	2.07	0.89	1.
time (sec)	N/A	0.03	0.001	0.045	0.964	0.829	0.097	1.171

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	58	165	63	58
normalized size	1	1.	0.95	0.79	1.04	2.95	1.12	1.04
time (sec)	N/A	0.051	0.019	0.044	1.488	0.993	0.189	1.16

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	70	252	65	70
normalized size	1	1.	0.94	0.81	1.11	4.	1.03	1.11
time (sec)	N/A	0.061	0.031	0.046	1.46	0.998	0.214	1.185

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	76	258	63	62
normalized size	1	1.	0.83	0.73	1.19	4.03	0.98	0.97
time (sec)	N/A	0.052	0.023	0.046	1.488	0.948	0.298	1.239

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	63	57	103	371	83	76
normalized size	1	1.	0.74	0.67	1.21	4.36	0.98	0.89
time (sec)	N/A	0.063	0.045	0.05	1.446	0.963	0.251	1.182

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	75	100	274	92	100
normalized size	1	1.	1.	0.78	1.04	2.85	0.96	1.04
time (sec)	N/A	0.071	0.003	0.044	0.983	0.823	0.126	1.177

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	65	86	217	78	86
normalized size	1	1.	1.	0.79	1.05	2.65	0.95	1.05
time (sec)	N/A	0.056	0.002	0.045	0.97	0.832	0.085	1.14

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	73	166	65	73
normalized size	1	1.	1.	0.81	1.07	2.44	0.96	1.07
time (sec)	N/A	0.051	0.002	0.043	0.957	0.85	0.086	1.194

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	59	122	53	59
normalized size	1	1.	1.	0.8	1.05	2.18	0.95	1.05
time (sec)	N/A	0.032	0.001	0.043	0.967	0.882	0.108	1.204

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	72	219	76	72
normalized size	1	1.	0.9	0.77	1.03	3.13	1.09	1.03
time (sec)	N/A	0.053	0.022	0.046	1.466	0.991	0.149	1.187

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	84	321	78	84
normalized size	1	1.	1.	0.79	1.09	4.17	1.01	1.09
time (sec)	N/A	0.072	0.027	0.046	1.479	1.03	0.172	1.208

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	63	97	406	85	84
normalized size	1	1.	0.93	0.75	1.15	4.83	1.01	1.
time (sec)	N/A	0.087	0.037	0.048	1.442	1.012	0.216	1.212

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	85	252	87	85
normalized size	1	1.	0.86	0.76	1.01	3.	1.04	1.01
time (sec)	N/A	0.057	0.027	0.046	1.435	0.975	0.156	1.25

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	72	196	73	72
normalized size	1	1.	0.9	0.77	1.03	2.8	1.04	1.03
time (sec)	N/A	0.056	0.021	0.047	1.472	0.935	0.144	1.167

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	58	147	60	58
normalized size	1	1.	0.93	0.79	1.04	2.62	1.07	1.04
time (sec)	N/A	0.05	0.017	0.049	1.44	0.994	0.197	1.126

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	45	112	46	45
normalized size	1	1.	1.	0.81	1.07	2.67	1.1	1.07
time (sec)	N/A	0.035	0.01	0.044	1.431	1.051	0.124	1.198

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	80	207	83	80
normalized size	1	1.	1.	0.82	1.1	2.84	1.14	1.1
time (sec)	N/A	0.053	0.031	0.048	1.427	1.	0.268	1.265

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	105	377	102	105
normalized size	1	1.	1.	0.82	1.12	4.01	1.09	1.12
time (sec)	N/A	0.089	0.078	0.049	1.483	0.985	0.298	1.223

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	104	89	132	578	119	119
normalized size	1	1.	0.9	0.77	1.15	5.03	1.03	1.03
time (sec)	N/A	0.124	0.144	0.049	1.439	1.027	0.4	1.26

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	71	97	348	88	97
normalized size	1	1.	1.	0.78	1.07	3.82	0.97	1.07
time (sec)	N/A	0.086	0.05	0.048	1.425	0.974	0.245	1.154

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	61	84	292	75	84
normalized size	1	1.	1.	0.79	1.09	3.79	0.97	1.09
time (sec)	N/A	0.073	0.027	0.047	1.435	1.033	0.254	1.183

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	70	234	61	70
normalized size	1	1.	1.	0.81	1.11	3.71	0.97	1.11
time (sec)	N/A	0.063	0.03	0.045	1.424	1.026	0.171	1.19

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	49	138	41	49
normalized size	1	1.	1.	0.79	1.14	3.21	0.95	1.14
time (sec)	N/A	0.026	0.014	0.047	1.432	0.988	0.206	1.139

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	77	105	363	102	105
normalized size	1	1.	1.	0.82	1.12	3.86	1.09	1.12
time (sec)	N/A	0.088	0.055	0.049	1.445	1.002	0.307	1.133

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	106	94	130	548	122	130
normalized size	1	1.	0.83	0.74	1.02	4.31	0.96	1.02
time (sec)	N/A	0.123	0.058	0.052	1.447	0.991	0.449	1.185

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	136	106	159	802	143	149
normalized size	1	1.	0.92	0.72	1.07	5.42	0.97	1.01
time (sec)	N/A	0.161	0.064	0.053	1.491	1.041	0.466	1.132

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	73	111	447	95	97
normalized size	1	1.	1.	0.74	1.13	4.56	0.97	0.99
time (sec)	N/A	0.113	0.037	0.054	1.454	0.905	0.278	1.148

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	97	377	82	84
normalized size	1	1.	1.	0.75	1.15	4.49	0.98	1.
time (sec)	N/A	0.087	0.036	0.05	1.454	0.985	0.3	1.131

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	76	239	61	62
normalized size	1	1.	0.8	0.73	1.19	3.73	0.95	0.97
time (sec)	N/A	0.053	0.029	0.047	1.439	1.007	0.276	1.291

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	47	76	225	61	62
normalized size	1	1.	0.8	0.73	1.19	3.52	0.95	0.97
time (sec)	N/A	0.033	0.028	0.046	1.452	1.05	0.192	1.202

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	99	89	132	556	122	119
normalized size	1	1.	0.86	0.77	1.15	4.83	1.06	1.03
time (sec)	N/A	0.124	0.155	0.053	1.432	1.025	0.397	1.16

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	136	106	157	763	143	149
normalized size	1	1.	0.85	0.66	0.98	4.77	0.89	0.93
time (sec)	N/A	0.16	0.102	0.056	1.466	1.03	0.535	1.156

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	151	118	186	1035	163	157
normalized size	1	1.	0.83	0.65	1.03	5.72	0.9	0.87
time (sec)	N/A	0.204	0.085	0.053	1.447	1.172	0.539	1.164

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	85	166	239	439	0	126
normalized size	1	1.	0.41	0.8	1.15	2.11	0.	0.61
time (sec)	N/A	0.309	0.291	0.069	1.484	1.408	0.	1.137

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	75	132	193	352	0	112
normalized size	1	1.	0.45	0.8	1.16	2.12	0.	0.67
time (sec)	N/A	0.181	0.172	0.059	1.472	1.523	0.	1.327

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	65	98	147	257	0	99
normalized size	1	1.	0.52	0.79	1.19	2.07	0.	0.8
time (sec)	N/A	0.099	0.106	0.053	1.534	1.529	0.	1.196

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	55	64	101	207	0	85
normalized size	1	1.	0.67	0.78	1.23	2.52	0.	1.04
time (sec)	N/A	0.04	0.053	0.048	1.504	1.649	0.	1.164

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	185	2065	0	7056	0	0
normalized size	1	1.	1.06	11.87	0.	40.55	0.	0.
time (sec)	N/A	0.441	0.463	0.276	0.	4.393	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	214	16357	0	8142	0	0
normalized size	1	1.	1.14	87.01	0.	43.31	0.	0.
time (sec)	N/A	0.393	1.083	0.292	0.	5.824	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	299	44343	0	10298	0	0
normalized size	1	1.	1.34	198.85	0.	46.18	0.	0.
time (sec)	N/A	0.459	2.069	0.428	0.	6.156	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	95	185	278	532	0	139
normalized size	1	1.	0.41	0.8	1.2	2.3	0.	0.6
time (sec)	N/A	0.342	0.367	0.072	1.546	1.585	0.	1.174

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	85	151	232	423	0	126
normalized size	1	1.	0.45	0.8	1.23	2.24	0.	0.67
time (sec)	N/A	0.19	0.234	0.059	1.503	1.593	0.	1.16

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	75	117	186	342	0	112
normalized size	1	1.	0.51	0.8	1.27	2.33	0.	0.76
time (sec)	N/A	0.122	0.144	0.055	1.5	1.517	0.	1.154

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	65	83	140	257	0	99
normalized size	1	1.	0.62	0.79	1.33	2.45	0.	0.94
time (sec)	N/A	0.05	0.084	0.058	1.499	1.602	0.	1.248

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	310	3460	0	7468	0	0
normalized size	1	1.	1.57	17.56	0.	37.91	0.	0.
time (sec)	N/A	0.488	0.695	0.19	0.	4.324	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	530	28185	0	8892	0	0
normalized size	1	1.	2.28	121.49	0.	38.33	0.	0.
time (sec)	N/A	0.575	2.584	0.355	0.	5.108	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	1262	81552	0	9586	0	0
normalized size	1	1.	5.66	365.7	0.	42.99	0.	0.
time (sec)	N/A	0.433	5.388	0.607	0.	4.943	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	105	204	317	657	0	153
normalized size	1	1.	0.41	0.8	1.25	2.59	0.	0.6
time (sec)	N/A	0.373	0.454	0.08	1.534	1.407	0.	1.155

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	95	170	271	512	0	139
normalized size	1	1.	0.45	0.8	1.28	2.42	0.	0.66
time (sec)	N/A	0.22	0.288	0.061	1.48	1.358	0.	1.152

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	85	136	225	409	0	126
normalized size	1	1.	0.5	0.8	1.32	2.41	0.	0.74
time (sec)	N/A	0.13	0.19	0.053	1.474	1.377	0.	1.163

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	75	102	180	327	0	112
normalized size	1	1.	0.59	0.8	1.41	2.55	0.	0.88
time (sec)	N/A	0.062	0.108	0.049	1.438	1.386	0.	1.145

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	229	4860	0	7899	0	0
normalized size	1	1.	1.03	21.89	0.	35.58	0.	0.
time (sec)	N/A	0.539	1.07	0.179	0.	3.907	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	685	40028	0	9335	0	0
normalized size	1	1.	2.69	156.97	0.	36.61	0.	0.
time (sec)	N/A	0.66	1.699	0.414	0.	5.257	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	1009	119458	0	10927	0	0
normalized size	1	1.	3.59	425.12	0.	38.89	0.	0.
time (sec)	N/A	0.655	2.116	0.69	0.	5.495	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	75	147	200	355	0	112
normalized size	1	1.	0.41	0.79	1.08	1.92	0.	0.61
time (sec)	N/A	0.312	0.246	0.065	1.532	1.36	0.	1.18

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	65	113	154	266	0	99
normalized size	1	1.	0.45	0.79	1.08	1.86	0.	0.69
time (sec)	N/A	0.168	0.132	0.054	1.449	1.34	0.	1.195

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	79	108	211	0	85
normalized size	1	1.	0.54	0.78	1.07	2.09	0.	0.84
time (sec)	N/A	0.088	0.076	0.051	1.489	1.377	0.	1.165

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	62	163	0	72
normalized size	1	1.	0.76	0.76	1.05	2.76	0.	1.22
time (sec)	N/A	0.033	0.038	0.051	1.532	1.321	0.	1.156

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	176	684	0	7101	0	0
normalized size	1	1.	1.19	4.62	0.	47.98	0.	0.
time (sec)	N/A	0.314	0.347	0.108	0.	4.685	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	287	5225	0	8625	0	0
normalized size	1	1.	1.53	27.79	0.	45.88	0.	0.
time (sec)	N/A	0.429	1.007	0.153	0.	5.141	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	1279	13040	0	9927	0	0
normalized size	1	1.	5.74	58.48	0.	44.52	0.	0.
time (sec)	N/A	0.469	6.238	0.191	0.	5.178	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	95	166	200	385	0	111
normalized size	1	1.	0.57	1.	1.2	2.32	0.	0.67
time (sec)	N/A	0.204	0.377	0.066	1.495	1.416	0.	1.198

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	65	132	154	311	0	97
normalized size	1	1.	0.52	1.06	1.24	2.51	0.	0.78
time (sec)	N/A	0.127	0.229	0.059	1.526	1.327	0.	1.206

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	55	98	108	251	0	84
normalized size	1	1.	0.67	1.2	1.32	3.06	0.	1.02
time (sec)	N/A	0.071	0.141	0.052	1.484	1.3	0.	1.181

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	64	62	209	0	72
normalized size	1	1.	1.	1.42	1.38	4.64	0.	1.6
time (sec)	N/A	0.029	0.077	0.049	1.509	1.301	0.	1.229

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	202	718	0	7618	0	0
normalized size	1	1.	1.15	4.08	0.	43.28	0.	0.
time (sec)	N/A	0.408	1.364	0.124	0.	4.791	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	740	5942	0	9287	0	0
normalized size	1	1.	3.51	28.16	0.	44.01	0.	0.
time (sec)	N/A	0.473	1.488	0.179	0.	5.232	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	231	18981	0	11105	0	0
normalized size	1	1.	0.94	77.16	0.	45.14	0.	0.
time (sec)	N/A	0.525	2.25	0.266	0.	5.473	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	75	214	342	441	0	109
normalized size	1	1.	0.51	1.46	2.33	3.	0.	0.74
time (sec)	N/A	0.167	0.518	0.071	1.92	1.373	0.	1.15

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	65	180	296	370	0	97
normalized size	1	1.	0.62	1.71	2.82	3.52	0.	0.92
time (sec)	N/A	0.105	0.335	0.057	1.501	1.371	0.	1.224

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	146	250	302	0	82
normalized size	1	1.	0.81	2.15	3.68	4.44	0.	1.21
time (sec)	N/A	0.061	0.241	0.052	1.776	1.371	0.	1.269

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	132	0	39
normalized size	1	1.	0.7	0.64	1.7	2.81	0.	0.83
time (sec)	N/A	0.022	0.104	0.045	1.077	1.341	0.	1.157

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	218	751	0	8280	0	0
normalized size	1	1.	1.1	3.77	0.	41.61	0.	0.
time (sec)	N/A	0.456	0.91	0.121	0.	4.947	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	296	5975	0	8498	0	0
normalized size	1	1.	1.26	25.53	0.	36.32	0.	0.
time (sec)	N/A	0.543	1.162	0.175	0.	4.836	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	242	19014	0	11142	0	0
normalized size	1	1.	0.9	70.68	0.	41.42	0.	0.
time (sec)	N/A	0.589	1.971	0.277	0.	5.318	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	657	1429	0	2866	0	861
normalized size	1	1.	1.51	3.28	0.	6.57	0.	1.97
time (sec)	N/A	0.789	0.965	0.063	0.	2.833	0.	1.302

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	173	453	0	1061	0	286
normalized size	1	1.	0.99	2.59	0.	6.06	0.	1.63
time (sec)	N/A	0.164	0.293	0.052	0.	1.495	0.	1.16

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0
normalized size	1	1.	0.97	13.97	0.	0.	0.	0.
time (sec)	N/A	1.051	1.255	0.433	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	488	555	22287	0	0	0	0
normalized size	1	1.	1.14	45.67	0.	0.	0.	0.
time (sec)	N/A	2.929	5.449	0.371	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	564	829	2458	0	5115	0	1553
normalized size	1	1.	1.47	4.36	0.	9.07	0.	2.75
time (sec)	N/A	0.935	1.828	0.063	0.	9.817	0.	1.339

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	392	862	0	1947	0	563
normalized size	1	1.	1.66	3.65	0.	8.25	0.	2.39
time (sec)	N/A	0.23	0.703	0.053	0.	2.634	0.	1.294

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	679	678	1232	22523	0	0	0	0
normalized size	1	1.	1.81	33.17	0.	0.	0.	0.
time (sec)	N/A	11.033	4.857	0.329	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	704	704	2843	72576	0	0	0	0
normalized size	1	1.	4.04	103.09	0.	0.	0.	0.
time (sec)	N/A	11.95	6.82	0.369	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	671	669	4727	178044	0	0	0	0
normalized size	1	1.	7.04	265.34	0.	0.	0.	0.
time (sec)	N/A	11.597	7.296	0.36	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	717	717	615	1930	0	3661	0	1112
normalized size	1	1.	0.86	2.69	0.	5.11	0.	1.55
time (sec)	N/A	2.71	1.405	0.073	0.	7.39	0.	1.369

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	251	706	0	1457	0	410
normalized size	1	1.	0.79	2.23	0.	4.61	0.	1.3
time (sec)	N/A	0.626	0.537	0.062	0.	2.845	0.	1.396

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	96	185	0	549	0	132
normalized size	1	1.	0.83	1.59	0.	4.73	0.	1.14
time (sec)	N/A	0.111	0.151	0.062	0.	1.929	0.	1.291

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	22873	0	0
normalized size	1	1.	1.01	2.03	0.	61.16	0.	0.
time (sec)	N/A	0.579	1.508	0.359	0.	45.018	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	789	787	1377	3858	0	0	0	0
normalized size	1	1.	1.75	4.89	0.	0.	0.	0.
time (sec)	N/A	8.21	6.734	0.329	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	745	2827	0	6743	0	1484
normalized size	1	1.	1.15	4.36	0.	10.39	0.	2.29
time (sec)	N/A	2.106	1.685	0.069	0.	15.911	0.	1.382

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	288	1011	0	2750	0	549
normalized size	1	1.	0.93	3.27	0.	8.9	0.	1.78
time (sec)	N/A	0.447	0.757	0.067	0.	9.595	0.	1.658

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	249	0	941	0	165
normalized size	1	1.	1.02	2.24	0.	8.48	0.	1.49
time (sec)	N/A	0.08	0.328	0.052	0.	4.45	0.	1.502

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0
normalized size	1	1.	1.05	6.15	0.	0.	0.	0.
time (sec)	N/A	1.829	6.792	0.355	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	891	891	872	4635	0	8338	0	1891
normalized size	1	1.	0.98	5.2	0.	9.36	0.	2.12
time (sec)	N/A	1.768	2.376	0.071	0.	48.355	0.	1.541

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	387	1786	0	3252	0	792
normalized size	1	1.	0.87	4.02	0.	7.32	0.	1.78
time (sec)	N/A	0.451	1.315	0.061	0.	27.422	0.	1.388

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	147	185	0	614	0	356
normalized size	1	1.	1.12	1.41	0.	4.69	0.	2.72
time (sec)	N/A	0.085	0.395	0.052	0.	23.233	0.	1.364

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	144	0	424	0	277
normalized size	1	1.	1.59	2.82	0.	8.31	0.	5.43
time (sec)	N/A	0.066	0.04	0.108	0.	1.91	0.	1.494

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1432	1432	670	929	0	0	0	0
normalized size	1	1.	0.47	0.65	0.	0.	0.	0.
time (sec)	N/A	6.219	2.487	0.836	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	652	652	390	420	0	0	0	0
normalized size	1	1.	0.6	0.64	0.	0.	0.	0.
time (sec)	N/A	0.677	0.62	3.535	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [14] had the largest ratio of [0.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.	29	0.138
2	A	2	2	1.	31	0.065
3	A	2	2	1.	27	0.074
4	A	4	4	1.	27	0.148
5	A	5	5	1.	27	0.185
6	A	6	5	1.	27	0.185
7	A	4	4	1.	31	0.129
8	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	2	2	1.	31	0.065
10	A	2	2	1.	34	0.059
11	A	3	3	1.	22	0.136
12	A	6	5	1.	18	0.278
13	A	3	3	1.	26	0.115
14	A	16	12	1.	27	0.444
15	A	2	1	1.	23	0.043
16	A	2	1	1.	23	0.043
17	A	2	1	1.	23	0.043
18	A	2	1	1.	21	0.048
19	A	6	5	1.	23	0.217
20	A	4	4	1.	23	0.174
21	A	5	5	1.	23	0.217
22	A	2	1	1.	25	0.04
23	A	2	1	1.	25	0.04
24	A	2	1	1.	25	0.04
25	A	2	1	1.	23	0.043
26	A	6	5	1.	25	0.2
27	A	7	6	1.	25	0.24
28	A	5	4	1.	25	0.16
29	A	6	5	1.	25	0.2
30	A	2	1	1.	25	0.04
31	A	2	1	1.	25	0.04
32	A	2	1	1.	25	0.04
33	A	2	1	1.	23	0.043
34	A	6	5	1.	25	0.2
35	A	7	6	1.	25	0.24
36	A	8	6	1.	25	0.24
37	A	6	5	1.	25	0.2
38	A	6	5	1.	25	0.2
39	A	6	5	1.	25	0.2
40	A	6	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	9	5	1.	25	0.2
42	A	10	6	1.	25	0.24
43	A	11	7	1.	25	0.28
44	A	7	6	1.	25	0.24
45	A	7	6	1.	25	0.24
46	A	7	6	1.	25	0.24
47	A	4	4	1.	23	0.174
48	A	10	6	1.	25	0.24
49	A	11	7	1.	25	0.28
50	A	12	7	1.	25	0.28
51	A	8	6	1.	25	0.24
52	A	8	6	1.	25	0.24
53	A	5	4	1.	25	0.16
54	A	5	5	1.	23	0.217
55	A	11	7	1.	25	0.28
56	A	12	7	1.	25	0.28
57	A	13	7	1.	25	0.28
58	A	11	5	1.	27	0.185
59	A	9	5	1.	27	0.185
60	A	7	5	1.	27	0.185
61	A	5	5	1.	25	0.2
62	A	8	7	1.	27	0.259
63	A	6	5	1.	27	0.185
64	A	7	6	1.	27	0.222
65	A	12	5	1.	27	0.185
66	A	10	5	1.	27	0.185
67	A	8	5	1.	27	0.185
68	A	6	5	1.	25	0.2
69	A	9	8	1.	27	0.296
70	A	10	9	1.	27	0.333
71	A	7	6	1.	27	0.222
72	A	13	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	11	5	1.	27	0.185
74	A	9	5	1.	27	0.185
75	A	7	5	1.	25	0.2
76	A	10	9	1.	27	0.333
77	A	11	9	1.	27	0.333
78	A	11	10	1.	27	0.37
79	A	10	4	1.	27	0.148
80	A	8	4	1.	27	0.148
81	A	6	4	1.	27	0.148
82	A	4	4	1.	25	0.16
83	A	5	4	1.	27	0.148
84	A	6	5	1.	27	0.185
85	A	7	6	1.	27	0.222
86	A	9	5	1.	27	0.185
87	A	7	5	1.	27	0.185
88	A	5	5	1.	27	0.185
89	A	4	4	1.	25	0.16
90	A	6	5	1.	27	0.185
91	A	7	6	1.	27	0.222
92	A	8	6	1.	27	0.222
93	A	8	5	1.	27	0.185
94	A	6	5	1.	27	0.185
95	A	5	4	1.	27	0.148
96	A	3	3	1.	25	0.12
97	A	7	6	1.	27	0.222
98	A	8	6	1.	27	0.222
99	A	9	6	1.	27	0.222
100	A	7	5	1.	27	0.185
101	A	5	5	1.	25	0.2
102	A	8	5	1.	27	0.185
103	A	6	4	1.	27	0.148
104	A	8	5	1.	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	6	5	1.	25	0.2
106	A	9	6	1.	27	0.222
107	A	10	7	1.	27	0.259
108	A	7	5	1.	27	0.185
109	A	8	4	1.	27	0.148
110	A	6	4	1.	27	0.148
111	A	4	4	1.	25	0.16
112	A	5	3	1.	27	0.111
113	A	6	4	1.	27	0.148
114	A	7	5	1.	27	0.185
115	A	5	5	1.	27	0.185
116	A	4	4	1.	25	0.16
117	A	6	4	1.	27	0.148
118	A	6	5	1.	27	0.185
119	A	5	4	1.	27	0.148
120	A	3	3	1.	25	0.12
121	A	5	4	1.	27	0.148
122	A	3	3	1.	29	0.103
123	A	3	3	1.	29	0.103

Chapter 3

Listing of integrals

3.1
$$\int \frac{a+bx+\frac{bf^2x^2}{e}}{\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=102

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2])])/(8*f^(3/2))

Rubi [A] time = 0.0944484, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af - b\left(\frac{4df}{e} + e\right)\right) \tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}} + \frac{bx\sqrt{d+ex+fx^2}}{2e} + \frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]

[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2])])/(8*f^(3/2))

)])]/(8*f^(3/2))

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx + \frac{bfx^2}{e}}{\sqrt{d + ex + fx^2}} dx &= \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\int \frac{\left(\frac{2a - bd}{e}\right)f + \frac{bf}{2}}{\sqrt{d + ex + fx^2}} dx}{2f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\left(-be + 8af - \frac{4bdf}{e}\right) \int \frac{1}{\sqrt{d + ex + fx^2}} dx}{8f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} + \frac{\left(-be + 8af - \frac{4bdf}{e}\right) \text{Subst}\left(\int \frac{1}{4f - x^2} dx, x, \frac{e + 2fx}{\sqrt{d + ex + fx^2}}\right)}{4f} \\
&= \frac{b\sqrt{d + ex + fx^2}}{4f} + \frac{bx\sqrt{d + ex + fx^2}}{2e} - \frac{\left(be - 8af + \frac{4bdf}{e}\right) \tanh^{-1}\left(\frac{e + 2fx}{2\sqrt{f}\sqrt{d + ex + fx^2}}\right)}{8f^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.191387, size = 87, normalized size = 0.85

$$\frac{2b\sqrt{f}(e + 2fx)\sqrt{d + x(e + fx)} - (b(4df + e^2) - 8aef) \tanh^{-1}\left(\frac{e + 2fx}{2\sqrt{f}\sqrt{d + x(e + fx)}}\right)}{8ef^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2], x]

[Out] (2*b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] - (-8*a*e*f + b*(e^2 + 4*d*f))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + x*(e + f*x)])]/(8*e*f^(3/2))

Maple [A] time = 0.052, size = 136, normalized size = 1.3

$$\frac{bx}{2e}\sqrt{fx^2 + ex + d} + \frac{b}{4f}\sqrt{fx^2 + ex + d} - \frac{be}{8} \ln\left(\left(\frac{e}{2} + fx\right) \frac{1}{\sqrt{f}} + \sqrt{fx^2 + ex + d}\right) f^{-\frac{3}{2}} - \frac{bd}{2e} \ln\left(\left(\frac{e}{2} + fx\right) \frac{1}{\sqrt{f}} + \sqrt{fx^2 + ex + d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2), x)

[Out] 1/2*b*x*(f*x^2+e*x+d)^(1/2)/e+1/4*b*(f*x^2+e*x+d)^(1/2)/f-1/8*e*b/f^(3/2)*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))-1/2/e*b/f^(1/2)*d*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))+a*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))

$/f^{(1/2)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.15693, size = 494, normalized size = 4.84

$$\left[\frac{(be^2 + 4(bd - 2ae)f)\sqrt{f} \log(-8f^2x^2 - 8efx - e^2 - 4\sqrt{fx^2 + ex + d}(2fx + e)\sqrt{f} - 4df) - 4(2bf^2x + bef)\sqrt{fx^2 + ex + d}}{16ef^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] $[-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*\text{sqrt}(f)*\log(-8*f^2*x^2 - 8*e*f*x - e^2 - 4*\text{sqrt}(f*x^2 + e*x + d)*(2*f*x + e)*\text{sqrt}(f) - 4*d*f) - 4*(2*b*f^2*x + b*e*f)*\text{sqrt}(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*\text{sqrt}(f)*\arctan(1/2*\text{sqrt}(f*x^2 + e*x + d)*(2*f*x + e)*\text{sqrt}(f)/(f^2*x^2 + e*f*x + d*f)) + 2*(2*b*f^2*x + b*e*f)*\text{sqrt}(f*x^2 + e*x + d))/(e*f^2)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}} dx + \int \frac{bex}{\sqrt{d+ex+fx^2}} dx + \int \frac{bf^2x^2}{\sqrt{d+ex+fx^2}} dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)

[Out] (Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e

Giac [A] time = 1.30205, size = 113, normalized size = 1.11

$$\frac{1}{4} \sqrt{fx^2 + xe + d} \left(2bx e^{(-1)} + \frac{b}{f} \right) + \frac{(4bdf - 8afe + be^2)e^{(-1)} \log\left(\left| -2(\sqrt{f}x - \sqrt{fx^2 + xe + d})\sqrt{f} - e \right| \right)}{8f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(f*x^2 + x*e + d)*(2*b*x*e^(-1) + b/f) + 1/8*(4*b*d*f - 8*a*f*e + b*e^2)*e^(-1)*log(abs(-2*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*sqrt(f) - e))/f^(3/2)

$$3.2 \quad \int \frac{1}{\sqrt{d+ex+fx^2} \left(a+bx+\frac{bf x^2}{e} \right)} dx$$

Optimal. Leaf size=82

$$\frac{2\sqrt{e} \tanh^{-1} \left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

[Out] $(-2*\text{Sqrt}[e]*\text{ArcTanh}[(\text{Sqrt}[b*d - a*e]*(e + 2*f*x))/(\text{Sqrt}[e]*\text{Sqrt}[b*e - 4*a*f]*\text{Sqrt}[d + e*x + f*x^2])])/(\text{Sqrt}[b*d - a*e]*\text{Sqrt}[b*e - 4*a*f])$

Rubi [A] time = 0.110623, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {982, 208}

$$\frac{2\sqrt{e} \tanh^{-1} \left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(\text{Sqrt}[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]$

[Out] $(-2*\text{Sqrt}[e]*\text{ArcTanh}[(\text{Sqrt}[b*d - a*e]*(e + 2*f*x))/(\text{Sqrt}[e]*\text{Sqrt}[b*e - 4*a*f]*\text{Sqrt}[d + e*x + f*x^2])])/(\text{Sqrt}[b*d - a*e]*\text{Sqrt}[b*e - 4*a*f])$

Rule 982

$\text{Int}[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*\text{Sqrt}[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] \rightarrow \text{Dist}[-2*e, \text{Subst}[\text{Int}[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/\text{Sqrt}[d + e*x + f*x^2]], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{EqQ}[c*e - b*f, 0]$

Rule 208

$\text{Int}(((a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2} \left(a+bx+\frac{bf x^2}{e} \right)} dx = - \left((2e) \text{Subst} \left(\int \frac{1}{e(be-4af)-(bd-ae)x^2} dx, x, \frac{e+2fx}{\sqrt{d+ex+fx^2}} \right) \right)$$

$$= - \frac{2\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}} \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Mathematica [B] time = 0.388, size = 178, normalized size = 2.17

$$\frac{\sqrt{e} \left(\tanh^{-1} \left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}} \right) + \tanh^{-1} \left(\frac{\sqrt{b}(e^2-4df)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}} \right) \right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]

[Out] (Sqrt[e]*(ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + ArcTanh[(Sqrt[b]*(e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]))/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])

Maple [B] time = 0.388, size = 491, normalized size = 6.

$$-e \ln \left(\left(-2 \frac{ae-bd}{b} + \frac{1}{b} \sqrt{-be(4af-be)} \left(x - \frac{1}{2bf} (-be + \sqrt{-be(4af-be)}) \right) \right) + 2 \sqrt{-\frac{ae-bd}{b}} \sqrt{x - 1/2 \frac{-be + \sqrt{-be(4af-be)}}{bf}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)

[Out] -e/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b

$$\begin{aligned} & *e)^{(1/2)}/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)-(a*e-b*d)/b)^{(1/2)} \\ &)/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f))+e/(-b*e*(4*a*f-b*e))^{(1/2)}/(\\ & -(a*e-b*d)/b)^{(1/2)}*\ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b \\ & *e+(-b*e*(4*a*f-b*e))^{(1/2)})/b/f)+2*(-(a*e-b*d)/b)^{(1/2)}*((x+1/2*(b*e+(-b*e \\ & *(4*a*f-b*e))^{(1/2)})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{(1/2)}/b*(x+1/2*(b*e+(-b*e \\ & *(4*a*f-b*e))^{(1/2)})/b/f)-(a*e-b*d)/b)^{(1/2)})/(x+1/2*(b*e+(-b*e*(4*a*f-b*e)) \\ & ^{(1/2)})/b/f)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.13968, size = 2226, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*\sqrt{e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)}*\log((8*b^2*d^2*e^4 - \\ & 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - \\ & 8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*b \\ & e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a \\ & *b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^ \\ & 2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)* \\ & f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^6 - 1 \\ & 6*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 + 2*a^ \\ & 2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16*(a*b^2*d \\ & ^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2 + 3*a^2*b* \\ & e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2 - a^3*d*e^3 \\ &)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4*(b^3*d^2*e^ \\ & 2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)*f)*x^2 - 4*(\\ & 3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a*b^2*e^6 + 32* \end{aligned}$$

$$(a^2 b d^2 e - a^3 d e^2) f^3 - 40(a b^2 d^2 e^2 - 2 a^2 b d e^3 + a^3 e^4) f^2 + 2(4 b^3 d^2 e^3 - 11 a b^2 d e^4 + 7 a^2 b e^5) f) x) \sqrt{f x^2 + e x + d} \sqrt{e/(b^2 d e - a b e^2 - 4(a b d - a^2 e) f)}) / (b^2 f^2 x^4 + 2 b^2 e f x^3 + 2 a b e^2 x + a^2 e^2 + (b^2 e^2 + 2 a b e f) x^2), -\sqrt{-e/(b^2 d e - a b e^2 - 4(a b d - a^2 e) f)} \arctan(-1/2(2 b d e^2 - a e^3 - 4 a d e f + (b e^2 f + 4(b d - 2 a e) f^2) x^2 + (b e^3 + 4(b d e - 2 a e^2) f) x) \sqrt{f x^2 + e x + d} \sqrt{-e/(b^2 d e - a b e^2 - 4(a b d - a^2 e) f)}) / (2 e f^2 x^3 + 3 e^2 f x^2 + d e^2 + (e^3 + 2 d e f) x)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e \int \frac{1}{a e \sqrt{d + e x + f x^2} + b e x \sqrt{d + e x + f x^2} + b f x^2 \sqrt{d + e x + f x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)

[Out] e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b*f*x**2*sqrt(d + e*x + f*x**2)), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")

[Out] Timed out

3.3

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx$$

Optimal. Leaf size=66

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])

Rubi [A] time = 0.0790939, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {982, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]

[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx = - \left((2b) \text{Subst} \left(\int \frac{1}{b(b^2-4cd) - (ab-bd)x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}} \right) \right)$$

$$= - \frac{2 \tanh^{-1} \left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}} \right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Mathematica [B] time = 0.225642, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1} \left(\frac{4ac-2cx\sqrt{b^2-4cd}-b(\sqrt{b^2-4cd}+b)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right) + \tanh^{-1} \left(\frac{-2c(2a+x\sqrt{b^2-4cd})-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}} \right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)), x]

[Out] (ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])] + ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])

Maple [B] time = 0.327, size = 307, normalized size = 4.7

$$\ln \left(\left(2a - 2d - \sqrt{b^2 - 4cd} \left(x + \frac{1}{2c} (\sqrt{b^2 - 4cd} + b) \right) + 2\sqrt{a-d} \sqrt{\left(x + \frac{1}{2} \frac{\sqrt{b^2 - 4cd} + b}{c} \right)^2 c - \sqrt{b^2 - 4cd} \left(x + \frac{1}{2} \frac{\sqrt{b^2 - 4cd} + b}{c} \right)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2), x)

[Out] 1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/

$$(x-1/2*(-b+(b^2-4*c*d)^{(1/2)})/c)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.45844, size = 1755, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{2} \log\left(\frac{(8a^2b^4 + (b^4c^2 + 24ab^2c^3 + 16a^2c^4 + 128c^4d^2 - 32(b^2c^3 + 4ac^4)d)x^4 + 2(b^5c + 24ab^3c^2 + 16a^2b^2c^3 + 128bc^3d^2 - 32(b^3c^2 + 4ab^2c^3)d)x^3 + (b^4 + 24ab^2c + 16a^2c^2)d^2 + (b^6 + 32ab^4c + 48a^2b^2c^2 + 32(5b^2c^2 + 4ac^3)d^2 - 2(19b^4c + 104ab^2c^2 + 48a^2c^3)d)x^2 - 4(2ab^3 + 2(b^2c^2 + 4ac^3 - 8c^3d)x^3 + 3(b^3c + 4ab^2c - 8bc^2d)x^2 - (b^3 + 4ab^2c)d + (b^4 + 8ab^2c - 2(5b^2c + 4ac^2)d)x)\sqrt{ab^2 + 4cd^2 - (b^2 + 4ac)d}\sqrt{c^2x^2 + bx + a} - 8(ab^4 + 4a^2b^2c)d + 2(4ab^5 + 16a^2b^3c + 16(b^3c + 4ab^2c^2)d^2 - (3b^5 + 40ab^3c + 48a^2b^2c^2)d)x)/(c^2x^4 + 2b^2cx^3 + 2bd^2x + (b^2 + 2cd)x^2 + d^2)}{\sqrt{ab^2 + 4cd^2 - (b^2 + 4ac)d}}, -\sqrt{-ab^2 - 4cd^2 + (b^2 + 4ac)d} \arctan\left(\frac{-1/2(2ab^2 + (b^2c + 4ac^2 - 8c^2d)x^2 - (b^2 + 4ac)d + (b^3 + 4ab^2c - 8bc^2d)x)\sqrt{-ab^2 - 4cd^2 + (b^2 + 4ac)d}\sqrt{c^2x^2 + bx + a}}{a^2b^3 + 4ab^2cd^2 + 2(a^2b^2c^2 + 4c^3d^2 - (b^2c^2 + 4ac^3)d)x^3 + 3(ab^3c + 4b^2cd^2 - (b^3c + 4ab^2c^2)d)x^2 - (ab^3 + 4a^2b^2c)d + (ab^4 + 2a^2b^2c + 4(b^2c + 2ac^2)d^2 - (b^4 + 6ab^2c + 8a^2c^2)d)x}\right) \right]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx + cx^2} (bx + cx^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2), x)

[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)

Giac [B] time = 14.6685, size = 620, normalized size = 9.39

$$\log\left(\frac{\left(-3ab^2c + 4a^2c^2 + 2b^2cd - (b^2c + 4ac^2 - 8c^2d + 4\sqrt{ab^2 - b^2d - 4acd + 4cd^2c^{\frac{3}{2}}})\left(\sqrt{cx} - \sqrt{cx^2 + bx + a}\right)^2 - \sqrt{ab^2 - b^2d - 4acd + 4cd^2c^{\frac{3}{2}}}\right)}{\sqrt{ab^2 - b^2d - 4acd + 4cd^2c^{\frac{3}{2}}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] log(abs(-3*a*b^2*c + 4*a^2*c^2 + 2*b^2*c*d - (b^2*c + 4*a*c^2 - 8*c^2*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*c^(3/2))*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c) - (b^3*sqrt(c) + 4*a*b*c^(3/2) - 8*b*c^(3/2)*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b*c)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - log(abs(-3*a*b^2*c + 4*a^2*c^2 + 2*b^2*c*d - (b^2*c + 4*a*c^2 - 8*c^2*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))*c^(3/2))*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c) - (b^3*sqrt(c) + 4*a*b*c^(3/2) - 8*b*c^(3/2)*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b*c)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)

$$3.4 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2} dx$$

Optimal. Leaf size=129

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)))

Rubi [A] time = 0.169022, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 12, 982, 208}

$$\frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} - \frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2), x]

[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)))

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*


```
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 982

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(
x_)^2]), x_Symbol] :=> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{c^2(a-d)^2(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} - \frac{(b^2+4c(a-2d)) \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)} dx}{2(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b(b^2+4c(a-2d))) \operatorname{Subst}\left(\int \frac{1}{b(b^2-4cd)-at}\right)}{(a-d)(b^2-4cd)} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(d+bx+cx^2)} + \frac{(b^2+4c(a-2d)) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}}
\end{aligned}$$

Mathematica [B] time = 0.909816, size = 296, normalized size = 2.29

$$\frac{1}{2} \left(\frac{8c(b+2cx)\sqrt{a+x(b+cx)}}{(a-d)(4cd-b^2)(\sqrt{b^2-4cd}-b-2cx)(\sqrt{b^2-4cd}+b+2cx)} - \frac{(4c(a-2d)+b^2) \tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b(\sqrt{b^2-4cd}+4c\sqrt{a-d}\sqrt{a+x(b+cx)})}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{(a-d)^{3/2}(b^2-4cd)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2), x]

[Out]
$$\frac{((-8*c*(b+2*c*x)*\text{Sqrt}[a+x*(b+c*x)])/((a-d)*(-b^2+4*c*d)*(-b+\text{Sqrt}[b^2-4*c*d]-2*c*x)*(b+\text{Sqrt}[b^2-4*c*d]+2*c*x)) - ((b^2+4*c*(a-2*d))*\text{ArcTanh}[(4*a*c-b*(b+\text{Sqrt}[b^2-4*c*d]) - 2*c*\text{Sqrt}[b^2-4*c*d]*x)/(4*c*\text{Sqrt}[a-d]*\text{Sqrt}[a+x*(b+c*x)])])/(a-d)^{(3/2)*(b^2-4*c*d)^{(3/2)} - ((b^2+4*c*(a-2*d))*\text{ArcTanh}[(b^2-b*\text{Sqrt}[b^2-4*c*d]-2*c*(2*a+\text{Sqrt}[b^2-4*c*d]*x))/(4*c*\text{Sqrt}[a-d]*\text{Sqrt}[a+x*(b+c*x)])])/(a-d)^{(3/2)*(b^2-4*c*d)^{(3/2)}}}{2}$$

Maple [B] time = 0.301, size = 829, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2), x)

[Out]
$$\begin{aligned} & -2/(b^2-4*c*d)^{(3/2)*c/(a-d)^{(1/2)*\ln((2*a-2*d-(b^2-4*c*d)^{(1/2)*(x+1/2*((b^2-4*c*d)^{(1/2)+b)/c)+2*(a-d)^{(1/2)*((x+1/2*((b^2-4*c*d)^{(1/2)+b)/c})^2*c-(b^2-4*c*d)^{(1/2)*(x+1/2*((b^2-4*c*d)^{(1/2)+b)/c)+a-d)^{(1/2)))/(x+1/2*((b^2-4*c*d)^{(1/2)+b)/c)))+2/(b^2-4*c*d)^{(3/2)*c/(a-d)^{(1/2)*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)+2*(a-d)^{(1/2)*((x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c})^2*c+(b^2-4*c*d)^{(1/2)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)+a-d)^{(1/2)))/(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)))-1/(b^2-4*c*d)/(a-d)/(x-1/2/c*(b^2-4*c*d)^{(1/2)+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c})^2*c+(b^2-4*c*d)^{(1/2)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)+a-d)^{(1/2)+1/2/(b^2-4*c*d)^{(1/2)/(a-d)^{(3/2)*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)+2*(a-d)^{(1/2)*((x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c})^2*c+(b^2-4*c*d)^{(1/2)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)+a-d)^{(1/2)))/(x-1/2*(-b+(b^2-4*c*d)^{(1/2))/c)))-1/(b^2-4*c*d)/(a-d)/(x+1/2/c*(b^2-4*c*d)^{(1/2)+1/2*b/c)*((x+1/2*((b^2-4*c*d)^{(1/2)+b)/c})^2*c-(b^2-4*c*d)^{(1/2)*(x+1/2*((b^2-4*c*d)^{(1/2)+b)/c)+a-d)^{(1/2)-1/2/(} \end{aligned}$$

$$b^2-4*c*d)^{(1/2)}/(a-d)^{(3/2)}*\ln((2*a-2*d-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)}*((x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)})/(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(cx^2 + bx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)

Fricas [B] time = 3.38433, size = 3297, normalized size = 25.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2) - 4*(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c

$$\begin{aligned}
&) * d^2 + (a^2 * b^4 * c + 16 * c^3 * d^4 - 8 * (b^2 * c^2 + 4 * a * c^3) * d^3 + (b^4 * c + 16 * a \\
& * b^2 * c^2 + 16 * a^2 * c^3) * d^2 - 2 * (a * b^4 * c + 4 * a^2 * b^2 * c^2) * d) * x^2 + (a^2 * b^5 \\
& + 16 * b * c^2 * d^4 - 8 * (b^3 * c + 4 * a * b * c^2) * d^3 + (b^5 + 16 * a * b^3 * c + 16 * a^2 * b * c \\
& ^2) * d^2 - 2 * (a * b^5 + 4 * a^2 * b^3 * c) * d) * x), -1/2 * (\text{sqrt}(-a * b^2 - 4 * c * d^2 + (b^2 \\
& + 4 * a * c) * d) * (8 * c * d^2 - (b^2 * c + 4 * a * c^2 - 8 * c^2 * d) * x^2 - (b^2 + 4 * a * c) * d - \\
& (b^3 + 4 * a * b * c - 8 * b * c * d) * x) * \arctan(-1/2 * (2 * a * b^2 + (b^2 * c + 4 * a * c^2 - 8 * c \\
& ^2 * d) * x^2 - (b^2 + 4 * a * c) * d + (b^3 + 4 * a * b * c - 8 * b * c * d) * x) * \text{sqrt}(-a * b^2 - 4 * \\
& c * d^2 + (b^2 + 4 * a * c) * d) * \text{sqrt}(c * x^2 + b * x + a) / (a^2 * b^3 + 4 * a * b * c * d^2 + 2 * (\\
& a * b^2 * c^2 + 4 * c^3 * d^2 - (b^2 * c^2 + 4 * a * c^3) * d) * x^3 + 3 * (a * b^3 * c + 4 * b * c^2 * d \\
& ^2 - (b^3 * c + 4 * a * b * c^2) * d) * x^2 - (a * b^3 + 4 * a^2 * b * c) * d + (a * b^4 + 2 * a^2 * b^2 * \\
& c + 4 * (b^2 * c + 2 * a * c^2) * d^2 - (b^4 + 6 * a * b^2 * c + 8 * a^2 * c^2) * d) * x)) + 2 * (a \\
& * b^3 + 4 * b * c * d^2 - (b^3 + 4 * a * b * c) * d + 2 * (a * b^2 * c + 4 * c^2 * d^2 - (b^2 * c + 4 * \\
& a * c^2) * d) * x) * \text{sqrt}(c * x^2 + b * x + a) / (a^2 * b^4 * d + 16 * c^2 * d^5 - 8 * (b^2 * c + 4 * \\
& a * c^2) * d^4 + (b^4 + 16 * a * b^2 * c + 16 * a^2 * c^2) * d^3 - 2 * (a * b^4 + 4 * a^2 * b^2 * c) * \\
& d^2 + (a^2 * b^4 * c + 16 * c^3 * d^4 - 8 * (b^2 * c^2 + 4 * a * c^3) * d^3 + (b^4 * c + 16 * a * b \\
& ^2 * c^2 + 16 * a^2 * c^3) * d^2 - 2 * (a * b^4 * c + 4 * a^2 * b^2 * c^2) * d) * x^2 + (a^2 * b^5 + \\
& 16 * b * c^2 * d^4 - 8 * (b^3 * c + 4 * a * b * c^2) * d^3 + (b^5 + 16 * a * b^3 * c + 16 * a^2 * b * c^2 \\
&) * d^2 - 2 * (a * b^5 + 4 * a^2 * b^3 * c) * d) * x)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

sage₀x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] sage₀x

$$3.5 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3} dx$$

Optimal. Leaf size=224

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}} + \frac{3(b+2cx)(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)}$$

[Out] $-\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}\right) - \left(\frac{(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\operatorname{ArcTanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}}\right)$

Rubi [A] time = 0.426872, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {974, 1060, 12, 982, 208}

$$\frac{(16c^2(3a^2 - 8ad + 8d^2) + 8b^2c(a - 4d) + 3b^4) \tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}} + \frac{3(b+2cx)(4c(a-2d)+b^2)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3), x]

[Out] $-\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}\right) - \left(\frac{(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\operatorname{ArcTanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}(b^2-4cd)^{5/2}}\right)$

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p+1)*(d + e*x + f*x^2)^(q+1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)))*(p+1), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)))*(p+1), Int[(a + b*x + c*x^2)^(p+1)*(d + e*x + f*x^2)^q*Sim

```
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 982

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(
x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
```

]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3} dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(3b^2+12ac-16cd)-4bc^3(a-d)x-4c^4(a-d)}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2} dx}{2c^2(a-d)^2(b^2-4cd)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)} \\
 &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)(b^2-4cd)(d+bx+cx^2)^2} + \frac{3(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2(b^2-4cd)^2(d+bx+cx^2)}
 \end{aligned}$$

Mathematica [B] time = 6.37555, size = 1748, normalized size = 7.8

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3), x]

[Out] (-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)])

$$\begin{aligned}
& 2 - 4*c*d)^2*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)] + (6*c^2*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b^2 - 4*a*c - b*\text{Sqrt}[b^2 - 4*c*d] - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / (\text{Sqrt}[a - d]*(b^2 - 4*c*d)^{(5/2)}*\text{Sqrt}[a + x*(b + c*x)]) + (3*c*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b^2 - 4*a*c - b*\text{Sqrt}[b^2 - 4*c*d] - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / (2*(a - d)^{(3/2)}*(b^2 - 4*c*d)^{(3/2)}*\text{Sqrt}[a + x*(b + c*x)]) + (6*c^2*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / (\text{Sqrt}[a - d]*(b^2 - 4*c*d)^{(5/2)}*\text{Sqrt}[a + x*(b + c*x)]) + (3*c*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / (2*(a - d)^{(3/2)}*(b^2 - 4*c*d)^{(3/2)}*\text{Sqrt}[a + x*(b + c*x)]) + (4*c^3*\text{Sqrt}[a + b*x + c*x^2]*(-((2*c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + 2*\text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2]) / ((4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*(-b + \text{Sqrt}[b^2 - 4*c*d] - 2*c*x))) + (4*c*\text{Sqrt}[a - d]*(b*(-2*c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + 2*\text{Sqrt}[b^2 - 4*c*d])) - 2*(4*a*c^3 - c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]))*(b + 2*\text{Sqrt}[b^2 - 4*c*d]))*\text{ArcTanh}[(4*a*c - b*(-b + \text{Sqrt}[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / ((4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 4*c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)))) / ((b^2 - 4*c*d)^{(3/2)}*(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x)]) + (4*c^3*\text{Sqrt}[a + b*x + c*x^2]*(-((2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) - 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2]) / ((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x))) + (4*c*\text{Sqrt}[a - d]*(b*(2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d])) - 2*(4*a*c^3 + c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]))*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]) / ((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))) / ((b^2 - 4*c*d)^{(3/2)}*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x)])
\end{aligned}$$

Maple [B] time = 0.242, size = 1884, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2), x)

[Out]
$$\begin{aligned}
& -1/2/(b^2-4*c*d)^{(3/2)}/(a-d)/(x-1/2/c*(b^2-4*c*d)^{(1/2)}+1/2*b/c)^2*((x-1/2* \\
& (-b+(b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)} \\
&))/c)+a-d)^{(1/2)}+3/4/(b^2-4*c*d)/(a-d)^2/(x-1/2/c*(b^2-4*c*d)^{(1/2)}+1/2*b/c \\
&)*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4 \\
& *c*d)^{(1/2)))/c)+a-d)^{(1/2)}-3/8/(b^2-4*c*d)^{(3/2)}/(a-d)^{(5/2)}*\ln((2*a-2*d+(b \\
& ^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)+2*(a-d)^{(1/2)}*((x-1/2*(-b+ \\
& (b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c \\
&)+a-d)^{(1/2)))/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))*b^2+3/2/(b^2-4*c*d)^{(3/2)}/(\\
& a-d)^{(5/2)}*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)+2 \\
& *(a-d)^{(1/2)}*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d)^{(1/2)}*(x-1/2 \\
& *(-b+(b^2-4*c*d)^{(1/2)))/c)+a-d)^{(1/2)))/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c))*c* \\
& d-1/(b^2-4*c*d)^{(3/2)}*c/(a-d)^{(3/2)}*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(- \\
& b+(b^2-4*c*d)^{(1/2)))/c)+2*(a-d)^{(1/2)}*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)^2*c \\
& +(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)+a-d)^{(1/2)))/(x-1/2*(-b+ \\
& (b^2-4*c*d)^{(1/2)))/c))+6*c^2/(b^2-4*c*d)^{(5/2)}/(a-d)^{(1/2)}*\ln((2*a-2*d-(b^2 \\
& -4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)}*((x+1/2*((b^2-4 \\
& *c*d)^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d \\
&)^2)/(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c))+3/(b^2-4*c*d)^2*c/(a-d)/(x-1/2/c* \\
& (b^2-4*c*d)^{(1/2)}+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d \\
&)^2)/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)+a-d)^{(1/2)}+1/2/(b^2-4*c*d)^{(3/2)}/(\\
& a-d)/(x+1/2/c*(b^2-4*c*d)^{(1/2)}+1/2*b/c)^2*((x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c) \\
& ^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)}+3/4/(b^2- \\
& 4*c*d)/(a-d)^2/(x+1/2/c*(b^2-4*c*d)^{(1/2)}+1/2*b/c)*((x+1/2*((b^2-4*c*d)^{(1/2)} \\
& +b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)}+3 \\
& /8/(b^2-4*c*d)^{(3/2)}/(a-d)^{(5/2)}*\ln((2*a-2*d-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2 \\
& -4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)}*((x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)^2*c-(b^2 \\
& -4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)))/(x+1/2*((b^2-4*c* \\
& d)^{(1/2)}+b)/c))*b^2-3/2/(b^2-4*c*d)^{(3/2)}/(a-d)^{(5/2)}*\ln((2*a-2*d-(b^2-4*c* \\
& d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)}*((x+1/2*((b^2-4*c*d) \\
& ^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+a-d)^{(1/2)} \\
&)/(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c))*c*d+1/(b^2-4*c*d)^{(3/2)}*c/(a-d)^{(3/2)}* \\
& \ln((2*a-2*d-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)+2*(a-d)^{(1/2)} \\
& *((x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d) \\
& ^{(1/2)}+b)/c)+a-d)^{(1/2)))/(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/c))-6*c^2/(b^2-4*c*d) \\
& ^{(5/2)}/(a-d)^{(1/2)}*\ln((2*a-2*d+(b^2-4*c*d)^{(1/2)}*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)} \\
&))/c)+2*(a-d)^{(1/2)}*((x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)^2*c+(b^2-4*c*d)^{(1/2)} \\
&)*(x-1/2*(-b+(b^2-4*c*d)^{(1/2)))/c)+a-d)^{(1/2)))/(x-1/2*(-b+(b^2-4*c*d)^{(1/2)} \\
&))/c))+3/(b^2-4*c*d)^2*c/(a-d)/(x+1/2/c*(b^2-4*c*d)^{(1/2)}+1/2*b/c)*((x+1/2* \\
& (b^2-4*c*d)^{(1/2)}+b)/c)^2*c-(b^2-4*c*d)^{(1/2)}*(x+1/2*((b^2-4*c*d)^{(1/2)}+b)/ \\
& c)+a-d)^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(cx^2 + bx + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)

Fricas [B] time = 19.0583, size = 8058, normalized size = 35.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3

$$\begin{aligned}
& *c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + \\
& 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2* \\
& b^2*c^2)*d)*x)*\sqrt{c*x^2 + b*x + a)/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c \\
& ^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36* \\
& a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16* \\
& a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + \\
& 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1 \\
& 44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b \\
& ^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b \\
& ^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12 \\
& *(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144* \\
& a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3* \\
& c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - \\
& 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + \\
& 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a \\
& *b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8 \\
& *a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b \\
& *c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2 \\
& *b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(\\
& a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2) \\
& *x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d \\
& ^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c \\
& + 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)* \\
& x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^ \\
& 2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c \\
& ^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (\\
& 3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*\sqrt{-a*b^2 - 4*c*d^2 + (b^2 + 4*a* \\
& c)*d)*\arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c \\
&)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*\sqrt{-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d} \\
& *\sqrt{c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - \\
& (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2) \\
& *d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2) \\
&)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4 \\
& - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + \\
& 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 \\
& + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32 \\
& *b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2* \\
& b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^ \\
& 3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3) \\
&)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*\sqrt{c*x^2 + b*x + a)/(\\
& a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^ \\
& 2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3) \\
& *d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^ \\
& 5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^ \\
& 5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a
\end{aligned}$$

```

*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^
4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6
- 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*
b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2
)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c
^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2
*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^
3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3
*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6
+ 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a
^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*
d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.6 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^4} dx$$

Optimal. Leaf size=328

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+8d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{8(a-d)^3(b^2-4cd)^3(bx+cx^2+d)}$$

[Out] $-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(3(a-d)(b^2-4cd)(d+bx+cx^2)^3)} + \frac{(5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2})}{(12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2)} - \frac{((15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))(b+2cx)\sqrt{a+bx+cx^2})}{(24(a-d)^3(b^2-4cd)^3(d+bx+cx^2))} + \frac{((b^2+4c(a-2d))(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+8d^2))\text{ArcTanh}[\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}])}{(8(a-d)^{7/2}(b^2-4cd)^{7/2})}$

Rubi [A] time = 0.970484, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {974, 1060, 12, 982, 208}

$$\frac{(b+2cx)(16c^2(15a^2-44ad+44d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{24(a-d)^3(b^2-4cd)^3(bx+cx^2+d)} + \frac{(4c(a-2d)+b^2)(16c^2(5a^2-8ad+8d^2)+8b^2c(7a-22d)+15b^4)\sqrt{a+bx+cx^2}}{8(a-d)^3(b^2-4cd)^3(bx+cx^2+d)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4), x]

[Out] $-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(3(a-d)(b^2-4cd)(d+bx+cx^2)^3)} + \frac{(5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2})}{(12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2)} - \frac{((15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))(b+2cx)\sqrt{a+bx+cx^2})}{(24(a-d)^3(b^2-4cd)^3(d+bx+cx^2))} + \frac{((b^2+4c(a-2d))(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+8d^2))\text{ArcTanh}[\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}])}{(8(a-d)^{7/2}(b^2-4cd)^{7/2})}$

Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^4} dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)(5b^2+20ac-24cd)-8bc^3(a-d)x-8c^4(a-d)}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3} dx}{3c^2(a-d)^2(b^2-4cd)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(d+bx+cx^2)^3} + \frac{5(b^2+4c(a-2d))(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2(b^2-4cd)^2(d+bx+cx^2)^2}$$

Mathematica [B] time = 6.58038, size = 3386, normalized size = 10.32

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]

[Out]
$$\begin{aligned} & (-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - \text{Sqrt}[b^2 - 4*c*d] \\ & + 2*c*x)^3*\text{Sqrt}[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*(b \\ & ^2 - 4*c*d)^{5/2}*(b - \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)^2*\text{Sqrt}[a + x*(b + c*x)]) \\ & - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - \text{Sqrt}[b^2 - 4*c*d] \\ & + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b \\ & ^2 - 4*c*d)^2*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)^3*\text{Sqrt}[a + x*(b + c*x)]) - (8 \\ & *c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^{5/2}*(b + \text{Sqrt}[b^2 - 4*c*d] \\ & + 2*c*x)^2*\text{Sqrt}[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b \\ & ^2 - 4*c*d)^3*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)]) - (20* \\ & c^3*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b^2 - 4*a*c - b*\text{Sqrt}[b^2 - 4*c*d] - 2*c* \\ & \text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/(\text{Sqrt}[a - d] \\ & *(b^2 - 4*c*d)^{7/2}*\text{Sqrt}[a + x*(b + c*x)]) - (5*c^2*\text{Sqrt}[a + b*x + c*x^2]* \\ & \text{ArcTanh}[(b^2 - 4*a*c - b*\text{Sqrt}[b^2 - 4*c*d] - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c* \\ & \text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/((a - d)^{3/2}*(b^2 - 4*c*d)^{5/2}*\text{Sqr} \\ & \text{t}[a + x*(b + c*x)]) - (20*c^3*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*c - b*(b + \\ & \text{Sqrt}[b^2 - 4*c*d]) - 2*c*\text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b* \\ & x + c*x^2])))/(\text{Sqrt}[a - d]*(b^2 - 4*c*d)^{7/2}*\text{Sqrt}[a + x*(b + c*x)]) - (5* \\ & c^2*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(4*a*c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c* \\ & \text{Sqrt}[b^2 - 4*c*d]*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/((a - d)^{3/ \\ & 2}*(b^2 - 4*c*d)^{5/2}*\text{Sqrt}[a + x*(b + c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^ \\ & 2]*(-(((2*c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + 2*\text{Sqrt}[b^2 - 4*c*d]))*S \\ & \text{qrt}[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \\ & \text{Sqrt}[b^2 - 4*c*d])^2)*(-b + \text{Sqrt}[b^2 - 4*c*d] - 2*c*x))) + (4*c*\text{Sqrt}[a - d] \\ & *(b*(-2*c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + 2*\text{Sqrt}[b^2 - 4*c*d])) - 2 \\ & *(4*a*c^3 - c^2*(-b + \text{Sqrt}[b^2 - 4*c*d]))*(b + 2*\text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTan} \\ & \text{h}[(-4*a*c - b*(-b + \text{Sqrt}[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + \text{Sqrt}[b^2 - 4*c* \\ & d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])))/((4*a*c^2 + 2*b*c*(-b + S \\ & \text{qrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + S \\ & \text{qrt}[b^2 - 4*c*d]) + 4*c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((b^2 - 4*c*d)^{5/2} \\ & *(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)* \\ & \text{Sqrt}[a + x*(b + c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^2]*(-((4*c^2*(-b + \text{Sqrt} \\ & [b^2 - 4*c*d]) + 2*c^2*(2*b + 3*\text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/ \\ & (2*(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2 \\ &)*(-b + \text{Sqrt}[b^2 - 4*c*d] - 2*c*x)^2) - (-(((10*c^3*\text{Sqrt}[b^2 - 4*c*d]*(-b \\ & + \text{Sqrt}[b^2 - 4*c*d]) - 2*c^3*(10*b^2 - 16*a*c - 24*c*d + 5*b*\text{Sqrt}[b^2 - 4*c \\ & *d]))*\text{Sqrt}[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c \\ & *(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*(-b + \text{Sqrt}[b^2 - 4*c*d] - 2*c*x))) + (4*c*\text{Sqrt} \\ & [a - d]*(b*(10*c^3*\text{Sqrt}[b^2 - 4*c*d]*(-b + \text{Sqrt}[b^2 - 4*c*d]) - 2*c^3*(10*b \\ & ^2 - 16*a*c - 24*c*d + 5*b*\text{Sqrt}[b^2 - 4*c*d])) - 2*(-20*a*c^4*\text{Sqrt}[b^2 - 4* \\ & c*d] + c^3*(-b + \text{Sqrt}[b^2 - 4*c*d]))*(10*b^2 - 16*a*c - 24*c*d + 5*b*\text{Sqrt}[b^ \\ & \end{aligned}$$

$$\begin{aligned}
& 2 - 4*c*d)))*\text{ArcTanh}[(-4*a*c - b*(-b + \text{Sqrt}[b^2 - 4*c*d]) - (2*b*c + 2*c*(\\
& -b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]/((4*a \\
& *c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a \\
& *c^2 + 8*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + 4*c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2)))/(\\
& 2*(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \text{Sqrt}[b^2 - 4*c*d])^2) \\
&))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 + 2*b*c*(-b + \text{Sqrt}[b^2 - 4*c*d]) + c*(-b + \\
& \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^2 \\
&]*(-(((2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) - 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqr} \\
& \text{t}[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt} \\
& [b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x))) + (4*c*\text{Sqrt}[a - d]*(b*(\\
& 2*c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]) + 2*c^2*(b + \text{Sqrt}[b^2 - 4*c*d])) - 2*(4*a*c \\
& ^3 + c^2*(b - 2*\text{Sqrt}[b^2 - 4*c*d]))*(b + \text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTanh}[(4*a*c \\
& - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(4 \\
& *c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4 \\
& *c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4*c \\
& *d]) + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((b^2 - 4*c*d)^(5/2)*(4*a*c^2 - 2* \\
& b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b + \\
& c*x)]) - (16*c^4*\text{Sqrt}[a + b*x + c*x^2]*(-((2*c^2*(2*b - 3*\text{Sqrt}[b^2 - 4*c*d] \\
&) - 4*c^2*(b + \text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c*x^2])/((2*(4*a*c^2 - 2*b \\
& *c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(b + \text{Sqrt}[b^2 - 4 \\
& *c*d] + 2*c*x)^2) - (-(((10*c^3*\text{Sqrt}[b^2 - 4*c*d])*(b + \text{Sqrt}[b^2 - 4*c*d]) + \\
& 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d]))*\text{Sqrt}[a + b*x + c \\
& *x^2])/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d] \\
&)^2)*(b + \text{Sqrt}[b^2 - 4*c*d] + 2*c*x))) + (4*c*\text{Sqrt}[a - d]*(b*(-10*c^3*\text{Sqrt}[\\
& b^2 - 4*c*d])*(b + \text{Sqrt}[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5* \\
& b*\text{Sqrt}[b^2 - 4*c*d])) - 2*(-20*a*c^4*\text{Sqrt}[b^2 - 4*c*d] + c^3*(b + \text{Sqrt}[b^2 \\
& - 4*c*d])*(10*b^2 - 16*a*c - 24*c*d - 5*b*\text{Sqrt}[b^2 - 4*c*d])))*\text{ArcTanh}[(4*a \\
& *c - b*(b + \text{Sqrt}[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + \text{Sqrt}[b^2 - 4*c*d]))*x)/(\\
& (4*c*\text{Sqrt}[a - d]*\text{Sqrt}[a + b*x + c*x^2])]/((4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 - \\
& 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + \text{Sqrt}[b^2 - 4 \\
& *c*d]) + 4*c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((2*(4*a*c^2 - 2*b*c*(b + \text{Sqrt}[b^2 \\
& - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)))/((3*(b^2 - 4*c*d)^2*(4*a*c^2 - \\
& 2*b*c*(b + \text{Sqrt}[b^2 - 4*c*d]) + c*(b + \text{Sqrt}[b^2 - 4*c*d])^2)*\text{Sqrt}[a + x*(b \\
& + c*x)])
\end{aligned}$$

Maple [B] time = 0.259, size = 3695, normalized size = 11.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2), x)$

[Out]
$$\begin{aligned}
& -5/8/(b^2-4cd)^2/(a-d)^3/(x+1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2}*b^2-7/3/(b^2-4cd)^2*c/(a-d)^2/(x+1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2}-2/(b^2-4cd)^{5/2}*c/(a-d)/(x+1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)^2*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2}-3/2/(b^2-4cd)^{5/2}*c/(a-d)^{5/2})*\ln((2*a-2*d-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+2*(a-d)^{1/2})*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2})/(x+1/2*((b^2-4cd)^{1/2}+b)/c))*b^2+6/(b^2-4cd)^{5/2}*c^2/(a-d)^{5/2}*\ln((2*a-2*d-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+2*(a-d)^{1/2})*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2})/(x+1/2*((b^2-4cd)^{1/2}+b)/c))*d+5/4/(b^2-4cd)^{3/2}/(a-d)^{7/2}*\ln((2*a-2*d-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+2*(a-d)^{1/2})*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2})/(x+1/2*((b^2-4cd)^{1/2}+b)/c))*c*d-5/4/(b^2-4cd)^{3/2}/(a-d)^{7/2}*\ln((2*a-2*d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2})*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+ \\
& a-d)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))*c*d+2/(b^2-4cd)^{5/2})*c/(a-d)/(x-1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)^2*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+a-d)^{1/2}-3/(b^2-4cd)^{5/2}*c^2/(a-d)^{3/2}*\ln((2*a-2*d-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+2*(a-d)^{1/2})*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2})/(x+1/2*((b^2-4cd)^{1/2}+b)/c))-20*c^3/(b^2-4cd)^{7/2}/(a-d)^{1/2}*\ln((2*a-2*d-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+2*(a-d)^{1/2})*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+ \\
& a-d)^{1/2})/(x+1/2*((b^2-4cd)^{1/2}+b)/c))+5/12/(b^2-4cd)^{3/2}/(a-d)^2/(x-1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)^2*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+a-d)^{1/2}+5/16/(b^2-4cd)^{3/2}/(a-d)^{7/2}*\ln((2*a-2*d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2})*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+ \\
& a-d)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))*b^2-3/4/(b^2-4cd)^{3/2}/(a-d)^{5/2})*c*\ln((2*a-2*d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2})*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+a-d)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))-1/3/(b^2-4cd)^2/(a-d)/(x-1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)^3*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+a-d)^{1/2}+3/(b^2-4cd)^{5/2}*c^2/(a-d)^{3/2})* \\
& \ln((2*a-2*d+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+2*(a-d)^{1/2})*((x-1/2*(-b+(b^2-4cd)^{1/2}))/c)^2*c+(b^2-4cd)^{1/2}*(x-1/2*(-b+(b^2-4cd)^{1/2}))/c)+a-d)^{1/2})/(x-1/2*(-b+(b^2-4cd)^{1/2}))/c))-5/12/(b^2-4cd)^{3/2}/(a-d)^2/(x+1/2/c*(b^2-4cd)^{1/2}+1/2*b/c)^2*((x+1/2*((b^2-4cd)^{1/2}+b)/c)^2*c-(b^2-4cd)^{1/2}*(x+1/2*((b^2-4cd)^{1/2}+b)/c)+a-d)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& (1/2) - 5/16 / (b^2 - 4cd)^{3/2} / (a-d)^{7/2} * \ln((2a-2d - (b^2 - 4cd)^{1/2}) * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + 2 * (a-d)^{1/2} * ((x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c)^{2c} - (b^2 - 4cd)^{1/2} * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + a-d)^{1/2} / (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) * b^2 + 3/4 / (b^2 - 4cd)^{3/2} / (a-d)^{5/2} * c * \ln((2a-2d - (b^2 - 4cd)^{1/2}) * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + 2 * (a-d)^{1/2} * ((x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c)^{2c} - (b^2 - 4cd)^{1/2} * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + a-d)^{1/2} / (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) - 1/3 / (b^2 - 4cd)^2 / (a-d) / (x+1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c)^3 * ((x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c)^{2c} - (b^2 - 4cd)^{1/2} * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + a-d)^{1/2} + 20 * c^3 / (b^2 - 4cd)^{7/2} / (a-d)^{1/2} * \ln((2a-2d + (b^2 - 4cd)^{1/2}) * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + 2 * (a-d)^{1/2} * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} / (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) - 5/8 / (b^2 - 4cd)^2 / (a-d)^3 / (x-1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} * b^2 - 10 / (b^2 - 4cd)^3 * c^2 / (a-d) / (x-1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} + 5/2 / (b^2 - 4cd)^2 / (a-d)^3 / (x+1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c)^{2c} - (b^2 - 4cd)^{1/2} * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + a-d)^{1/2} * c * d + 5/2 / (b^2 - 4cd)^2 / (a-d)^3 / (x-1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} * c * d - 7/3 / (b^2 - 4cd)^2 * c / (a-d)^2 / (x-1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} + 3/2 / (b^2 - 4cd)^{5/2} * c / (a-d)^{5/2} * \ln((2a-2d + (b^2 - 4cd)^{1/2}) * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + 2 * (a-d)^{1/2} * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} / (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) * b^2 - 6 / (b^2 - 4cd)^{5/2} * c^2 / (a-d)^{5/2} * \ln((2a-2d + (b^2 - 4cd)^{1/2}) * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + 2 * (a-d)^{1/2} * ((x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c)^{2c} + (b^2 - 4cd)^{1/2} * (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) + a-d)^{1/2} / (x-1/2 * (-b + (b^2 - 4cd)^{1/2})/c) * d - 10 / (b^2 - 4cd)^3 * c^2 / (a-d) / (x+1/2 / c * (b^2 - 4cd)^{1/2} + 1/2 * b/c) * ((x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c)^{2c} - (b^2 - 4cd)^{1/2} * (x+1/2 * ((b^2 - 4cd)^{1/2} + b)/c) + a-d)^{1/2}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(cx^2 + bx + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.7 \quad \int \frac{1}{\sqrt{d+ex+fx^2}(ae+bex+bf x^2)^2} dx$$

Optimal. Leaf size=162

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

[Out] $-\left(\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}\right) - \left(\frac{(8aef - b(4df + e^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right]}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}\right)$

Rubi [A] time = 0.305681, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {974, 12, 982, 208}

$$\frac{(8aef - b(4df + e^2)) \tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out] $-\left(\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)}\right) - \left(\frac{(8aef - b(4df + e^2)) \operatorname{ArcTanh}\left[\frac{\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right]}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}\right)$

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p+1)*(d + e*x + f*x^2)^(q+1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p+1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p+1)), Int[(a + b*x + c*x^2)^(p+1)*(d + e*x + f*x^2)^q* Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p+1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p+1) - c*d*(p+2)) - e*(b^2*c*e - 2*a*c^2*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 982

```

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :=> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex+fx^2}(ae+bex+bf x^2)^2} dx &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{\int \frac{b(bd-ae)f^2(8aef-b(e^2+4df))}{2\sqrt{d+ex+fx^2}(ae+bex+bf x^2)} dx}{be(bd-ae)^2 f^2 (be-4af)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} + \frac{(8aef-b(e^2+4df)) \int \frac{1}{\sqrt{d+ex+fx^2}}}{2e(bd-ae)(be-4af)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \operatorname{Subst}\left(\int \frac{1}{e(\sqrt{d+ex+fx^2})}\right)}{(bd-ae)} \\
&= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)(ae+bex+bf x^2)} - \frac{(8aef-b(e^2+4df)) \tanh^{-1}\left(\frac{1}{\sqrt{e}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)}
\end{aligned}$$

Mathematica [B] time = 2.12731, size = 490, normalized size = 3.02

$$2f \left(\frac{(b(4df+e^2)-8aef) \tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{e \tanh^{-1}\left(\frac{\sqrt{b}(e^2-4df)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}(e^2-4df)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}} - \frac{b}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} \right) \frac{1}{e^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2), x]

[Out] (2*f*(-((Sqrt[b]*Sqrt[e]*Sqrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*(-Sqrt[e]*Sqrt[b*e - 4*a*f]) + Sqrt[b]*(e + 2*f*x))) - (Sqrt[b]*Sqrt[e]*Sqrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*(Sqrt[e]*Sqrt[b*e - 4*a*f] + Sqrt[b]*(e + 2*f*x))) - ((-8*a*e*f + b*(e^2 + 4*d*f))*ArcTanh[(-Sqrt[b]*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x)]/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])))/(4*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) - (e*ArcTanh[(Sqrt[b]*(e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])))/(4*(b*d - a*e)^(3/2)*f*Sqrt[b*e - 4*a*f]) + ArcTanh[(-Sqrt[b]*(e^2 - 4*d*f)) + Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x)]/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)]))/(Sqrt[b*d - a*e]*(b*e - 4*a*f)^(3/2)))/e^(3/2)

Maple [B] time = 0.352, size = 1377, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2), x)

[Out] -2/e/(4*a*f-b*e)*f/(-b*e*(4*a*f-b*e))^(1/2)/(-a*e-b*d)/b^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)-(a*e-b*d)/b)^(1/2)/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)-(a*e-b*d)/b)^(1/2)+1/2/b/e/(4*a*f-b*e)*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)))/b/f)^2*f+(-b*e*(4*a*f-b

$$\begin{aligned}
 & *e))^{\frac{1}{2}}/b*(x-\frac{1}{2}*(-b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)-(a*e-b*d)/b)^{\frac{1}{2}} \\
 &)/(x-\frac{1}{2}*(-b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f))+2/e/(4*a*f-b*e)*f/(-b*e*(4*a \\
 & *f-b*e))^{\frac{1}{2}}/(-a*e-b*d)/b)^{\frac{1}{2}}*\ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^{\frac{1}{2}}/ \\
 & b*(x+\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)+2*(-a*e-b*d)/b)^{\frac{1}{2}}*((\\
 & x+\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{\frac{1}{2}}/b*(x \\
 & +\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)-(a*e-b*d)/b)^{\frac{1}{2}}/(x+\frac{1}{2}*(b*e+(- \\
 & b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f))-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+\frac{1}{2}*e/f+\frac{1}{2}/b/f* \\
 & (-b*e*(4*a*f-b*e))^{\frac{1}{2}})*((x+\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)^2*f-(\\
 & -b*e*(4*a*f-b*e))^{\frac{1}{2}}/b*(x+\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)-(a*e-b \\
 & *d)/b)^{\frac{1}{2}}-1/2/b/e/(4*a*f-b*e)*(-b*e*(4*a*f-b*e))^{\frac{1}{2}}/(a*e-b*d)/(-a*e- \\
 & b*d)/b)^{\frac{1}{2}}*\ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^{\frac{1}{2}}/b*(x+\frac{1}{2}*(b*e+(-b \\
 & *e*(4*a*f-b*e))^{\frac{1}{2}})/b/f)+2*(-a*e-b*d)/b)^{\frac{1}{2}}*((x+\frac{1}{2}*(b*e+(-b*e*(4*a* \\
 & f-b*e))^{\frac{1}{2}})/b/f)^2*f-(-b*e*(4*a*f-b*e))^{\frac{1}{2}}/b*(x+\frac{1}{2}*(b*e+(-b*e*(4*a*f \\
 & -b*e))^{\frac{1}{2}})/b/f)-(a*e-b*d)/b)^{\frac{1}{2}}/(x+\frac{1}{2}*(b*e+(-b*e*(4*a*f-b*e))^{\frac{1}{2}} \\
 &)/b/f))
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bf^2x^2 + bex + ae)^2 \sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)

Fricas [B] time = 8.12704, size = 4059, normalized size = 25.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] [-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3

$$\begin{aligned}
& + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*\log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*e^6 \\
& + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*e^2) \\
& *f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d^2*e \\
& - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3 + (b^ \\
& 2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*a*b*d*e \\
& ^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*\sqrt{b^2*d* \\
& e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*d*e^2*f + \\
& 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e - 2*a*e^2) \\
& *f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)*\sqrt{f*x^2 \\
& + e*x + d} - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^ \\
& 6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 + \\
& 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b \\
& ^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*b^2*d*e^2 - a^2 \\
& *b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 - a*b^2*e^3)*f)*x \\
&)*\sqrt{f*x^2 + e*x + d})/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6 + a^3*b^2*e^7 + 1 \\
& 6*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(a^2*b^3*d^2*e^2 - \\
& 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3 \\
& *b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^3*e^6)*f)*x^2 - 8*(a^2 \\
& *b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*d^2*e^5 - 2*a*b^4*d*e^ \\
& 6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*f^2 - \\
& 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*x), 1/2*(\sqrt{-b^2*d*e \\
& ^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d - \\
& 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3 + 4*(b^2*d*e - 2*a \\
& *b*e^2)*f)*x)*\arctan(-1/2*\sqrt{-b^2*d*e^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2) \\
& *f)*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 + \\
& (b*e^3 + 4*(b*d*e - 2*a*e^2)*f)*x)*\sqrt{f*x^2 + e*x + d}/(b^2*d^2*e^3 - a*b \\
& *d*e^4 - 2*(4*(a*b*d*e - a^2*e^2)*f^3 - (b^2*d*e^2 - a*b*e^3)*f^2)*x^3 - 3* \\
& (4*(a*b*d*e^2 - a^2*e^3)*f^2 - (b^2*d*e^3 - a*b*e^4)*f)*x^2 - 4*(a*b*d^2*e^ \\
& 2 - a^2*d*e^3)*f + (b^2*d*e^4 - a*b*e^5 - 8*(a*b*d^2*e - a^2*d*e^2)*f^2 + 2 \\
& *(b^2*d^2*e^2 - 3*a*b*d*e^3 + 2*a^2*e^4)*f)*x)) - 2*(b^3*d*e^3 - a*b^2*e^4 \\
& - 4*(a*b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d \\
& *e^2 - a*b^2*e^3)*f)*x)*\sqrt{f*x^2 + e*x + d})/(a*b^4*d^2*e^5 - 2*a^2*b^3*d \\
& *e^6 + a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (\\
& 16*(a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - \\
& 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^ \\
& 3*e^6)*f)*x^2 - 8*(a^2*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5* \\
& d^2*e^5 - 2*a*b^4*d*e^6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e \\
& ^4 + a^4*b*e^5)*f^2 - 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)* \\
& x)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex + fx^2} (ae + bex + bfx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2), x)

[Out] Integral(1/(sqrt(d + e*x + f*x**2)*(a*e + b*e*x + b*f*x**2)**2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.8 \quad \int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]

Rubi [A] time = 0.0172824, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]

[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]

Rule 982

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(4 + 2x + x^2)\sqrt{5 + 2x + x^2}} dx = -\left(4 \operatorname{Subst}\left(\int \frac{1}{-24 - 2x^2} dx, x, \frac{2 + 2x}{\sqrt{5 + 2x + x^2}}\right)\right)$$

$$= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.0677789, size = 84, normalized size = 3.

$$\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right) - \tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]), x]

[Out] ((-I/2)*(ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]))/Sqrt[3]

Maple [A] time = 0.048, size = 27, normalized size = 1.

$$\frac{\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}(2x+2)}{6} \frac{1}{\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2), x)

[Out] 1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + 2x + 5}(x^2 + 2x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)

Fricas [A] time = 1.03401, size = 123, normalized size = 4.39

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \sqrt{x^2 + 2x + 5}(x + 1) - \frac{1}{3} \sqrt{3}(x^2 + 2x + 4)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(x^2 + 2*x + 4))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 2x + 4) \sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)

[Out] Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)

Giac [B] time = 1.14215, size = 70, normalized size = 2.5

$$-\frac{1}{3} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3}(x - \sqrt{x^2 + 2x + 5} + 2)\right) + \frac{1}{3} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3}(x - \sqrt{x^2 + 2x + 5})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))

3.9 $\int \left(a + \frac{ex}{2} + cx^2\right)^p \left(2a + ex + 2cx^2\right)^q dx$

Optimal. Leaf size=136

$$\frac{2^{q+1} \left(-\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}} \right)^{-p-q-1} (2a+2cx^2+ex)^{p+q+1} {}_2F_1 \left(-p-q, p+q+1; p+q+2; \frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}} \right)}{(p+q+1)\sqrt{e^2-16ac}}$$

[Out] $-\left(2^{(1+q)} \cdot \left(-\left(\frac{e - \sqrt{-16ac + e^2} + 4cx}{\sqrt{-16ac + e^2}}\right)\right)^{-1-p-q} \cdot (2a + ex + 2cx^2)^{1+p+q} \cdot \text{Hypergeometric2F1}[-p-q, 1+p+q, 2+p+q, \frac{e + \sqrt{-16ac + e^2} + 4cx}{2\sqrt{-16ac + e^2}}]\right) / \left(\sqrt{-16ac + e^2} \cdot (1+p+q)\right)$

Rubi [A] time = 0.102988, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {967, 624}

$$\frac{2^{q+1} \left(-\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}} \right)^{-p-q-1} (2a+2cx^2+ex)^{p+q+1} {}_2F_1 \left(-p-q, p+q+1; p+q+2; \frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}} \right)}{(p+q+1)\sqrt{e^2-16ac}}$$

Antiderivative was successfully verified.

[In] Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]

[Out] $-\left(2^{(1+q)} \cdot \left(-\left(\frac{e - \sqrt{-16ac + e^2} + 4cx}{\sqrt{-16ac + e^2}}\right)\right)^{-1-p-q} \cdot (2a + ex + 2cx^2)^{1+p+q} \cdot \text{Hypergeometric2F1}[-p-q, 1+p+q, 2+p+q, \frac{e + \sqrt{-16ac + e^2} + 4cx}{2\sqrt{-16ac + e^2}}]\right) / \left(\sqrt{-16ac + e^2} \cdot (1+p+q)\right)$

Rule 967

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[(c/f)^p, Int[(d + e*x + f*x^2)^(p+q), x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && (!IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2
- 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int \left(a + \frac{ex}{2} + cx^2\right)^p (2a + ex + 2cx^2)^q dx = 2^{-p} \int (2a + ex + 2cx^2)^{p+q} dx$$

$$= \frac{2^{1+q} \left(-\frac{e-\sqrt{-16ac+e^2+4cx}}{\sqrt{-16ac+e^2}}\right)^{-1-p-q} (2a + ex + 2cx^2)^{1+p+q} {}_2F_1\left(-p-q, 1+p+q, 1+p+q; \frac{-e-4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{\sqrt{-16ac+e^2}(1+p+q)}$$

Mathematica [A] time = 0.138064, size = 142, normalized size = 1.04

$$\frac{2^{q-2} \left(-\sqrt{e^2-16ac} + 4cx + e\right) \left(\frac{\sqrt{e^2-16ac+4cx+e}}{\sqrt{e^2-16ac}}\right)^{-p-q} (2a + x(2cx + e))^{p+q} {}_2F_1\left(-p-q, p+q+1; p+q+2; \frac{-e-4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{c(p+q+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]

[Out] (2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x)/(2*Sqrt[-16*a*c + e^2])]/(c*(1 + p + q))

Maple [F] time = 2.905, size = 0, normalized size = 0.

$$\int \left(a + \frac{ex}{2} + cx^2\right)^p (2cx^2 + ex + 2a)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)

[Out] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (2cx^2 + ex + 2a)^q \left(cx^2 + \frac{1}{2}ex + a \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")

[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2cx^2 + ex + 2a\right)^q \left(cx^2 + \frac{1}{2}ex + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")

[Out] integral((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (2cx^2 + ex + 2a)^q \left(cx^2 + \frac{1}{2}ex + a \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

$$3.10 \quad \int \left(a + \frac{cex}{f} + cx^2 \right)^p \left(\frac{af}{c} + ex + fx^2 \right)^q dx$$

Optimal. Leaf size=200

$$\frac{\sqrt{c} 2^{p+q+1} \left(a + \frac{cex}{f} + cx^2 \right)^p \left(\frac{af}{c} + ex + fx^2 \right)^{q+1} \left(-\frac{\sqrt{c} \left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}} + e + 2fx \right)}{\sqrt{ce^2-4af^2}} \right)^{-p-q-1} {}_2F_1 \left(-p-q, p+q+1; p+q+2; \frac{\sqrt{c} \left(e + 2fx + \sqrt{\frac{ce^2-4af^2}{c}} \right)}{2\sqrt{ce^2-4af^2}} \right)}{(p+q+1)\sqrt{ce^2-4af^2}}$$

[Out] $-\left(2^{(1+p+q)}\sqrt{c}\left(-\left(\frac{\sqrt{c}\left(e-\sqrt{c^2e^2-4af^2}\right)}{\sqrt{c}}+2fx\right)\right)\sqrt{c^2e^2-4af^2}\right)^{-1-p-q}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{1+q}\text{Hypergeometric2F1}\left[-p-q, 1+p+q, 2+p+q, \frac{\sqrt{c}\left(e+\frac{\sqrt{c^2e^2-4af^2}}{\sqrt{c}}\right)}{2\sqrt{c^2e^2-4af^2}}\right]\right)/\left(\sqrt{c^2e^2-4af^2}\left(1+p+q\right)\right)$

Rubi [A] time = 0.134029, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {968, 624}

$$\frac{\sqrt{c} 2^{p+q+1} \left(a + \frac{cex}{f} + cx^2 \right)^p \left(\frac{af}{c} + ex + fx^2 \right)^{q+1} \left(-\frac{\sqrt{c} \left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}} + e + 2fx \right)}{\sqrt{ce^2-4af^2}} \right)^{-p-q-1} {}_2F_1 \left(-p-q, p+q+1; p+q+2; \frac{\sqrt{c} \left(e + 2fx + \sqrt{\frac{ce^2-4af^2}{c}} \right)}{2\sqrt{ce^2-4af^2}} \right)}{(p+q+1)\sqrt{ce^2-4af^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^q, x\right]$

[Out] $-\left(2^{(1+p+q)}\sqrt{c}\left(-\left(\frac{\sqrt{c}\left(e-\sqrt{c^2e^2-4af^2}\right)}{\sqrt{c}}+2fx\right)\right)\sqrt{c^2e^2-4af^2}\right)^{-1-p-q}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{1+q}\text{Hypergeometric2F1}\left[-p-q, 1+p+q, 2+p+q, \frac{\sqrt{c}\left(e+\frac{\sqrt{c^2e^2-4af^2}}{\sqrt{c}}\right)}{2\sqrt{c^2e^2-4af^2}}\right]\right)/\left(\sqrt{c^2e^2-4af^2}\left(1+p+q\right)\right)$

Rule 968

$\text{Int}\left[\left((a_{.}) + (b_{.})(x_{.}) + (c_{.})(x_{.})^2\right)^{p_{.}} \left(\left(d_{.}\right) + (e_{.})(x_{.}) + (f_{.})(x_{.})^2\right)^{q_{.}}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\left(a_{\text{IntPart}}\right) \left(a + bx + cx^2\right)^{\text{FracPart}[p]} / \left(\right.\right.$

$d^{\text{IntPart}[p]}(d + e*x + f*x^2)^{\text{FracPart}[p]}$, $\text{Int}[(d + e*x + f*x^2)^{(p + q)}, x]$, $x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, p, q\}, x$ && $\text{EqQ}[c*d - a*f, 0]$ && $\text{EqQ}[b*d - a*e, 0]$ && $!\text{IntegerQ}[p]$ && $!\text{IntegerQ}[q]$ && $!\text{GtQ}[c/f, 0]$

Rule 624

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}]$, $x_Symbol]$:> $\text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}$, $-\text{Simp}[(a + b*x + c*x^2)^{(p + 1)}*\text{Hypergeometric2F1}[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)]]/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^{(p + 1)})$, $x]$ /; $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $!\text{IntegerQ}[4*p]$

Rubi steps

$$\int \left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^q dx = \left(\left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^{-p}\right) \int \left(\frac{af}{c} + ex + fx^2\right)^{p+q} dx$$

$$= \frac{2^{1+p+q} \sqrt{c} \left(-\frac{\sqrt{c} \left(e - \frac{\sqrt{ce^2 - 4af^2}}{\sqrt{c}} + 2fx\right)}{\sqrt{ce^2 - 4af^2}}\right)^{-1-p-q} \left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^{1+q}}{\sqrt{ce^2 - 4af^2}(1 + p + q)}$$

Mathematica [A] time = 0.238339, size = 172, normalized size = 0.86

$$\frac{2^{p+q-1} \left(\sqrt{c}(e + 2fx) - \sqrt{ce^2 - 4af^2}\right) \left(a + \frac{cx(e+fx)}{f}\right)^p \left(\frac{af}{c} + x(e + fx)\right)^q \left(\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2 - 4af^2}} + 1\right)^{-p-q} {}_2F_1\left(-p - q, p + q + 1; p + q + 1; -\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2 - 4af^2}}\right)}{\sqrt{c}f(p + q + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}\left[\left(a + \frac{cex}{f} + cx^2\right)^p \left(\frac{af}{c} + ex + fx^2\right)^q, x\right]$

[Out] $(2^{-1 + p + q} * ((af)/c + x*(e + fx))^q * (a + (c*x*(e + fx))/f)^p * (-\text{Sqrt}[c*e^2 - 4*a*f^2] + \text{Sqrt}[c]*(e + 2*f*x)) * (1 + (\text{Sqrt}[c]*(e + 2*f*x))/\text{Sqrt}[c*e^2 - 4*a*f^2])^{-(p + q)} * \text{Hypergeometric2F1}[-p - q, 1 + p + q, 2 + p + q, 1/2 - (\text{Sqrt}[c]*(e + 2*f*x))/(2*\text{Sqrt}[c*e^2 - 4*a*f^2])]) / (\text{Sqrt}[c]*f*(1 + p + q))$

Maple [F] time = 3.482, size = 0, normalized size = 0.

$$\int \left(a + \frac{cex}{f} + cx^2 \right)^p \left(\frac{af}{c} + ex + fx^2 \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)

[Out] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + \frac{cex}{f} + a \right)^p \left(fx^2 + ex + \frac{af}{c} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")

[Out] integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(\frac{cfx^2 + cex + af}{c} \right)^q \left(\frac{cfx^2 + cex + af}{f} \right)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")

[Out] integral(((c*f*x^2 + c*e*x + a*f)/c)^q*((c*f*x^2 + c*e*x + a*f)/f)^p, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + \frac{cex}{f} + a \right)^p \left(fx^2 + ex + \frac{af}{c} \right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")`

[Out] `integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)`

$$3.11 \quad \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=48

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1} \sinh^{-1}(x)}{x+1}$$

[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[x])/(1 + x)

Rubi [A] time = 0.0153268, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1} + \frac{\sqrt{x^2+2x+1} \sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[x])/(1 + x)

Rule 970

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx &= \frac{\sqrt{1+2x+x^2} \int \frac{2+2x}{\sqrt{1+x^2}} dx}{2+2x} \\ &= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{(2\sqrt{1+2x+x^2}) \int \frac{1}{\sqrt{1+x^2}} dx}{2+2x} \\ &= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\sqrt{1+2x+x^2} \sinh^{-1}(x)}{1+x} \end{aligned}$$

Mathematica [A] time = 0.0176484, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2} \left(\sqrt{x^2+1} + \sinh^{-1}(x) \right)}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)

Maple [C] time = 0.168, size = 16, normalized size = 0.3

$$\text{csgn}(1+x) \left(\text{Arcsinh}(x) + \sqrt{x^2+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2), x)

[Out] csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)

Fricas [A] time = 0.866251, size = 55, normalized size = 1.15

$$\sqrt{x^2 + 1} - \log(-x + \sqrt{x^2 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)

[Out] Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)

Giac [A] time = 1.18228, size = 66, normalized size = 1.38

$$-\left(\sqrt{2} - \log\left(\sqrt{2} + 1\right)\right)\operatorname{sgn}(x + 1) - \log\left(-x + \sqrt{x^2 + 1}\right)\operatorname{sgn}(x + 1) + \sqrt{x^2 + 1}\operatorname{sgn}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")


```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x + 1) + sqrt(x^2 + 1)*sgn(x + 1)
```

$$3.12 \quad \int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx$$

Optimal. Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8

Rubi [A] time = 0.0514388, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)} - \frac{1}{8} \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - \frac{5}{8} \tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]

[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
+ q + 4)))]*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x]
;/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[
q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}} dx &= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \int \frac{3+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} + \frac{1}{8} \int \frac{1}{(1+x)\sqrt{-1+x+x^2}} dx - \frac{5}{8} \int \frac{1}{(-1+x)\sqrt{-1+x+x^2}} dx \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-4-x^2} dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}} \right) + \frac{5}{4} \text{Subst} \left(\int \frac{1}{4-x^2} dx, \right. \\
&= \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{8} \tan^{-1} \left(\frac{3+x}{2\sqrt{-1+x+x^2}} \right) - \frac{5}{8} \tanh^{-1} \left(\frac{1-3x}{2\sqrt{-1+x+x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.0834212, size = 66, normalized size = 0.94

$$\frac{1}{8} \left(-\frac{4\sqrt{x^2+x-1}}{x^2-1} - \tan^{-1} \left(\frac{x+3}{2\sqrt{x^2+x-1}} \right) - 5 \tanh^{-1} \left(\frac{1-3x}{2\sqrt{x^2+x-1}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]

[Out] ((-4*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]
] - 5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8

Maple [A] time = 0.059, size = 84, normalized size = 1.2

$$\frac{1}{8} \arctan \left(\frac{-3-x}{2} \frac{1}{\sqrt{(1+x)^2-2-x}} \right) - \frac{1}{-4+4x} \sqrt{(-1+x)^2-2+3x} + \frac{5}{8} \operatorname{Artanh} \left(\frac{3x-1}{2} \frac{1}{\sqrt{(-1+x)^2-2+3x}} \right) + \frac{1}{4+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)

[Out] 1/8*arctan(1/2*(-3-x)/((1+x)^2-2-x)^(1/2))-1/4/(-1+x)*((-1+x)^2-2+3*x)^(1/2)
) + 5/8*arctanh(1/2*(3*x-1)/((-1+x)^2-2+3*x)^(1/2)) + 1/4/(1+x)*((1+x)^2-2-x)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+x-1}(x^2-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)

Fricas [A] time = 0.808353, size = 235, normalized size = 3.36

$$\frac{2(x^2 - 1) \arctan(-x + \sqrt{x^2 + x - 1} - 1) + 5(x^2 - 1) \log(-x + \sqrt{x^2 + x - 1} + 2) - 5(x^2 - 1) \log(-x + \sqrt{x^2 + x - 1})}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")

[Out] 1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1))) - 4*sqrt(x^2 + x - 1))/(x^2 - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)^2 (x+1)^2 \sqrt{x^2+x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)

[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)

Giac [B] time = 1.16836, size = 193, normalized size = 2.76

$$\frac{2(x - \sqrt{x^2 + x - 1})^3 + 3(x - \sqrt{x^2 + x - 1})^2 - x + \sqrt{x^2 + x - 1} - 1}{2\left(\left(x - \sqrt{x^2 + x - 1}\right)^4 - 2\left(x - \sqrt{x^2 + x - 1}\right)^2 - 4x + 4\sqrt{x^2 + x - 1}\right)} + \frac{1}{4} \arctan(-x + \sqrt{x^2 + x - 1} - 1) + \frac{5}{8} \log\left(\left|-x + \sqrt{x^2 + x - 1} + 2\right|\right) - \frac{5}{8} \log\left(\left|-x + \sqrt{x^2 + x - 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")

[Out] 1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2 - 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8*log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x - 1)))

$$3.13 \quad \int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}} dx$$

Optimal. Leaf size=1077

result too large to display

```
[Out] -(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[((1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]]/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2)]/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))]
```

Rubi [A] time = 3.09899, antiderivative size = 1077, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} =$

0.115, Rules used = {993, 936, 1103}

$$\sqrt[4]{db^2 + \sqrt{b^2 - 4ac}db - 2a(cd - af)} \left(b + 2cx + \sqrt{b^2 - 4ac}\right)^{3/2} \sqrt{2a + \left(b + \sqrt{b^2 - 4ac}\right)x} \sqrt{\frac{\left(4ac - \left(b + \sqrt{b^2 - 4ac}\right)^2\right)^2 (fx^2 + d)}{\left(4fa^2 + \left(b + \sqrt{b^2 - 4ac}\right)^2 d\right)\left(b + 2cx + \sqrt{b^2 - 4ac}\right)}}$$

$$\left(4ac - \left(b + \sqrt{b^2 - 4ac}\right)^2\right)^{4/2} \sqrt{\dots}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]

[Out] -((((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^(2*(d + f*x^2)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2)/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))]

Rule 993

Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2

```
*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a
+ (b + r)*x]*Sqrt[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ
[b^2 - 4*a*c, 0]
```

Rule 936

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] := Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/
((c*f^2 + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}} dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\right) \int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\sqrt{d+fx^2}}}{\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(2(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\sqrt{\frac{(4ac-(b+\sqrt{b^2-4ac})^2)(d+fx^2)}{\left((b+\sqrt{b^2-4ac})^2d+4a^2f\right)(b+\sqrt{b^2-4ac})}}\right)}{(4ac-(b+\sqrt{b^2-4ac})^2)}$$

$$= \frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}d-2a(cd-af)}(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}}{\dots}$$

Mathematica [C] time = 1.52761, size = 600, normalized size = 0.56

$$\frac{2\sqrt{2}(\sqrt{fx}-i\sqrt{d})\left(\sqrt{b^2-4ac}-b-2cx\right)\sqrt{\frac{c\sqrt{b^2-4ac}(\sqrt{fx}+i\sqrt{d})}{(\sqrt{b^2-4ac}-b-2cx)(\sqrt{f}(\sqrt{b^2-4ac}+b)-2ic\sqrt{d})}}\sqrt{\frac{c(-i\sqrt{d}(\sqrt{b^2-4ac}+2cx)+\sqrt{f}(x\sqrt{b^2-4ac}-2a)+b)}{(\sqrt{b^2-4ac}-b-2cx)(\sqrt{f}(\sqrt{b^2-4ac}+b)+2ic\sqrt{d})}}}{\sqrt{d+fx^2}\sqrt{a+x(b+cx)}\left(\sqrt{f}(\sqrt{b^2-4ac}-b)-2ic\sqrt{d}\right)\sqrt{\frac{c\sqrt{b^2-4ac}(\sqrt{fx}+i\sqrt{d})}{(\sqrt{b^2-4ac}-b-2cx)(\sqrt{f}(\sqrt{b^2-4ac}+b)-2ic\sqrt{d})}}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]

[Out] (-2*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*((-I)*Sqrt[d] + Sqrt[f]*x)*Sqrt[-((c*Sqrt[b^2 - 4*a*c]*(I*Sqrt[d] + Sqrt[f]*x))/((-2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))] * Sqrt[(c*((-I)*Sqrt[d]*(Sqrt[b^2 - 4*a*c] + 2*c*x) + Sqrt[f]*(-2*a + Sqrt[b^2 - 4*a*c]*x) + b*((-I)*Sqrt[d] - Sqrt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))] * EllipticF[ArcSin[Sqrt[(((

$$\frac{-2Ic\sqrt{d} + (-b + \sqrt{b^2 - 4ac})\sqrt{f}}{(2Ic\sqrt{d} + (b + \sqrt{b^2 - 4ac})\sqrt{f})(-b + \sqrt{b^2 - 4ac} - 2cx)} \cdot \frac{(cd - I\sqrt{b^2 - 4ac}\sqrt{d}\sqrt{f} + af)}{(cd + I\sqrt{b^2 - 4ac}\sqrt{d}\sqrt{f} + af)} \cdot \frac{(-2Ic\sqrt{d} + (-b + \sqrt{b^2 - 4ac})\sqrt{f})\sqrt{(I\sqrt{b^2 - 4ac}(\sqrt{d} + I\sqrt{f}x))}}{(2Ic\sqrt{d} + (b + \sqrt{b^2 - 4ac})\sqrt{f})(-b + \sqrt{b^2 - 4ac} - 2cx)} \cdot \sqrt{d + fx^2}\sqrt{a + x(b + cx)}$$

Maple [A] time = 0.618, size = 661, normalized size = 0.6

$$\frac{(bf^2x^2 + 2x^2cf\sqrt{-df} + \sqrt{-4ac + b^2}f^2x^2 + 2xbf\sqrt{-df} - 4cxfd + 2xf\sqrt{-4ac + b^2}\sqrt{-df} - bdf - 2cd\sqrt{-df} - \sqrt{-4ac})}{4\sqrt{-df}\left(f\sqrt{-4ac + b^2} - 2\sqrt{-df}c + bf\right)\sqrt{cfx^4 + bfx^3 + afx^2 + cd x^2 + bdx + ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x)

[Out] $4*(b*f^2*x^2+2*x^2*c*f*(-d*f)^{(1/2)}+(-4*a*c+b^2)^{(1/2)}*f^2*x^2+2*x*b*f*(-d*f)^{(1/2)}-4*c*x*f*d+2*x*f*(-4*a*c+b^2)^{(1/2)}*(-d*f)^{(1/2)}-b*d*f-2*c*d*(-d*f)^{(1/2)}-(-4*a*c+b^2)^{(1/2)}*d*f)*\text{EllipticF}((-f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c+b*f)*(-f*x+(-d*f)^{(1/2)})/(f*(-4*a*c+b^2)^{(1/2)}+2*(-d*f)^{(1/2)}*c+b*f)/(f*x+(-d*f)^{(1/2)})^2, ((f*(-4*a*c+b^2)^{(1/2)}+2*(-d*f)^{(1/2)}*c-b*f)*(f*(-4*a*c+b^2)^{(1/2)}+2*(-d*f)^{(1/2)}*c+b*f)/(f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c-b*f)/(f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c+b*f))^2)*((d*f)^{(1/2)}*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/f/(f*(-4*a*c+b^2)^{(1/2)}+2*(-d*f)^{(1/2)}*c+b*f)/(f*x+(-d*f)^{(1/2)})^2)*((d*f)^{(1/2)}*(-b-2*c*x+(-4*a*c+b^2)^{(1/2)})/f/(f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c-b*f)/(f*x+(-d*f)^{(1/2)})^2)*(-f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c+b*f)*(-f*x+(-d*f)^{(1/2)})/(f*(-4*a*c+b^2)^{(1/2)}+2*(-d*f)^{(1/2)}*c+b*f)/(f*x+(-d*f)^{(1/2)})^2)*(c*x^2+b*x+a)^{(1/2)}*(f*x^2+d)^{(1/2)}/(1/c/f*(-f*x+(-d*f)^{(1/2)})*(f*x+(-d*f)^{(1/2)})*(-b-2*c*x+(-4*a*c+b^2)^{(1/2)})*(b+2*c*x+(-4*a*c+b^2)^{(1/2)}))^2/(d*f)^{(1/2)}/(f*(-4*a*c+b^2)^{(1/2)}-2*(-d*f)^{(1/2)}*c+b*f)/(c*f*x^4+b*f*x^3+a*f*x^2+c*d*x^2+b*d*x+a*d)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + d}}{cfx^4 + bfx^3 + bdx + (cd + af)x^2 + ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x + (c*d + a*f)*x^2 + a*d), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + fx^2}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+d)**(1/2),x)`

[Out] `Integral(1/(sqrt(d + f*x**2)*sqrt(a + b*x + c*x**2)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

$$3.14 \quad \int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx$$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2

Rubi [A] time = 0.209648, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2} \sin^{-1}(x+2)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2), x]

[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2

Rule 989

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 1028

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(2*h*d - g*e)/e, Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && NeQ[2*h*d - g*e, 0]

Rule 986

Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1026

Int[(x_)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 - 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]

Rule 1161

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[g, Subst[Int[1/(a + (c*d - a*f)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] && EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} dx &= -\left(\frac{1}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx\right) - \frac{1}{2} \int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -4-2x \right) + \frac{1}{2} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx + \frac{3}{2} \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{4} \int \frac{-6-4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx + \frac{1}{4} \int \frac{4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{3-3x^2} dx, x, \frac{x}{\sqrt{-3-4x-x^2}} \right) - \int \frac{1}{\sqrt{-3-4x-x^2}} dx \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - 8 \text{Subst} \left(\int \frac{1+3x^2}{-4-8x^2-36x^4} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{\frac{1}{3} - \frac{2x}{3} + x^2} dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-\frac{8}{9} - x^2} dx, x, \frac{2}{3} \left(-1 + \frac{3+x}{\sqrt{-3-4x-x^2}} \right) \right) \\
&= -\frac{1}{2} \sin^{-1}(2+x) - \frac{\tan^{-1} \left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{-3-4x-x^2}} \right)
\end{aligned}$$

Mathematica [C] time = 0.400879, size = 159, normalized size = 1.62

$$\frac{1}{4} \left(-i\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}\sqrt{-x^2-4x-3}}} \right) + i\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}\sqrt{-x^2-4x-3}}} \right) - 2 \sin^{-1}(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2), x]

[Out] (-2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])]/4

Maple [B] time = 0.112, size = 341, normalized size = 3.5

$$-\frac{\arcsin(2+x)}{2} + \frac{\sqrt{4}\sqrt{3}}{12} \sqrt{3 \frac{x^2}{(-3/2-x)^2} - 12} \left(\sqrt{2} \arctan \left(\frac{\sqrt{2}}{6} \sqrt{3 \frac{x^2}{(-3/2-x)^2} - 12} \right) - \operatorname{Artanh} \left(3 \frac{x}{-3/2-x} \frac{1}{\sqrt{3 \frac{x^2}{(-3/2-x)^2} - 12}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3), x)`

[Out] `-1/2*arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3), x, algorithm="maxima")`

[Out] `integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)`

Fricas [A] time = 0.974786, size = 441, normalized size = 4.5

$$-\frac{1}{4} \sqrt{2} \arctan \left(\frac{\sqrt{2}x + 3 \sqrt{2} \sqrt{-x^2 - 4x - 3}}{2(2x + 3)} \right) - \frac{1}{4} \sqrt{2} \arctan \left(-\frac{\sqrt{2}x - 3 \sqrt{2} \sqrt{-x^2 - 4x - 3}}{2(2x + 3)} \right) + \frac{1}{2} \arctan \left(\frac{\sqrt{-x^2 - 4x - 3}}{x^2 + 4x + 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")

[Out]
$$-1/4*\sqrt{2}*\arctan(1/2*(\sqrt{2}*x + 3*\sqrt{2}*\sqrt{-x^2 - 4*x - 3})/(2*x + 3)) - 1/4*\sqrt{2}*\arctan(-1/2*(\sqrt{2}*x - 3*\sqrt{2}*\sqrt{-x^2 - 4*x - 3})/(2*x + 3)) + 1/2*\arctan(\sqrt{-x^2 - 4*x - 3}*(x + 2)/(x^2 + 4*x + 3)) + 1/8*\log(-(2*\sqrt{-x^2 - 4*x - 3})*x + 4*x + 3)/x^2) - 1/8*\log((2*\sqrt{-x^2 - 4*x - 3})*x - 4*x - 3)/x^2)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2 + 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)

[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)

Giac [B] time = 1.17704, size = 231, normalized size = 2.36

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\arcsin(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")

[Out]
$$-1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(3*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 1)) - 1/2*\sqrt{2}*\arctan(1/2*\sqrt{2}*((\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 1)) - 1/2*\arcsin(x + 2) - 1/4*\log(2*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + 3*(\sqrt{-x^2 - 4*x - 3} - 1)^2/(x + 2)^2 + 1) + 1/4*\log(2*(\sqrt{-x^2 - 4*x - 3} - 1)/(x + 2) + (\sqrt{-x^2 - 4*x - 3} - 1)^2/(x + 2)^2 + 3))$$

$$3.15 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=68

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 1176*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11

Rubi [A] time = 0.0491625, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4, x]

[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 1176*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^4 dx &= \int (48 + 272x + 1064x^2 + 2624x^3 + 5099x^4 + 7131x^5 + 8232x^6 + 6830x^7 + 5075x^8 + 1176x^9 + 475x^{10} + 1250x^{11}) dx \\ &= 48x + 136x^2 + \frac{1064x^3}{3} + 656x^4 + \frac{5099x^5}{5} + \frac{2377x^6}{2} + 1176x^7 + \frac{3415x^8}{4} + \frac{5075x^9}{9} + \frac{475x^{10}}{2} + \frac{1250x^{11}}{11} \end{aligned}$$

Mathematica [A] time = 0.0026852, size = 68, normalized size = 1.

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]

[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 1176*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11

Maple [A] time = 0.044, size = 55, normalized size = 0.8

$$48x + 136x^2 + \frac{1064x^3}{3} + 656x^4 + \frac{5099x^5}{5} + \frac{2377x^6}{2} + 1176x^7 + \frac{3415x^8}{4} + \frac{5075x^9}{9} + \frac{475x^{10}}{2} + \frac{1250x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x)

[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5075/9*x^9+475/2*x^10+1250/11*x^11

Maxima [A] time = 0.972346, size = 73, normalized size = 1.07

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")

[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x

Fricas [A] time = 0.847187, size = 176, normalized size = 2.59

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x

Sympy [A] time = 0.084379, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11} + \frac{475x^{10}}{2} + \frac{5075x^9}{9} + \frac{3415x^8}{4} + 1176x^7 + \frac{2377x^6}{2} + \frac{5099x^5}{5} + 656x^4 + \frac{1064x^3}{3} + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)

[Out] 1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377*x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x

Giac [A] time = 1.15386, size = 73, normalized size = 1.07

$$\frac{1250}{11}x^{11} + \frac{475}{2}x^{10} + \frac{5075}{9}x^9 + \frac{3415}{4}x^8 + 1176x^7 + \frac{2377}{2}x^6 + \frac{5099}{5}x^5 + 656x^4 + \frac{1064}{3}x^3 + 136x^2 + 48x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x

3.16 $\int (3 - x + 2x^2)(2 + 3x + 5x^2)^3 dx$

Optimal. Leaf size=56

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

[Out] $24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^7)/7 + (325*x^8)/8 + (250*x^9)/9$

Rubi [A] time = 0.0377753, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]

[Out] $24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^7)/7 + (325*x^8)/8 + (250*x^9)/9$

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^3 dx &= \int (24 + 100x + 322x^2 + 579x^3 + 876x^4 + 804x^5 + 720x^6 + 325x^7 + 250x^8) dx \\ &= 24x + 50x^2 + \frac{322x^3}{3} + \frac{579x^4}{4} + \frac{876x^5}{5} + 134x^6 + \frac{720x^7}{7} + \frac{325x^8}{8} + \frac{250x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0017287, size = 56, normalized size = 1.

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]

[Out] $24x + 50x^2 + \frac{322x^3}{3} + \frac{579x^4}{4} + \frac{876x^5}{5} + 134x^6 + \frac{720x^7}{7} + \frac{325x^8}{8} + \frac{250x^9}{9}$

Maple [A] time = 0.044, size = 45, normalized size = 0.8

$$24x + 50x^2 + \frac{322x^3}{3} + \frac{579x^4}{4} + \frac{876x^5}{5} + 134x^6 + \frac{720x^7}{7} + \frac{325x^8}{8} + \frac{250x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x)

[Out] $24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9*x^9$

Maxima [A] time = 0.983348, size = 59, normalized size = 1.05

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] $250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3*x^3 + 50*x^2 + 24*x$

Fricas [A] time = 0.716507, size = 131, normalized size = 2.34

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3*x^3 + 50*x^2 + 24*x

Sympy [A] time = 0.084274, size = 53, normalized size = 0.95

$$\frac{250x^9}{9} + \frac{325x^8}{8} + \frac{720x^7}{7} + 134x^6 + \frac{876x^5}{5} + \frac{579x^4}{4} + \frac{322x^3}{3} + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)

[Out] 250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 + 322*x**3/3 + 50*x**2 + 24*x

Giac [A] time = 1.17455, size = 59, normalized size = 1.05

$$\frac{250}{9}x^9 + \frac{325}{8}x^8 + \frac{720}{7}x^7 + 134x^6 + \frac{876}{5}x^5 + \frac{579}{4}x^4 + \frac{322}{3}x^3 + 50x^2 + 24x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3*x^3 + 50*x^2 + 24*x

$$3.17 \quad \int (3 - x + 2x^2)(2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=44

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^7)/7

Rubi [A] time = 0.0285789, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]

[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^7)/7

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2)^2 dx &= \int (12 + 32x + 83x^2 + 85x^3 + 103x^4 + 35x^5 + 50x^6) dx \\ &= 12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.001421, size = 44, normalized size = 1.

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]

[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^7)/7

Maple [A] time = 0.042, size = 35, normalized size = 0.8

$$12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x)

[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7

Maxima [A] time = 0.960582, size = 46, normalized size = 1.05

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x

Fricas [A] time = 0.711843, size = 96, normalized size = 2.18

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] $50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x$

Sympy [A] time = 0.079116, size = 41, normalized size = 0.93

$$\frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)`

[Out] $50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x$

Giac [A] time = 1.18458, size = 46, normalized size = 1.05

$$\frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")`

[Out] $50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x$

3.18 $\int (3 - x + 2x^2)(2 + 3x + 5x^2) dx$

Optimal. Leaf size=30

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

[Out] $6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5$

Rubi [A] time = 0.0161242, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1657}

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2), x]$

[Out] $6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5$

Rule 1657

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] :> \text{Int}[\text{Expand Integrand}[Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)(2 + 3x + 5x^2) dx &= \int (6 + 7x + 16x^2 + x^3 + 10x^4) dx \\ &= 6x + \frac{7x^2}{2} + \frac{16x^3}{3} + \frac{x^4}{4} + 2x^5 \end{aligned}$$

Mathematica [A] time = 0.0009675, size = 30, normalized size = 1.

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]

[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5

Maple [A] time = 0.043, size = 25, normalized size = 0.8

$$6x + \frac{7x^2}{2} + \frac{16x^3}{3} + \frac{x^4}{4} + 2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)*(5*x^2+3*x+2),x)

[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5

Maxima [A] time = 0.99311, size = 32, normalized size = 1.07

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")

[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x

Fricas [A] time = 0.71146, size = 59, normalized size = 1.97

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")

[Out] $2x^5 + 1/4x^4 + 16/3x^3 + 7/2x^2 + 6x$

Sympy [A] time = 0.062864, size = 26, normalized size = 0.87

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)`

[Out] $2x^{**5} + x^{**4}/4 + 16x^{**3}/3 + 7x^{**2}/2 + 6x$

Giac [A] time = 1.17373, size = 32, normalized size = 1.07

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")`

[Out] $2x^5 + 1/4x^4 + 16/3x^3 + 7/2x^2 + 6x$

$$3.19 \quad \int \frac{3-x+2x^2}{2+3x+5x^2} dx$$

Optimal. Leaf size=42

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x + 5*x^2])/50

Rubi [A] time = 0.0402874, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2), x]

[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x + 5*x^2])/50

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{3-x+2x^2}{2+3x+5x^2} dx &= \int \left(\frac{2}{5} + \frac{11(1-x)}{5(2+3x+5x^2)} \right) dx \\
 &= \frac{2x}{5} + \frac{11}{5} \int \frac{1-x}{2+3x+5x^2} dx \\
 &= \frac{2x}{5} - \frac{11}{50} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{143}{50} \int \frac{1}{2+3x+5x^2} dx \\
 &= \frac{2x}{5} - \frac{11}{50} \log(2+3x+5x^2) - \frac{143}{25} \text{Subst} \left(\int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
 &= \frac{2x}{5} + \frac{143 \tan^{-1} \left(\frac{3+10x}{\sqrt{31}} \right)}{25\sqrt{31}} - \frac{11}{50} \log(2+3x+5x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0154453, size = 42, normalized size = 1.

$$-\frac{11}{50} \log(5x^2 + 3x + 2) + \frac{2x}{5} + \frac{143 \tan^{-1} \left(\frac{10x+3}{\sqrt{31}} \right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2), x]
```


[Out] $(2x)/5 + (143 \cdot \text{ArcTan}[(3 + 10x)/\text{Sqrt}[31]])/(25 \cdot \text{Sqrt}[31]) - (11 \cdot \text{Log}[2 + 3x + 5x^2])/50$

Maple [A] time = 0.049, size = 34, normalized size = 0.8

$$\frac{2x}{5} - \frac{11 \ln(5x^2 + 3x + 2)}{50} + \frac{143\sqrt{31}}{775} \arctan\left(\frac{(3 + 10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)/(5*x^2+3*x+2), x)`

[Out] $2/5x - 11/50 \ln(5x^2 + 3x + 2) + 143/775 \arctan(1/31(3 + 10x) \cdot 31^{1/2}) \cdot 31^{1/2}$

Maxima [A] time = 1.4844, size = 45, normalized size = 1.07

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{2}{5}x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)/(5*x^2+3*x+2), x, algorithm="maxima")`

[Out] $143/775 \cdot \text{sqrt}(31) \cdot \arctan(1/31 \cdot \text{sqrt}(31) \cdot (10x + 3)) + 2/5x - 11/50 \cdot \log(5x^2 + 3x + 2)$

Fricas [A] time = 0.83428, size = 119, normalized size = 2.83

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{2}{5}x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)/(5*x^2+3*x+2), x, algorithm="fricas")`

[Out] $143/775 \cdot \text{sqrt}(31) \cdot \arctan(1/31 \cdot \text{sqrt}(31) \cdot (10x + 3)) + 2/5x - 11/50 \cdot \log(5x^2 + 3x + 2)$

Sympy [A] time = 0.15425, size = 49, normalized size = 1.17

$$\frac{2x}{5} - \frac{11 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{50} + \frac{143\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)

[Out] 2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/775

Giac [A] time = 1.16842, size = 45, normalized size = 1.07

$$\frac{143}{775} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{2}{5}x - \frac{11}{50} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")

[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2 + 3*x + 2)

$$3.20 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])

Rubi [A] time = 0.0267706, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2, x]

[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{3-x+2x^2}{(2+3x+5x^2)^2} dx &= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{1}{31} \int \frac{41}{2+3x+5x^2} dx \\ &= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{41}{31} \int \frac{1}{2+3x+5x^2} dx \\ &= \frac{11(7+13x)}{155(2+3x+5x^2)} - \frac{82}{31} \text{Subst} \left(\int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\ &= \frac{11(7+13x)}{155(2+3x+5x^2)} + \frac{82 \tan^{-1} \left(\frac{3+10x}{\sqrt{31}} \right)}{31\sqrt{31}} \end{aligned}$$

Mathematica [A] time = 0.0150616, size = 43, normalized size = 1.

$$\frac{11(13x+7)}{155(5x^2+3x+2)} + \frac{82 \tan^{-1} \left(\frac{10x+3}{\sqrt{31}} \right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])
```

Maple [A] time = 0.046, size = 34, normalized size = 0.8

$$\left(\frac{143x}{775} + \frac{77}{775}\right)\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)^{-1} + \frac{82\sqrt{31}}{961} \arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)

[Out] (143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A] time = 1.46585, size = 49, normalized size = 1.14

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{11(13x+7)}{155(5x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2 + 3*x + 2)

Fricas [A] time = 0.85103, size = 146, normalized size = 3.4

$$\frac{410\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+4433x+2387}{4805(5x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4433*x + 2387)/(5*x^2 + 3*x + 2)

Sympy [A] time = 0.213093, size = 42, normalized size = 0.98

$$\frac{143x + 77}{775x^2 + 465x + 310} + \frac{82\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)

[Out] (143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/961

Giac [A] time = 1.22253, size = 49, normalized size = 1.14

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{11(13x + 7)}{155(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2 + 3*x + 2)

$$3.21 \quad \int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])

Rubi [A] time = 0.0370305, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610(5x^2+3x+2)} + \frac{11(13x+7)}{310(5x^2+3x+2)^2} + \frac{1106 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3, x]

[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx &= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{553}{5(2+3x+5x^2)^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553}{310} \int \frac{1}{(2+3x+5x^2)^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{553}{961} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} - \frac{1106}{961} \text{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3+10x\right) \\
&= \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{1106 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

Mathematica [A] time = 0.0249325, size = 53, normalized size = 0.83

$$\frac{31(5530x^3+4977x^2+4094x+1141)}{(5x^2+3x+2)^2} + \frac{2212\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{59582}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3, x]

[Out] ((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212* $\sqrt{31}$ *ArcTan[(3 + 10*x)/ $\sqrt{31}$])/59582

Maple [A] time = 0.044, size = 47, normalized size = 0.7

$$25 \frac{1}{(5x^2 + 3x + 2)^2} \left(\frac{553x^3}{4805} + \frac{4977x^2}{48050} + \frac{2047x}{24025} + \frac{1141}{48050} \right) + \frac{1106\sqrt{31}}{29791} \arctan\left(\frac{(3 + 10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^3, x)

[Out] 25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+1106/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A] time = 1.47233, size = 76, normalized size = 1.19

$$\frac{1106}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3, x, algorithm="maxima")

[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

Fricas [A] time = 0.801471, size = 242, normalized size = 3.78

$$\frac{171430x^3 + 2212\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 154287x^2 + 126914x + 35371}{59582(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

Sympy [A] time = 0.280638, size = 63, normalized size = 0.98

$$\frac{5530x^3 + 4977x^2 + 4094x + 1141}{48050x^4 + 57660x^3 + 55738x^2 + 23064x + 7688} + \frac{1106\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)

[Out] (5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x**2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/29791

Giac [A] time = 1.25107, size = 62, normalized size = 0.97

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{5530x^3 + 4977x^2 + 4094x + 1141}{1922(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2

$$3.22 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=80

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3}$$

[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 + (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (875*x^12)/3 + (2500*x^13)/13

Rubi [A] time = 0.0599792, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]

[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 + (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (875*x^12)/3 + (2500*x^13)/13

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 dx &= \int (144 + 768x + 3016x^2 + 7352x^3 + 14801x^4 + 21542x^5 + 27763x^6 + 26520x^7 \\ &= 144x + 384x^2 + \frac{3016x^3}{3} + 1838x^4 + \frac{14801x^5}{5} + \frac{10771x^6}{3} + \frac{27763x^7}{7} + 3315x^8 \end{aligned}$$

Mathematica [A] time = 0.0029768, size = 80, normalized size = 1.

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} +$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]

[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 + (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (875*x^12)/3 + (2500*x^13)/13

Maple [A] time = 0.043, size = 65, normalized size = 0.8

$$144x + 384x^2 + \frac{3016x^3}{3} + 1838x^4 + \frac{14801x^5}{5} + \frac{10771x^6}{3} + \frac{27763x^7}{7} + 3315x^8 + \frac{24859x^9}{9} + 1571x^{10} + \frac{11525x^{11}}{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x)

[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13

Maxima [A] time = 0.956925, size = 86, normalized size = 1.08

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")

[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384*x^2 + 144*x

Fricas [A] time = 0.62232, size = 221, normalized size = 2.76

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384*x^2 + 144*x

Sympy [A] time = 0.133479, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)

[Out] 2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 + 3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016*x**3/3 + 384*x**2 + 144*x

Giac [A] time = 1.19878, size = 86, normalized size = 1.08

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384*x^2 + 144*x

$$3.23 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=66

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11

Rubi [A] time = 0.0481998, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]

[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^3 dx &= \int (72 + 276x + 914x^2 + 1615x^3 + 2693x^4 + 2694x^5 + 3108x^6 + 1863x^7 + 1865x^8 + 72x^9) dx \\ &= 72x + 138x^2 + \frac{914x^3}{3} + \frac{1615x^4}{4} + \frac{2693x^5}{5} + 449x^6 + 444x^7 + \frac{1863x^8}{8} + \frac{1865x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0021084, size = 66, normalized size = 1.

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]

[Out] $72x + 138x^2 + \frac{914x^3}{3} + \frac{1615x^4}{4} + \frac{2693x^5}{5} + 449x^6 + 444x^7 + \frac{1863x^8}{8} + \frac{1865x^9}{9} + 40x^{10} + \frac{500x^{11}}{11}$

Maple [A] time = 0.042, size = 55, normalized size = 0.8

$$72x + 138x^2 + \frac{914x^3}{3} + \frac{1615x^4}{4} + \frac{2693x^5}{5} + 449x^6 + 444x^7 + \frac{1863x^8}{8} + \frac{1865x^9}{9} + 40x^{10} + \frac{500x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x)

[Out] $72x+138x^2+914/3x^3+1615/4x^4+2693/5x^5+449x^6+444x^7+1863/8x^8+1865/9x^9+40x^{10}+500/11x^{11}$

Maxima [A] time = 0.964319, size = 73, normalized size = 1.11

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] $500/11x^{11} + 40x^{10} + 1865/9x^9 + 1863/8x^8 + 444x^7 + 449x^6 + 2693/5x^5 + 1615/4x^4 + 914/3x^3 + 138x^2 + 72x$

Fricas [A] time = 0.734116, size = 167, normalized size = 2.53

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x

Sympy [A] time = 0.137197, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)

[Out] 500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6 + 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x

Giac [A] time = 1.16031, size = 73, normalized size = 1.11

$$\frac{500}{11} x^{11} + 40x^{10} + \frac{1865}{9} x^9 + \frac{1863}{8} x^8 + 444x^7 + 449x^6 + \frac{2693}{5} x^5 + \frac{1615}{4} x^4 + \frac{914}{3} x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x

$$3.24 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=54

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 + (5*x^8)/2 + (100*x^9)/9

Rubi [A] time = 0.042492, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]

[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 + (5*x^8)/2 + (100*x^9)/9

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^2 dx &= \int (36 + 84x + 241x^2 + 236x^3 + 390x^4 + 172x^5 + 321x^6 + 20x^7 + 100x^8) dx \\ &= 36x + 42x^2 + \frac{241x^3}{3} + 59x^4 + 78x^5 + \frac{86x^6}{3} + \frac{321x^7}{7} + \frac{5x^8}{2} + \frac{100x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0017383, size = 54, normalized size = 1.

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]

[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 + (5*x^8)/2 + (100*x^9)/9

Maple [A] time = 0.044, size = 45, normalized size = 0.8

$$36x + 42x^2 + \frac{241x^3}{3} + 59x^4 + 78x^5 + \frac{86x^6}{3} + \frac{321x^7}{7} + \frac{5x^8}{2} + \frac{100x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x)

[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9

Maxima [A] time = 0.967055, size = 59, normalized size = 1.09

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 + 42*x^2 + 36*x

Fricas [A] time = 0.904191, size = 122, normalized size = 2.26

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] $100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 + 42*x^2 + 36*x$

Sympy [A] time = 0.108465, size = 51, normalized size = 0.94

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)`

[Out] $100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x**3/3 + 42*x**2 + 36*x$

Giac [A] time = 1.19494, size = 59, normalized size = 1.09

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")`

[Out] $100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 + 42*x^2 + 36*x$

$$3.25 \quad \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=46

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

[Out] $18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/7$

Rubi [A] time = 0.0296107, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2), x]

[Out] $18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/7$

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^2 (2 + 3x + 5x^2) dx &= \int (18 + 15x + 53x^2 + x^3 + 61x^4 - 8x^5 + 20x^6) dx \\ &= 18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.000904, size = 46, normalized size = 1.

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2), x]

[Out] $18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7}$

Maple [A] time = 0.045, size = 35, normalized size = 0.8

$$18x + \frac{15x^2}{2} + \frac{53x^3}{3} + \frac{x^4}{4} + \frac{61x^5}{5} - \frac{4x^6}{3} + \frac{20x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2), x)

[Out] $18x + 15/2x^2 + 53/3x^3 + 1/4x^4 + 61/5x^5 - 4/3x^6 + 20/7x^7$

Maxima [A] time = 0.964242, size = 46, normalized size = 1.

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2), x, algorithm="maxima")

[Out] $20/7x^7 - 4/3x^6 + 61/5x^5 + 1/4x^4 + 53/3x^3 + 15/2x^2 + 18x$

Fricas [A] time = 0.82888, size = 95, normalized size = 2.07

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2), x, algorithm="fricas")

[Out] $20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x$

Sympy [A] time = 0.097347, size = 41, normalized size = 0.89

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)`

[Out] $20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x$

Giac [A] time = 1.17085, size = 46, normalized size = 1.

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")`

[Out] $20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x$

$$3.26 \quad \int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx$$

Optimal. Leaf size=56

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250

Rubi [A] time = 0.0513899, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15} - \frac{16x^2}{25} - \frac{1573 \log(5x^2 + 3x + 2)}{1250} + \frac{381x}{125} + \frac{8349 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2), x]

[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^2}{2+3x+5x^2} dx &= \int \left(\frac{381}{125} - \frac{32x}{25} + \frac{4x^2}{5} + \frac{121(3-13x)}{125(2+3x+5x^2)} \right) dx \\
 &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{121}{125} \int \frac{3-13x}{2+3x+5x^2} dx \\
 &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573}{1250} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{8349}{1250} \int \frac{1}{2+3x+5x^2} dx \\
 &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573 \log(2+3x+5x^2)}{1250} - \frac{8349}{625} \text{Subst} \left(\int \frac{1}{-31-x^2} dx, x, 3+10x \right) \\
 &= \frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} + \frac{8349 \tan^{-1} \left(\frac{3+10x}{\sqrt{31}} \right)}{625\sqrt{31}} - \frac{1573 \log(2+3x+5x^2)}{1250}
 \end{aligned}$$

Mathematica [A] time = 0.01929, size = 53, normalized size = 0.95

$$\frac{10x(100x^2 - 240x + 1143) - 4719 \log(5x^2 + 3x + 2)}{3750} + \frac{8349 \tan^{-1} \left(\frac{10x+3}{\sqrt{31}} \right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2), x]
```


[Out] $(8349 \operatorname{ArcTan}[(3 + 10x)/\sqrt{31}]) / (625 \sqrt{31}) + (10x(1143 - 240x + 100x^2) - 4719 \operatorname{Log}[2 + 3x + 5x^2]) / 3750$

Maple [A] time = 0.044, size = 44, normalized size = 0.8

$$\frac{381x}{125} - \frac{16x^2}{25} + \frac{4x^3}{15} - \frac{1573 \ln(5x^2 + 3x + 2)}{1250} + \frac{8349 \sqrt{31}}{19375} \arctan\left(\frac{(3 + 10x) \sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^2/(5*x^2+3*x+2),x)`

[Out] $381/125*x - 16/25*x^2 + 4/15*x^3 - 1573/1250*\ln(5*x^2+3*x+2) + 8349/19375*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

Maxima [A] time = 1.48807, size = 58, normalized size = 1.04

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{381}{125}x - \frac{1573}{1250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] $4/15*x^3 - 16/25*x^2 + 8349/19375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 381/125*x - 1573/1250*\log(5*x^2 + 3*x + 2)$

Fricas [A] time = 0.993161, size = 165, normalized size = 2.95

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{381}{125}x - \frac{1573}{1250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")`

[Out] $4/15*x^3 - 16/25*x^2 + 8349/19375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3))$
 $+ 381/125*x - 1573/1250*\log(5*x^2 + 3*x + 2)$

Sympy [A] time = 0.188934, size = 63, normalized size = 1.12

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)`

[Out] $4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*\log(x**2 + 3*x/5 + 2/5)/1250 + 8349*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/19375$

Giac [A] time = 1.16036, size = 58, normalized size = 1.04

$$\frac{4}{15}x^3 - \frac{16}{25}x^2 + \frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{381}{125}x - \frac{1573}{1250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")`

[Out] $4/15*x^3 - 16/25*x^2 + 8349/19375*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3))$
 $+ 381/125*x - 1573/1250*\log(5*x^2 + 3*x + 2)$

$$3.27 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 + 10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125

Rubi [A] time = 0.0608414, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2) + \frac{4x}{25} + \frac{41932 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]

[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 + 10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq

, x] && IGtQ[p, -2]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx &= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{4032}{25} - \frac{992x}{25} + \frac{124x^2}{5}}{2+3x+5x^2} dx \\
&= \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{1}{31} \int \left(\frac{124}{25} + \frac{44(86-31x)}{25(2+3x+5x^2)} \right) dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{44}{775} \int \frac{86-31x}{2+3x+5x^2} dx \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{20966 \int \frac{1}{2+3x+5x^2} dx}{3875} \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} - \frac{22}{125} \log(2+3x+5x^2) - \frac{41932 \operatorname{Subst} \left(\int \frac{1}{-31-x^2} dx, x, 3+10x \right)}{3875} \\
&= \frac{4x}{25} + \frac{121(61+69x)}{3875(2+3x+5x^2)} + \frac{41932 \tan^{-1} \left(\frac{3+10x}{\sqrt{31}} \right)}{3875\sqrt{31}} - \frac{22}{125} \log(2+3x+5x^2)
\end{aligned}$$

Mathematica [A] time = 0.0308515, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2} - 21142 \log(5x^2+3x+2) + 19220x + 41932\sqrt{31} \tan^{-1} \left(\frac{10x+3}{\sqrt{31}} \right)}{120125}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]

[Out] (19220*x + (3751*(61 + 69*x)))/(2 + 3*x + 5*x^2) + 41932*sqrt[31]*ArcTan[(3 + 10*x)/sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125

Maple [A] time = 0.046, size = 51, normalized size = 0.8

$$\frac{4x}{25} - \frac{11}{25} \left(-\frac{759x}{775} - \frac{671}{775} \right) \left(x^2 + \frac{3x}{5} + \frac{2}{5} \right)^{-1} - \frac{22 \ln(5x^2+3x+2)}{125} + \frac{41932\sqrt{31}}{120125} \arctan \left(\frac{(3+10x)\sqrt{31}}{31} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)

[Out] $4/25*x-11/25*(-759/775*x-671/775)/(x^2+3/5*x+2/5)-22/125*\ln(5*x^2+3*x+2)+41932/120125*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

Maxima [A] time = 1.45998, size = 70, normalized size = 1.11

$$\frac{41932}{120125} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{4}{25}x + \frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125} \log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] $41932/120125*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 4/25*x + 121/3875*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*\log(5*x^2 + 3*x + 2)$

Fricas [A] time = 0.997507, size = 252, normalized size = 4.

$$\frac{96100x^3 + 41932\sqrt{31}(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 57660x^2 - 21142(5x^2 + 3x + 2)\log(5x^2 + 3x + 2)}{120125(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")`

[Out] $1/120125*(96100*x^3 + 41932*\sqrt{31}*(5*x^2 + 3*x + 2)*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*\log(5*x^2 + 3*x + 2) + 297259*x + 228811)/(5*x^2 + 3*x + 2)$

Sympy [A] time = 0.213602, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{8349x + 7381}{19375x^2 + 11625x + 7750} - \frac{22 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{125} + \frac{41932\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)`

[Out] $4x/25 + (8349x + 7381)/(19375x^2 + 11625x + 7750) - 22\log(x^2 + 3x/5 + 2/5)/125 + 41932\sqrt{31}\operatorname{atan}(10\sqrt{31}x/31 + 3\sqrt{31}/31)/120125$

Giac [A] time = 1.18491, size = 70, normalized size = 1.11

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{4}{25}x + \frac{121(69x+61)}{3875(5x^2+3x+2)} - \frac{22}{125}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")`

[Out] $41932/120125\sqrt{31}\operatorname{arctan}(1/31\sqrt{31}(10x+3)) + 4/25x + 121/3875(69x+61)/(5x^2+3x+2) - 22/125\log(5x^2+3x+2)$

$$3.28 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{121(69x+61)}{7750(5x^2+3x+2)^2} + \frac{11(45710x+17557)}{240250(5x^2+3x+2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(240250*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])

Rubi [A] time = 0.0516227, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1660, 12, 618, 204}

$$\frac{121(69x+61)}{7750(5x^2+3x+2)^2} + \frac{11(45710x+17557)}{240250(5x^2+3x+2)} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]

[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(240250*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{48669}{125} - \frac{1984x}{25} + \frac{248x^2}{5}}{(2+3x+5x^2)^2} dx \\
 &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{\int \frac{4330}{2+3x+5x^2} dx}{1922} \\
 &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{2165}{961} \int \frac{1}{2+3x+5x^2} dx \\
 &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} - \frac{4330}{961} \text{Subst}\left(\int \frac{1}{-31-x^2} dx, x, 3+10x\right) \\
 &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{4330 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
 \end{aligned}$$

Mathematica [A] time = 0.0233833, size = 53, normalized size = 0.83

$$\frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2} + \frac{4330 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3, x]

[Out] $(11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2) + (4330*\text{ArcTan}[(3 + 10*x)/\text{Sqrt}[31]])/(961*\text{Sqrt}[31])$

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$25 \frac{1}{(5x^2 + 3x + 2)^2} \left(\frac{50281x^3}{120125} + \frac{494813x^2}{1201250} + \frac{184382x}{600625} + \frac{123013}{1201250} \right) + \frac{4330\sqrt{31}}{29791} \arctan\left(\frac{(3 + 10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)`

[Out] $25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5*x^2+3*x+2)^2+4330/29791*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

Maxima [A] time = 1.4875, size = 76, normalized size = 1.19

$$\frac{4330}{29791} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] $4330/29791*\text{sqrt}(31)*\arctan(1/31*\text{sqrt}(31)*(10*x + 3)) + 11/48050*(45710*x^3 + 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)$

Fricas [A] time = 0.948494, size = 258, normalized size = 4.03

$$\frac{15587110x^3 + 216500\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 15339203x^2 + 11431684x + 3}{1489550(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")`

[Out] $\frac{1}{1489550} \cdot (15587110x^3 + 216500\sqrt{31} \cdot (25x^4 + 30x^3 + 29x^2 + 12x + 4) \cdot \arctan(1/31 \cdot \sqrt{31} \cdot (10x + 3))) + 15339203x^2 + 11431684x + 3813403$
 $)/(25x^4 + 30x^3 + 29x^2 + 12x + 4)$

Sympy [A] time = 0.298147, size = 63, normalized size = 0.98

$$\frac{502810x^3 + 494813x^2 + 368764x + 123013}{1201250x^4 + 1441500x^3 + 1393450x^2 + 576600x + 192200} + \frac{4330\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)`

[Out] $(502810x^3 + 494813x^2 + 368764x + 123013)/(1201250x^4 + 1441500x^3 + 1393450x^2 + 576600x + 192200) + 4330\sqrt{31} \cdot \operatorname{atan}(10\sqrt{31}x/31 + 3\sqrt{31}/31)/29791$

Giac [A] time = 1.23913, size = 62, normalized size = 0.97

$$\frac{4330}{29791} \sqrt{31} \operatorname{arctan}\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{11(45710x^3 + 44983x^2 + 33524x + 11183)}{48050(5x^2 + 3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")`

[Out] $4330/29791 \cdot \sqrt{31} \cdot \operatorname{arctan}(1/31 \cdot \sqrt{31} \cdot (10x + 3)) + 11/48050 \cdot (45710x^3 + 44983x^2 + 33524x + 11183)/(5x^2 + 3x + 2)^2$

$$3.29 \quad \int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx$$

Optimal. Leaf size=85

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(120125*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])

Rubi [A] time = 0.0633293, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955(5x^2+3x+2)} + \frac{11(12060x+4579)}{120125(5x^2+3x+2)^2} + \frac{121(69x+61)}{11625(5x^2+3x+2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4, x]

[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(120125*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx &= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{1}{93} \int \frac{\frac{77178}{125} - \frac{2976x}{25} + \frac{372x^2}{5}}{(2+3x+5x^2)^3} dx \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{\int \frac{100128}{5(2+3x+5x^2)^2} dx}{5766} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688 \int \frac{1}{(2+3x+5x^2)^2} dx}{4805} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{33376 \int \frac{1}{2+3x+5x^2}}{29791} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} - \frac{66752 \operatorname{Subst}\left(\int \frac{1}{2+3x+5x^2}\right)}{29791} \\
&= \frac{121(61+69x)}{11625(2+3x+5x^2)^3} + \frac{11(4579+12060x)}{120125(2+3x+5x^2)^2} + \frac{16688(3+10x)}{148955(2+3x+5x^2)} + \frac{66752 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}
\end{aligned}$$

Mathematica [A] time = 0.0449281, size = 63, normalized size = 0.74

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3} + \frac{66752 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4, x]

[Out] (1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 12516000*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])

Maple [A] time = 0.05, size = 57, normalized size = 0.7

$$125 \frac{1}{(5x^2 + 3x + 2)^3} \left(\frac{33376x^5}{148955} + \frac{50064x^4}{148955} + \frac{7163932x^3}{18619375} + \frac{4260199x^2}{18619375} + \frac{1891636x}{18619375} + \frac{1259239}{55858125} \right) + \frac{66752\sqrt{31}}{923521} \operatorname{arctan}\left(\frac{3+10x}{\sqrt{31}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x)`

[Out] $125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/18619375*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$

Maxima [A] time = 1.44632, size = 103, normalized size = 1.21

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")`

[Out] $66752/923521*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 1/446865*(12516000*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)$

Fricas [A] time = 0.963056, size = 371, normalized size = 4.36

$$\frac{387996000x^5 + 581994000x^4 + 666245676x^3 + 1001280\sqrt{31}(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}{13852815(125x^6 + 225x^5 + 285x^4 + 207x^3 + 114x^2 + 36x + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")`

[Out] $1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*\sqrt{31}*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)$

Sympy [A] time = 0.251072, size = 83, normalized size = 0.98

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{55858125x^6 + 100544625x^5 + 127356525x^4 + 92501055x^3 + 50942610x^2 + 16087140x + 3574920} + \frac{66752\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{923521}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)

[Out] (12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x + 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/923521

Giac [A] time = 1.18208, size = 76, normalized size = 0.89

$$\frac{66752}{923521} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{446865(5x^2 + 3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(5*x^2 + 3*x + 2)^3

$$3.30 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=96

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6}$$

[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3

Rubi [A] time = 0.0705267, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]

[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^4 dx &= \int (432 + 2160x + 8568x^2 + 20576x^3 + 43083x^4 + 64529x^5 + 91349x^6 + 94881x^7 + 94881x^8 + 91349x^9 + 64529x^{10} + 43083x^{11} + 20576x^{12} + 2160x^{13} + 432x^{14}) dx \\ &= 432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083x^5}{5} + \frac{64529x^6}{6} + \frac{91349x^7}{7} + \frac{94881x^8}{8} \end{aligned}$$

Mathematica [A] time = 0.0029899, size = 96, normalized size = 1.

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} +$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]

[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3

Maple [A] time = 0.044, size = 75, normalized size = 0.8

$$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083x^5}{5} + \frac{64529x^6}{6} + \frac{91349x^7}{7} + \frac{94881x^8}{8} + \frac{103583x^9}{9} + \frac{75311x^{10}}{10} + \frac{68583x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x)

[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+2250/7*x^14+1000/3*x^15

Maxima [A] time = 0.983455, size = 100, normalized size = 1.04

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 + 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

Fricas [A] time = 0.823481, size = 274, normalized size = 2.85

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 + 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

Sympy [A] time = 0.126187, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)

[Out] 1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**11/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 64529*x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x

Giac [A] time = 1.17693, size = 100, normalized size = 1.04

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 + 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

$$3.31 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=82

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2$$

[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11 + 25*x^12 + (1000*x^13)/13

Rubi [A] time = 0.0555126, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]

[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11 + 25*x^12 + (1000*x^13)/13

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^3 dx &= \int (216 + 756x + 2610x^2 + 4483x^3 + 8292x^4 + 8619x^5 + 12016x^6 + 7869x^7 + 990x^8 + 3316x^9 + 3061x^{10} + 4830x^{11} + 1000x^{12} + 1000x^{13}) dx \\ &= 216x + 378x^2 + 870x^3 + \frac{4483x^4}{4} + \frac{8292x^5}{5} + \frac{2873x^6}{2} + \frac{12016x^7}{7} + \frac{7869x^8}{8} + \frac{3316x^9}{3} + \frac{3061x^{10}}{10} + \frac{4830x^{11}}{11} + \frac{1000x^{12}}{12} + \frac{1000x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.0018334, size = 82, normalized size = 1.

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]

[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11 + 25*x^12 + (1000*x^13)/13

Maple [A] time = 0.045, size = 65, normalized size = 0.8

$$216x + 378x^2 + 870x^3 + \frac{4483x^4}{4} + \frac{8292x^5}{5} + \frac{2873x^6}{2} + \frac{12016x^7}{7} + \frac{7869x^8}{8} + \frac{3316x^9}{3} + \frac{3061x^{10}}{10} + \frac{4830x^{11}}{11} + 25x^{12} + \frac{1000x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x)

[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13

Maxima [A] time = 0.969923, size = 86, normalized size = 1.05

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^2 + 216*x

Fricas [A] time = 0.83181, size = 217, normalized size = 2.65

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^2 + 216*x

Sympy [A] time = 0.084503, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)

[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 7869*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x**3 + 378*x**2 + 216*x

Giac [A] time = 1.13975, size = 86, normalized size = 1.05

$$\frac{1000}{13}x^{13} + 25x^{12} + \frac{4830}{11}x^{11} + \frac{3061}{10}x^{10} + \frac{3316}{3}x^9 + \frac{7869}{8}x^8 + \frac{12016}{7}x^7 + \frac{2873}{2}x^6 + \frac{8292}{5}x^5 + \frac{4483}{4}x^4 + 870x^3 + 378x^2 + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^2 + 216*x

$$3.32 \quad \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=68

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (1571*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11

Rubi [A] time = 0.0505835, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {1657}

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]

[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (1571*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2)^2 dx &= \int (108 + 216x + 711x^2 + 635x^3 + 1416x^4 + 598x^5 + 1571x^6 + 83x^7 + 922x^8 - \\ &= 108x + 108x^2 + 237x^3 + \frac{635x^4}{4} + \frac{1416x^5}{5} + \frac{299x^6}{3} + \frac{1571x^7}{7} + \frac{83x^8}{8} + \frac{922x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0022274, size = 68, normalized size = 1.

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]

[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (1571*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11

Maple [A] time = 0.043, size = 55, normalized size = 0.8

$$108x + 108x^2 + 237x^3 + \frac{635x^4}{4} + \frac{1416x^5}{5} + \frac{299x^6}{3} + \frac{1571x^7}{7} + \frac{83x^8}{8} + \frac{922x^9}{9} - 6x^{10} + \frac{200x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x)

[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+922/9*x^9-6*x^10+200/11*x^11

Maxima [A] time = 0.956996, size = 73, normalized size = 1.07

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x

Fricas [A] time = 0.850299, size = 166, normalized size = 2.44

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x

Sympy [A] time = 0.085544, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)

[Out] 200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3 + 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x

Giac [A] time = 1.19404, size = 73, normalized size = 1.07

$$\frac{200}{11} x^{11} - 6x^{10} + \frac{922}{9} x^9 + \frac{83}{8} x^8 + \frac{1571}{7} x^7 + \frac{299}{3} x^6 + \frac{1416}{5} x^5 + \frac{635}{4} x^4 + 237 x^3 + 108 x^2 + 108 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x

3.33 $\int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx$

Optimal. Leaf size=56

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

[Out] $54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9$

Rubi [A] time = 0.0318655, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1657}

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2), x]

[Out] $54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9$

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 - x + 2x^2)^3 (2 + 3x + 5x^2) dx &= \int (54 + 27x + 180x^2 - 20x^3 + 288x^4 - 83x^5 + 190x^6 - 36x^7 + 40x^8) dx \\ &= 54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0012394, size = 56, normalized size = 1.

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2), x]

[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7 - (9*x^8)/2 + (40*x^9)/9

Maple [A] time = 0.043, size = 45, normalized size = 0.8

$$54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2), x)

[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9

Maxima [A] time = 0.967082, size = 59, normalized size = 1.05

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2), x, algorithm="maxima")

[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x

Fricas [A] time = 0.882358, size = 122, normalized size = 2.18

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2), x, algorithm="fricas")

[Out] $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

Sympy [A] time = 0.107796, size = 53, normalized size = 0.95

$$\frac{40x^9}{9} - \frac{9x^8}{2} + \frac{190x^7}{7} - \frac{83x^6}{6} + \frac{288x^5}{5} - 5x^4 + 60x^3 + \frac{27x^2}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)`

[Out] $40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x**3 + 27*x**2/2 + 54*x$

Giac [A] time = 1.20353, size = 59, normalized size = 1.05

$$\frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")`

[Out] $40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27/2*x^2 + 54*x$

$$3.34 \quad \int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx$$

Optimal. Leaf size=70

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/25 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 + 3*x + 5*x^2])/31250

Rubi [A] time = 0.0529201, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} - \frac{158389 \log(5x^2 + 3x + 2)}{31250} + \frac{49508x}{3125} + \frac{328757 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2), x]

[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/25 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 + 3*x + 5*x^2])/31250

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx &= \int \left(\frac{49508}{3125} - \frac{7451x}{625} + \frac{1222x^2}{125} - \frac{84x^3}{25} + \frac{8x^4}{5} - \frac{1331(11+119x)}{3125(2+3x+5x^2)} \right) dx \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{1331 \int \frac{11+119x}{2+3x+5x^2} dx}{3125} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \int \frac{3+10x}{2+3x+5x^2} dx}{31250} + \frac{328757 \int \frac{1}{2+3x+5x^2} dx}{31250} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \log(2+3x+5x^2)}{31250} - \frac{328757 \operatorname{Subst}\left(\int \frac{1}{-31x-1562} dx\right)}{1562} \\ &= \frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} + \frac{328757 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}} - \frac{158389 \log(2+3x+5x^2)}{31250} \end{aligned}$$

Mathematica [A] time = 0.0224648, size = 63, normalized size = 0.9

$$\frac{31(5x(6000x^4 - 15750x^3 + 61100x^2 - 111765x + 297048) - 475167 \log(5x^2 + 3x + 2)) + 1972542\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{2906250}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2), x]

[Out] (1972542*sqrt[31]*ArcTan[(3 + 10*x)/sqrt[31]] + 31*(5*x*(297048 - 111765*x + 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250

Maple [A] time = 0.046, size = 54, normalized size = 0.8

$$\frac{49508x}{3125} - \frac{7451x^2}{1250} + \frac{1222x^3}{375} - \frac{21x^4}{25} + \frac{8x^5}{25} - \frac{158389 \ln(5x^2 + 3x + 2)}{31250} + \frac{328757\sqrt{31}}{484375} \arctan\left(\frac{(3 + 10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2), x)

[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A] time = 1.46638, size = 72, normalized size = 1.03

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{49508}{3125}x - \frac{158389}{31250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2), x, algorithm="maxima")

[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2 + 3*x + 2)

Fricas [A] time = 0.991387, size = 219, normalized size = 3.13

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + \frac{49508}{3125}x - \frac{158389}{31250} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] $\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2 + 3x + 2)$

Sympy [A] time = 0.149317, size = 76, normalized size = 1.09

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)

[Out] $8x^{**5}/25 - 21x^{**4}/25 + 1222x^{**3}/375 - 7451x^{**2}/1250 + 49508x/3125 - 158389*\log(x^{**2} + 3*x/5 + 2/5)/31250 + 328757*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/484375$

Giac [A] time = 1.187, size = 72, normalized size = 1.03

$$\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")

[Out] $\frac{8}{25}x^5 - \frac{21}{25}x^4 + \frac{1222}{375}x^3 - \frac{7451}{1250}x^2 + \frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{49508}{3125}x - \frac{158389}{31250}\log(5x^2 + 3x + 2)$

$$3.35 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=77

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769 \log(5x^2 + 3x + 2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 + 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (10769*Log[2 + 3*x + 5*x^2])/6250

Rubi [A] time = 0.0722482, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x + 443)}{96875(5x^2 + 3x + 2)} - \frac{10769 \log(5x^2 + 3x + 2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]

[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 + 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (10769*Log[2 + 3*x + 5*x^2])/6250

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx &= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \frac{\frac{372701}{625} - \frac{230981x}{625} + \frac{37882x^2}{125} - \frac{2604x^3}{25} + \frac{248x^4}{5}}{2+3x+5x^2} dx \\
&= \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{1}{31} \int \left(\frac{45446}{625} - \frac{3348x}{125} + \frac{248x^2}{25} + \frac{121(2329-2759x)}{625(2+3x+5x^2)} \right) dx \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{121 \int \frac{2329-2759x}{2+3x+5x^2} dx}{19375} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \int \frac{3+10x}{2+3x+5x^2} dx}{6250} + \frac{3819607 \int \frac{1}{2+3x+5x^2} dx}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} - \frac{10769 \log(2+3x+5x^2)}{6250} - \frac{3819607 \operatorname{Subst}\left(\frac{1}{2+3x+5x^2}, x, \frac{3+10x}{\sqrt{31}}\right)}{193750} \\
&= \frac{1466x}{625} - \frac{54x^2}{125} + \frac{8x^3}{75} + \frac{1331(443+247x)}{96875(2+3x+5x^2)} + \frac{3819607 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}} - \frac{10769 \log(2+3x+5x^2)}{6250}
\end{aligned}$$

Mathematica [A] time = 0.026893, size = 77, normalized size = 1.

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769 \log(5x^2+3x+2)}{6250} + \frac{1466x}{625} + \frac{3819607 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2, x]

[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 + 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (10769*Log[2 + 3*x + 5*x^2])/6250

Maple [A] time = 0.046, size = 61, normalized size = 0.8

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} - \frac{121}{625} \left(-\frac{2717x}{775} - \frac{4873}{775} \right) \left(x^2 + \frac{3x}{5} + \frac{2}{5} \right)^{-1} - \frac{10769 \ln(5x^2+3x+2)}{6250} + \frac{3819607\sqrt{31}}{3003125} \arctan\left(\frac{3+10x}{\sqrt{31}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)

[Out] $\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{1466}{625}x - \frac{121}{625} \cdot \frac{(-2717/775x - 4873/775)}{(x^2 + 3/5x + 2/5)} - \frac{10769}{6250} \ln(5x^2 + 3x + 2) + \frac{3819607}{3003125} \arctan\left(\frac{1}{31} \sqrt{31}(3 + 10x)\right) \cdot 31^{1/2}$

Maxima [A] time = 1.47863, size = 84, normalized size = 1.09

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769}{6250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] $\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331}{96875} \cdot \frac{(247x+443)}{(5x^2+3x+2)} - \frac{10769}{6250} \log(5x^2+3x+2)$

Fricas [A] time = 1.03024, size = 321, normalized size = 4.17

$$\frac{9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31}(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 111226140x^2 - 27(5x^2 + 3x + 2)\log(5x^2 + 3x + 2) + 145678362x + 109671738}{18018750(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] $\frac{1}{18018750} \cdot (9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31}(5x^2 + 3x + 2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 111226140x^2 - 27(5x^2 + 3x + 2)\log(5x^2 + 3x + 2) + 145678362x + 109671738)$

Sympy [A] time = 0.17165, size = 78, normalized size = 1.01

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x + 589633}{484375x^2 + 290625x + 193750} - \frac{10769 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)

[Out] $8x^3/75 - 54x^2/125 + 1466x/625 + (328757x + 589633)/(484375x^2 + 290625x + 193750) - 10769\log(x^2 + 3x/5 + 2/5)/6250 + 3819607\sqrt{31}\arctan(10\sqrt{31}x/31 + 3\sqrt{31}/31)/3003125$

Giac [A] time = 1.20751, size = 84, normalized size = 1.09

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769}{6250}\log(5x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] $8/75x^3 - 54/125x^2 + 3819607/3003125\sqrt{31}\arctan(1/31\sqrt{31}(10x+3)) + 1466/625x + 1331/96875(247x+443)/(5x^2+3x+2) - 10769/6250\log(5x^2+3x+2)$

$$3.36 \quad \int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=84

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(188381 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sqrt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625

Rubi [A] time = 0.0865293, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x + 188381)}{6006250(5x^2 + 3x + 2)} + \frac{1331(247x + 443)}{193750(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2) + \frac{8x}{125} + \frac{11341176 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3, x]

[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(188381 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sqrt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} dx &= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{1}{62} \int \frac{\frac{4055767}{3125} - \frac{461962x}{625} + \frac{75764x^2}{125} - \frac{5208x^3}{25} + \frac{496x^4}{5}}{(2+3x+5x^2)^2} dx \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \frac{\frac{2222876}{125} - \frac{207576x}{125} + \frac{15376x^2}{25}}{2+3x+5x^2} dx}{1922} \\
&= \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{\int \left(\frac{15376}{125} + \frac{132(16607-1922x)}{125(2+3x+5x^2)} \right) dx}{1922} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{66 \int \frac{16607-1922x}{2+3x+5x^2} dx}{120125} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \int \frac{3+10x}{2+3x+5x^2} dx + \frac{567058}{625} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} - \frac{66}{625} \log(2+3x+5x^2) - \frac{11341176}{625} \\
&= \frac{8x}{125} + \frac{1331(443+247x)}{193750(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250(2+3x+5x^2)} + \frac{11341176 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}} - \frac{66}{625} \log
\end{aligned}$$

Mathematica [A] time = 0.037298, size = 78, normalized size = 0.93

$$\frac{3751(342840x+188381)}{5x^2+3x+2} + \frac{1279091(247x+443)}{(5x^2+3x+2)^2} - 19662060 \log(5x^2+3x+2) + 11916400x + 113411760\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)$$

186193750

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]

[Out] (11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381 + 342840*x))/(2 + 3*x + 5*x^2) + 113411760*sqrt[31]*ArcTan[(3 + 10*x)/sqrt[31]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750

Maple [A] time = 0.048, size = 63, normalized size = 0.8

$$\frac{8x}{125} - \frac{11}{5(5x^2 + 3x + 2)^2} \left(-\frac{377124x^3}{24025} - \frac{866987x^2}{48050} - \frac{293711x}{24025} - \frac{232243}{48050} \right) - \frac{66 \ln(5x^2 + 3x + 2)}{625} + \frac{11341176\sqrt{31}}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)

[Out] 8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/48050)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)

Maxima [A] time = 1.44198, size = 97, normalized size = 1.15

$$\frac{11341176}{18619375} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{8}{125} x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(25x^4 + 30x^3 + 29x^2 + 12x + 4)} - \frac{66}{625} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)

Fricas [A] time = 1.01191, size = 406, normalized size = 4.83

$$\frac{59582000x^5 + 71498400x^4 + 1355107960x^3 + 22682352\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right)}{37238750(25x^4 + 30x^3 + 29x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) +

$$1506812195x^2 - 3932412(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(5x^2 + 3x + 2) + 1011087630x + 395974315)/(25x^4 + 30x^3 + 29x^2 + 12x + 4)$$

Sympy [A] time = 0.216081, size = 85, normalized size = 1.01

$$\frac{8x}{125} + \frac{8296728x^3 + 9536857x^2 + 6461642x + 2554673}{6006250x^4 + 7207500x^3 + 6967250x^2 + 2883000x + 961000} - \frac{66 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}}{31}\right)}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)

[Out] 8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**4 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/5 + 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18619375

Giac [A] time = 1.21183, size = 84, normalized size = 1.

$$\frac{11341176}{18619375} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{8}{125} x + \frac{121(68568x^3 + 78817x^2 + 53402x + 21113)}{240250(5x^2 + 3x + 2)^2} - \frac{66}{625} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 66/625*log(5*x^2 + 3*x + 2)

$$3.37 \quad \int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$$

Optimal. Leaf size=84

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}}{256\sqrt{23}}$$

[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512

Rubi [A] time = 0.056764, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{307461}{512} \log(2x^2 - x + 3) + \frac{122691x}{128} + \frac{1156639 \tan^{-1}}{256\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2), x]

[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^4}{3 - x + 2x^2} dx &= \int \left(\frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(25 - 21x)}{128(3 - x + 2x^2)} \right) dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128} \int \frac{25 - 21x}{3 - x + 2x^2} dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \int \frac{-1 + 4x}{3 - x + 2x^2} dx \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512} \log(3 - x + 2x^2) \\ &= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}} \end{aligned}$$

Mathematica [A] time = 0.0268885, size = 72, normalized size = 0.86

$$\frac{x(120000x^6 + 406000x^5 + 623280x^4 + 262290x^3 - 594412x^2 - 603687x + 2576511)}{2688} + \frac{307461}{512} \log(2x^2 - x + 3) - \frac{1156639}{256\sqrt{23}} \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2), x]
```

[Out] $(x(2576511 - 603687x - 594412x^2 + 262290x^3 + 623280x^4 + 406000x^5 + 120000x^6))/2688 - (1156639 \operatorname{ArcTan}[-1 + 4x]/\sqrt{23}]/(256\sqrt{23}) + (307461 \operatorname{Log}[3 - x + 2x^2])/512$

Maple [A] time = 0.046, size = 64, normalized size = 0.8

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461 \ln(2x^2 - x + 3)}{512} - \frac{1156639\sqrt{23}}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((5x^2+3x+2)^4/(2x^2-x+3), x)$

[Out] $625/14x^7+3625/24x^6+1855/8x^5+6245/64x^4-21229/96x^3-28747/128x^2+122691/128x+307461/512*\ln(2x^2-x+3)-1156639/5888*23^{(1/2)}*\arctan(1/23*(-1+4x)*23^{(1/2)})$

Maxima [A] time = 1.43462, size = 85, normalized size = 1.01

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((5x^2+3x+2)^4/(2x^2-x+3), x, \operatorname{algorithm}="maxima")$

[Out] $625/14x^7 + 3625/24x^6 + 1855/8x^5 + 6245/64x^4 - 21229/96x^3 - 28747/128x^2 - 1156639/5888*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4x - 1)) + 122691/128x + 307461/512*\log(2x^2 - x + 3)$

Fricas [A] time = 0.975225, size = 252, normalized size = 3.

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/128*x + 307461/512*log(2*x^2 - x + 3)

Sympy [A] time = 0.156462, size = 87, normalized size = 1.04

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{512} - \frac{1156639\sqrt{23} \operatorname{atan}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)

[Out] 625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 28747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888

Giac [A] time = 1.25004, size = 85, normalized size = 1.01

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23} \operatorname{arctan}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x + \frac{307461}{512}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/128*x + 307461/512*log(2*x^2 - x + 3)

$$3.38 \quad \int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$$

Optimal. Leaf size=70

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

[Out] $(-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/128$

Rubi [A] time = 0.0560196, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} + \frac{1331}{128} \log(2x^2 - x + 3) - \frac{4795x}{32} - \frac{59895 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2), x]

[Out] $(-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/128$

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^3}{3 - x + 2x^2} dx &= \int \left(-\frac{4795}{32} - \frac{829x}{16} + \frac{965x^2}{8} + \frac{575x^3}{4} + \frac{125x^4}{2} + \frac{1331(11 + x)}{32(3 - x + 2x^2)} \right) dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{32} \int \frac{11 + x}{3 - x + 2x^2} dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \int \frac{-1 + 4x}{3 - x + 2x^2} dx + \frac{59895}{128} \int \frac{1}{3 - x + 2x^2} dx \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} + \frac{1331}{128} \log(3 - x + 2x^2) - \frac{59895}{64} \operatorname{Subst} \left(\int \frac{1}{-23 - 4x} dx \right) \\ &= -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} - \frac{59895 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{64\sqrt{23}} + \frac{1331}{128} \log(3 - x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.0210563, size = 63, normalized size = 0.9

$$\frac{1}{384} \left(4x(1200x^4 + 3450x^3 + 3860x^2 - 2487x - 14385) + 3993 \log(2x^2 - x + 3) \right) + \frac{59895 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2), x]
```


[Out] $(59895 \cdot \text{ArcTan}[-1 + 4x] / \sqrt{23}) / (64 \cdot \sqrt{23}) + (4x \cdot (-14385 - 2487x + 3860x^2 + 3450x^3 + 1200x^4) + 3993 \cdot \text{Log}[3 - x + 2x^2]) / 384$

Maple [A] time = 0.047, size = 54, normalized size = 0.8

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{1331 \ln(2x^2 - x + 3)}{128} + \frac{59895\sqrt{23}}{1472} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^3/(2*x^2-x+3),x)`

[Out] $25/2 \cdot x^5 + 575/16 \cdot x^4 + 965/24 \cdot x^3 - 829/32 \cdot x^2 - 4795/32 \cdot x + 1331/128 \cdot \ln(2x^2 - x + 3) + 59895/1472 \cdot 23^{(1/2)} \cdot \arctan(1/23 \cdot (-1 + 4x) \cdot 23^{(1/2)})$

Maxima [A] time = 1.47194, size = 72, normalized size = 1.03

$$\frac{25}{2} x^5 + \frac{575}{16} x^4 + \frac{965}{24} x^3 - \frac{829}{32} x^2 + \frac{59895}{1472} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{4795}{32} x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")`

[Out] $25/2 \cdot x^5 + 575/16 \cdot x^4 + 965/24 \cdot x^3 - 829/32 \cdot x^2 + 59895/1472 \cdot \text{sqrt}(23) \cdot \arctan(1/23 \cdot \text{sqrt}(23) \cdot (4x - 1)) - 4795/32 \cdot x + 1331/128 \cdot \log(2x^2 - x + 3)$

Fricas [A] time = 0.935331, size = 196, normalized size = 2.8

$$\frac{25}{2} x^5 + \frac{575}{16} x^4 + \frac{965}{24} x^3 - \frac{829}{32} x^2 + \frac{59895}{1472} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{4795}{32} x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")`

[Out] $25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 4795/32*x + 1331/128*\log(2*x^2 - x + 3)$

Sympy [A] time = 0.143687, size = 73, normalized size = 1.04

$$\frac{25x^5}{2} + \frac{575x^4}{16} + \frac{965x^3}{24} - \frac{829x^2}{32} - \frac{4795x}{32} + \frac{1331 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{59895\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)

[Out] $25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*\log(x**2 - x/2 + 3/2)/128 + 59895*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/1472$

Giac [A] time = 1.16731, size = 72, normalized size = 1.03

$$\frac{25}{2}x^5 + \frac{575}{16}x^4 + \frac{965}{24}x^3 - \frac{829}{32}x^2 + \frac{59895}{1472}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4795}{32}x + \frac{1331}{128} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")

[Out] $25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 4795/32*x + 1331/128*\log(2*x^2 - x + 3)$

$$3.39 \quad \int \frac{(2+3x+5x^2)^2}{3-x+2x^2} dx$$

Optimal. Leaf size=56

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[23]) - (363*Log[3 - x + 2*x^2])/32

Rubi [A] time = 0.0503026, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6} + \frac{85x^2}{8} - \frac{363}{32} \log(2x^2 - x + 3) + \frac{51x}{8} + \frac{847 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2), x]

[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[23]) - (363*Log[3 - x + 2*x^2])/32

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^2}{3 - x + 2x^2} dx &= \int \left(\frac{51}{8} + \frac{85x}{4} + \frac{25x^2}{2} - \frac{121(1 + 3x)}{8(3 - x + 2x^2)} \right) dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{121}{8} \int \frac{1 + 3x}{3 - x + 2x^2} dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \int \frac{-1 + 4x}{3 - x + 2x^2} dx - \frac{847}{32} \int \frac{1}{3 - x + 2x^2} dx \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} - \frac{363}{32} \log(3 - x + 2x^2) + \frac{847}{16} \text{Subst} \left(\int \frac{1}{-23 - x^2} dx, x, -1 + 4x \right) \\ &= \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} + \frac{847 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{16\sqrt{23}} - \frac{363}{32} \log(3 - x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.0168558, size = 52, normalized size = 0.93

$$\frac{1}{24}x(100x^2 + 255x + 153) - \frac{363}{32} \log(2x^2 - x + 3) - \frac{847 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2), x]
```

[Out] $(x*(153 + 255*x + 100*x^2))/24 - (847*\text{ArcTan}[(-1 + 4*x)/\text{Sqrt}[23]])/(16*\text{Sqrt}[23]) - (363*\text{Log}[3 - x + 2*x^2])/32$

Maple [A] time = 0.049, size = 44, normalized size = 0.8

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363 \ln(2x^2 - x + 3)}{32} - \frac{847\sqrt{23}}{368} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2/(2*x^2-x+3),x)`

[Out] $25/6*x^3+85/8*x^2+51/8*x-363/32*\ln(2*x^2-x+3)-847/368*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.43979, size = 58, normalized size = 1.04

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")`

[Out] $25/6*x^3 + 85/8*x^2 - 847/368*\text{sqrt}(23)*\arctan(1/23*\text{sqrt}(23)*(4*x - 1)) + 51/8*x - 363/32*\log(2*x^2 - x + 3)$

Fricas [A] time = 0.993869, size = 147, normalized size = 2.62

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")`

[Out] $25/6*x^3 + 85/8*x^2 - 847/368*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 51/8*x - 363/32*\log(2*x^2 - x + 3)$

Sympy [A] time = 0.197072, size = 60, normalized size = 1.07

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{847\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)`

[Out] $25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*\log(x**2 - x/2 + 3/2)/32 - 847*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/368$

Giac [A] time = 1.12605, size = 58, normalized size = 1.04

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")`

[Out] $25/6*x^3 + 85/8*x^2 - 847/368*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 51/8*x - 363/32*\log(2*x^2 - x + 3)$

$$3.40 \quad \int \frac{2+3x+5x^2}{3-x+2x^2} dx$$

Optimal. Leaf size=42

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*x^2])/8

Rubi [A] time = 0.0348584, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} + \frac{33 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2), x]

[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*x^2])/8

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{2+3x+5x^2}{3-x+2x^2} dx &= \int \left(\frac{5}{2} - \frac{11(1-x)}{2(3-x+2x^2)} \right) dx \\
 &= \frac{5x}{2} - \frac{11}{2} \int \frac{1-x}{3-x+2x^2} dx \\
 &= \frac{5x}{2} + \frac{11}{8} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{33}{8} \int \frac{1}{3-x+2x^2} dx \\
 &= \frac{5x}{2} + \frac{11}{8} \log(3-x+2x^2) + \frac{33}{4} \text{Subst} \left(\int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
 &= \frac{5x}{2} + \frac{33 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{4\sqrt{23}} + \frac{11}{8} \log(3-x+2x^2)
 \end{aligned}$$

Mathematica [A] time = 0.009824, size = 42, normalized size = 1.

$$\frac{11}{8} \log(2x^2 - x + 3) + \frac{5x}{2} - \frac{33 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2), x]
```


[Out] $(5x)/2 - (33 \operatorname{ArcTan}[(-1 + 4x)/\sqrt{23}])/(4\sqrt{23}) + (11 \operatorname{Log}[3 - x + 2x^2])/8$

Maple [A] time = 0.044, size = 34, normalized size = 0.8

$$\frac{5x}{2} + \frac{11 \ln(2x^2 - x + 3)}{8} - \frac{33\sqrt{23}}{92} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)/(2*x^2-x+3), x)`

[Out] $5/2*x + 11/8*\ln(2*x^2 - x + 3) - 33/92*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.43148, size = 45, normalized size = 1.07

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3), x, algorithm="maxima")`

[Out] $-33/92*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 5/2*x + 11/8*\log(2*x^2 - x + 3)$

Fricas [A] time = 1.05056, size = 112, normalized size = 2.67

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8}\log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3), x, algorithm="fricas")`

[Out] $-33/92*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 5/2*x + 11/8*\log(2*x^2 - x + 3)$

Sympy [A] time = 0.123831, size = 46, normalized size = 1.1

$$\frac{5x}{2} + \frac{11 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{33\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)

[Out] 5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/92

Giac [A] time = 1.19791, size = 45, normalized size = 1.07

$$-\frac{33}{92} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{5}{2}x + \frac{11}{8} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")

[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 - x + 3)

$$3.41 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$$

Optimal. Leaf size=73

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44

Rubi [A] time = 0.0533335, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) + \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]

[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44

Rule 980

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx &= \frac{1}{242} \int \frac{-11-22x}{3-x+2x^2} dx + \frac{1}{242} \int \frac{88+55x}{2+3x+5x^2} dx \\
 &= -\left(\frac{1}{44} \int \frac{-1+4x}{3-x+2x^2} dx\right) + \frac{1}{44} \int \frac{3+10x}{2+3x+5x^2} dx - \frac{3}{44} \int \frac{1}{3-x+2x^2} dx + \frac{13}{44} \int \frac{1}{2+3x+5x^2} dx \\
 &= -\frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2) + \frac{3}{22} \text{Subst}\left(\int \frac{1}{-23-x^2} dx, x, -1\right) \\
 &= \frac{3 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44} \log(3-x+2x^2) + \frac{1}{44} \log(2+3x+5x^2)
 \end{aligned}$$

Mathematica [A] time = 0.0307099, size = 73, normalized size = 1.

$$-\frac{1}{44} \log(2x^2 - x + 3) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{3 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)), x]`

[Out] $(-3 \operatorname{ArcTan}[-1 + 4x]/\operatorname{Sqrt}[23])/(22 \operatorname{Sqrt}[23]) + (13 \operatorname{ArcTan}[(3 + 10x)/\operatorname{Sqrt}[31]])/(22 \operatorname{Sqrt}[31]) - \operatorname{Log}[3 - x + 2x^2]/44 + \operatorname{Log}[2 + 3x + 5x^2]/44$

Maple [A] time = 0.048, size = 60, normalized size = 0.8

$$\frac{\ln(5x^2 + 3x + 2)}{44} + \frac{13\sqrt{31}}{682} \arctan\left(\frac{(3 + 10x)\sqrt{31}}{31}\right) - \frac{\ln(2x^2 - x + 3)}{44} - \frac{3\sqrt{23}}{506} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x)`

[Out] $1/44 \ln(5x^2 + 3x + 2) + 13/682 \arctan(1/31(3 + 10x) \cdot 31^{1/2}) \cdot 31^{1/2} - 1/44 \ln(2x^2 - x + 3) - 3/506 \cdot 23^{1/2} \arctan(1/23(-1 + 4x) \cdot 23^{1/2})$

Maxima [A] time = 1.42659, size = 80, normalized size = 1.1

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] $13/682 \sqrt{31} \arctan(1/31 \sqrt{31} (10x + 3)) - 3/506 \sqrt{23} \arctan(1/23 \sqrt{23} (4x - 1)) + 1/44 \log(5x^2 + 3x + 2) - 1/44 \log(2x^2 - x + 3)$

Fricas [A] time = 1.00017, size = 207, normalized size = 2.84

$$\frac{13}{682} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{3}{506} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{1}{44} \log(5x^2 + 3x + 2) - \frac{1}{44} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")`

[Out] $13/682*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 3/506*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/44*\log(5*x^2 + 3*x + 2) - 1/44*\log(2*x^2 - x + 3)$
)

Sympy [A] time = 0.267898, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{44} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{44} - \frac{3\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{506} + \frac{13\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)`

[Out] $-\log(x^2 - x/2 + 3/2)/44 + \log(x^2 + 3*x/5 + 2/5)/44 - 3*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/506 + 13*\sqrt{31}*\operatorname{atan}(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/682$

Giac [A] time = 1.26467, size = 80, normalized size = 1.1

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{1}{44}\log(5x^2+3x+2) - \frac{1}{44}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")`

[Out] $13/682*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 3/506*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/44*\log(5*x^2 + 3*x + 2) - 1/44*\log(2*x^2 - x + 3)$
)

$$3.42 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=94

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sqrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 - x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968

Rubi [A] time = 0.0886149, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2), x]

[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sqrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 - x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(

$2*p + 2*q + 5)*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(\text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1]) \&\& !\text{IGtQ}[q, 0]$

Rule 1072

$\text{Int}[(A_.) + (B_.)*(x_) + (C_.)*(x_)^2]/((a_) + (b_.)*(x_) + (c_.)*(x_)^2) * ((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol] \rightarrow \text{With}\{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2\}, \text{Dist}[1/q, \text{Int}[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + \text{Dist}[1/q, \text{Int}[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; \text{NeQ}[q, 0]] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[(d_) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} dx &= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{-1804+1397x-1430x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{7502} \\
&= \frac{4+65x}{682(2+3x+5x^2)} - \frac{\int \frac{18755-22506x}{3-x+2x^2} dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2} dx}{1815484} \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{3}{968} \int \frac{3+10x}{2+3x+5x^2} dx - \frac{7}{968} \int \frac{1}{2+3x+5x^2} dx \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{3}{968} \log(3-x+2x^2) - \frac{3}{968} \log(2+3x+5x^2) + \frac{7}{484} \operatorname{Su} \\
&= \frac{4+65x}{682(2+3x+5x^2)} + \frac{7 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968} \log(3-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0779902, size = 94, normalized size = 1.

$$\frac{65x+4}{682(5x^2+3x+2)} + \frac{3}{968} \log(2x^2-x+3) - \frac{3}{968} \log(5x^2+3x+2) - \frac{7 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)*(2+3*x+5*x^2)^2),x]

[Out] (4+65*x)/(682*(2+3*x+5*x^2)) - (7*ArcTan[(-1+4*x)/Sqrt[23]])/(484*Sqrt[23]) + (2891*ArcTan[(3+10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3-x+2*x^2])/968 - (3*Log[2+3*x+5*x^2])/968

Maple [A] time = 0.049, size = 77, normalized size = 0.8

$$-\frac{1}{484} \left(-\frac{286x}{31} - \frac{88}{155} \right) \left(x^2 + \frac{3x}{5} + \frac{2}{5} \right)^{-1} - \frac{3 \ln(5x^2+3x+2)}{968} + \frac{2891 \sqrt{31}}{465124} \arctan\left(\frac{(3+10x)\sqrt{31}}{31} \right) + \frac{3 \ln(2x^2-x+3)}{968}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x)

[Out] $-1/484*(-286/31*x-88/155)/(x^2+3/5*x+2/5)-3/968*\ln(5*x^2+3*x+2)+2891/465124$
 $*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}+3/968*\ln(2*x^2-x+3)-7/11132*23^{(1/2)}$
 $2)*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.48301, size = 105, normalized size = 1.12

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{65x + 4}{682(5x^2 + 3x + 2)} - \frac{3}{968} \log(5x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] $2891/465124*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 7/11132*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*$
 $\log(5*x^2 + 3*x + 2) + 3/968*\log(2*x^2 - x + 3)$

Fricas [A] time = 0.984673, size = 377, normalized size = 4.01

$$\frac{132986 \sqrt{31}(5x^2 + 3x + 2) \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - 13454 \sqrt{23}(5x^2 + 3x + 2) \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - 66309(5x^2 - x + 3) + 2039180x + 125488}{21395704(5x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")`

[Out] $1/21395704*(132986*\sqrt{31}*(5*x^2 + 3*x + 2)*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 13454*\sqrt{23}*(5*x^2 + 3*x + 2)*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 66309*(5*x^2 + 3*x + 2)*\log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*\log(2*x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)$

Sympy [A] time = 0.297665, size = 102, normalized size = 1.09

$$\frac{65x + 4}{3410x^2 + 2046x + 1364} + \frac{3 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} - \frac{3 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{7\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{11132} + \frac{2891\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{\sqrt{31}}{31}\right)}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)

[Out] (65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*log(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124

Giac [A] time = 1.22338, size = 105, normalized size = 1.12

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{65x + 4}{682(5x^2 + 3x + 2)} - \frac{3}{968} \log(5x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*log(5*x^2 - x + 3)

$$3.43 \quad \int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=115

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10232728\sqrt{31}}$$

[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x + 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTan[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Log[2 + 3*x + 5*x^2]/21296

Rubi [A] time = 0.124178, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364(5x^2+3x+2)^2} + \frac{21605x+7923}{465124(5x^2+3x+2)} - \frac{\log(2x^2-x+3)}{21296} + \frac{\log(5x^2+3x+2)}{21296} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10232728\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3), x]

[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x + 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTan[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Log[2 + 3*x + 5*x^2]/21296

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
)*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} dx &= \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-10456820x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{112560008} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2} dx}{27239521936} - \frac{\int \frac{-1}{2+3x+5x^2} dx}{102327} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2} dx}{21296} + \frac{\int \frac{3+10x}{2+3x+5x^2} dx}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \frac{\log(2+3x+5x^2)}{21296} \\
&= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{102327}
\end{aligned}$$

Mathematica [A] time = 0.143564, size = 104, normalized size = 0.9

$$\frac{31 \left(\frac{44(108025x^3+104430x^2+89144x+17210)}{(5x^2+3x+2)^2} - 961 \log(2x^2-x+3) + 961 \log(5x^2+3x+2) \right) + 1695586\sqrt{31} \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{634429136} +$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)*(2+3*x+5*x^2)^3),x]

[Out] (45*ArcTan[(-1+4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcTan[(10*x+3)/Sqrt[31]] + 31*((44*(17210+89144*x+104430*x^2+108025*x^3))/(2+3*x+5*x^2)^2 - 961*Log[3-x+2*x^2] + 961*Log[2+3*x+5*x^2]))/634429136

Maple [A] time = 0.049, size = 89, normalized size = 0.8

$$\frac{25}{10648(5x^2+3x+2)^2} \left(\frac{95062x^3}{961} + \frac{459492x^2}{4805} + \frac{1961168x}{24025} + \frac{75724}{4805} \right) + \frac{\ln(5x^2+3x+2)}{21296} + \frac{847793\sqrt{31}}{317214568} \arctan\left(\frac{10x+3}{\sqrt{31}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x)`

[Out] $25/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+3*x+2)^2+1/21296*\ln(5*x^2+3*x+2)+847793/317214568*\arctan(1/31*(3+10*x))*31^{(1/2)}*31^{(1/2)}-1/21296*\ln(2*x^2-x+3)+45/244904*23^{(1/2)}*\arctan(1/23*(-1+4*x))*23^{(1/2)}$

Maxima [A] time = 1.43887, size = 132, normalized size = 1.15

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{108025x^3 + 104430x^2 + 89144x + 17210}{465124(25x^4 + 30x^3 + 29x^2 + 12x + 4)} + \frac{1}{21296} \log(5x^2 + 3x + 2) - \frac{1}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] $847793/317214568*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 45/244904*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 + 89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*\log(5*x^2 + 3*x + 2) - 1/21296*\log(2*x^2 - x + 3)$

Fricas [A] time = 1.0267, size = 578, normalized size = 5.03

$$3388960300x^3 + 38998478\sqrt{31}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 2681190\sqrt{23}(25x^4 + 30x^3 + 29x^2 + 12x + 4)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 3276177960x^2 + 685193(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(5x^2 + 3x + 2) - 685193(25x^4 + 30x^3 + 29x^2 + 12x + 4)\log(2x^2 - x + 3) + 2796625568x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")`

[Out] $1/14591870128*(3388960300*x^3 + 38998478*\sqrt{31}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 2681190*\sqrt{23}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 3276177960*x^2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\log(5*x^2 + 3*x + 2) - 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\log(2*x^2 - x + 3) + 2796625568*x$

$$+ 539912120)/(25x^4 + 30x^3 + 29x^2 + 12x + 4)$$

Sympy [A] time = 0.399983, size = 119, normalized size = 1.03

$$\frac{108025x^3 + 104430x^2 + 89144x + 17210}{11628100x^4 + 13953720x^3 + 13488596x^2 + 5581488x + 1860496} - \frac{\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{45\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x - \sqrt{23}}{23}\right)}{244904} + \frac{847793\sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x + 3\sqrt{31}}{31}\right)}{31721456}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)

[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + log(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456

Giac [A] time = 1.25983, size = 119, normalized size = 1.03

$$\frac{847793}{317214568} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{45}{244904} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{108025x^3 + 104430x^2 + 89144x + 17210}{465124(5x^2 + 3x + 2)} - \frac{1}{21296} \log(5x^2 + 3x + 2) - \frac{1}{21296} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 + 89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/21296*log(2*x^2 - x + 3)

$$3.44 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=91

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4 - (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128

Rubi [A] time = 0.0863451, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} - \frac{13292697 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]

[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4 - (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2} dx &= -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23} \int \frac{\frac{832627}{64} - \frac{661181x}{64} - \frac{488267x^2}{32} + \frac{143635x^3}{16} + \frac{213325x^4}{8} + \frac{83375x^5}{4} + \frac{14375x^6}{2}}{3-x+2x^2} \\
&= -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23} \int \left(-\frac{2055257}{64} - \frac{27255x}{4} + \frac{224825x^2}{16} + \frac{48875x^3}{4} + \frac{14375x^4}{4} + \frac{1331}{32} \right) \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1331}{736} \int \frac{2629-529x}{3-x+2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128} \int \frac{-1+4x}{3-x+2x^2} \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128} \log(3-x+2x^2) \\
&= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{13292697 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.0499269, size = 91, normalized size = 1.

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128} \log(2x^2-x+3) - \frac{89359x}{64} + \frac{13292697 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]

[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4 - (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*x)/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128

Maple [A] time = 0.048, size = 71, normalized size = 0.8

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} - \frac{1331}{64} \left(\frac{869x}{92} + \frac{1111}{92} \right) \left(x^2 - \frac{x}{2} + \frac{3}{2} \right)^{-1} - \frac{30613 \ln(2x^2-x+3)}{128} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x)

[Out] $125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x-1331/64*(869/92*x+111/92)/(x^2-1/2*x+3/2)-30613/128*\ln(2*x^2-x+3)+13292697/33856*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.42549, size = 97, normalized size = 1.07

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")

[Out] $125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2*x^2 - x + 3) - 30613/128*\log(2*x^2 - x + 3)$

Fricas [A] time = 0.9736, size = 348, normalized size = 3.82

$$\frac{12696000x^7 + 47610000x^6 + 74800600x^5 - 20609840x^4 - 413058012x^3 + 79756182\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 193356906x^2 - 48582831(2x^2 - x + 3)\log(2x^2 - x + 3) - 930684489x - 102033129}{203136(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")

[Out] $1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 413058012*x^3 + 79756182*\sqrt{23}*(2*x^2 - x + 3)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*\log(2*x^2 - x + 3) - 930684489*x - 102033129)/(2*x^2 - x + 3)$

Sympy [A] time = 0.245003, size = 88, normalized size = 0.97

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} - \frac{1156639x + 1478741}{5888x^2 - 2944x + 8832} - \frac{30613 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23}}{33856} \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)

[Out] $125*x^{5/4} + 2125*x^{4/16} + 9775*x^{3/48} - 1185*x^{2/8} - 89359*x/64 - (1156639*x + 1478741)/(5888*x^{**2} - 2944*x + 8832) - 30613*\log(x^{**2} - x/2 + 3/2)/128 + 13292697*\sqrt{23}*atan(4*\sqrt{23}*x/23 - \sqrt{23}/23)/33856$

Giac [A] time = 1.1539, size = 97, normalized size = 1.07

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")

[Out] $125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*\sqrt{23})*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2*x^2 - x + 3) - 30613/128*\log(2*x^2 - x + 3)$

$$3.45 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=77

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x + 2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3 - x + 2*x^2])/32

Rubi [A] time = 0.0725653, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12} + \frac{175x^2}{4} - \frac{1331(17-45x)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} + \frac{223971 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]

[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x + 2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3 - x + 2*x^2])/32

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} dx &= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23} \int \frac{-\frac{25195}{16} - \frac{19067x}{16} + \frac{22195x^2}{8} + \frac{13225x^3}{4} + \frac{2875x^4}{2}}{3-x+2x^2} dx \\
&= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23} \int \left(\frac{21045}{16} + \frac{4025x}{2} + \frac{2875x^2}{4} - \frac{121(365+391x)}{8(3-x+2x^2)} \right) dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{121}{184} \int \frac{365+391x}{3-x+2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{223971}{736} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32} \log(3-x+2x^2) + \frac{223971}{368} \text{Subst} \left(\int \frac{1}{-x^2+x+3} dx \right) \\
&= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{223971 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{368\sqrt{23}} - \frac{2057}{32} \log(3-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0274481, size = 77, normalized size = 1.

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32} \log(2x^2-x+3) + \frac{915x}{16} - \frac{223971 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]

[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x + 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3 - x + 2*x^2])/32

Maple [A] time = 0.047, size = 61, normalized size = 0.8

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} - \frac{121}{16} \left(-\frac{495x}{92} + \frac{187}{92} \right) \left(x^2 - \frac{x}{2} + \frac{3}{2} \right)^{-1} - \frac{2057 \ln(2x^2 - x + 3)}{32} - \frac{223971 \sqrt{23}}{8464} \arctan \left(\frac{-1 - 4x}{\sqrt{23}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)

[Out] 125/12*x^3+175/4*x^2+915/16*x-121/16*(-495/92*x+187/92)/(x^2-1/2*x+3/2)-2057/32*ln(2*x^2-x+3)-223971/8464*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.43452, size = 84, normalized size = 1.09

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{915}{16}x + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")

[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 - x + 3)

Fricas [A] time = 1.03317, size = 292, normalized size = 3.79

$$\frac{1058000x^5 + 3914600x^4 + 5173620x^3 - 1343826\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 3761190x^2 - 3264459}{50784(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)

Sympy [A] time = 0.254475, size = 75, normalized size = 0.97

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)

[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x + 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/8464

Giac [A] time = 1.18254, size = 84, normalized size = 1.09

$$\frac{125}{12}x^3 + \frac{175}{4}x^2 - \frac{223971}{8464}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{915}{16}x + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")

[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 - x + 3)

$$3.46 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8

Rubi [A] time = 0.0629492, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} + \frac{1859 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]

[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8

Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

, x] && IGtQ[p, -2]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx &= \frac{121(19-7x)}{184(3-x+2x^2)} + \frac{1}{23} \int \frac{\frac{163}{4} + \frac{1955x}{4} + \frac{575x^2}{2}}{3-x+2x^2} dx \\
&= \frac{121(19-7x)}{184(3-x+2x^2)} + \frac{1}{23} \int \left(\frac{575}{4} - \frac{11(71-115x)}{2(3-x+2x^2)} \right) dx \\
&= \frac{25x}{4} + \frac{121(19-7x)}{184(3-x+2x^2)} - \frac{11}{46} \int \frac{71-115x}{3-x+2x^2} dx \\
&= \frac{25x}{4} + \frac{121(19-7x)}{184(3-x+2x^2)} + \frac{55}{8} \int \frac{-1+4x}{3-x+2x^2} dx - \frac{1859}{184} \int \frac{1}{3-x+2x^2} dx \\
&= \frac{25x}{4} + \frac{121(19-7x)}{184(3-x+2x^2)} + \frac{55}{8} \log(3-x+2x^2) + \frac{1859}{92} \text{Subst} \left(\int \frac{1}{-23-x^2} dx, x, -1+4x \right) \\
&= \frac{25x}{4} + \frac{121(19-7x)}{184(3-x+2x^2)} + \frac{1859 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{92\sqrt{23}} + \frac{55}{8} \log(3-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0301396, size = 63, normalized size = 1.

$$-\frac{121(7x-19)}{184(2x^2-x+3)} + \frac{55}{8} \log(2x^2-x+3) + \frac{25x}{4} - \frac{1859 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]

[Out] (25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*x)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8

Maple [A] time = 0.045, size = 51, normalized size = 0.8

$$\frac{25x}{4} + \frac{11}{4} \left(-\frac{77x}{92} + \frac{209}{92} \right) \left(x^2 - \frac{x}{2} + \frac{3}{2} \right)^{-1} + \frac{55 \ln(2x^2 - x + 3)}{8} - \frac{1859\sqrt{23}}{2116} \arctan \left(\frac{(-1 + 4x)\sqrt{23}}{23} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x)

[Out] $25/4*x+11/4*(-77/92*x+209/92)/(x^2-1/2*x+3/2)+55/8*\ln(2*x^2-x+3)-1859/2116*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.42417, size = 70, normalized size = 1.11

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x-\frac{121(7x-19)}{184(2x^2-x+3)}+\frac{55}{8}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")`

[Out] $-1859/2116*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 25/4*x - 121/184*(7*x - 19)/(2*x^2 - x + 3) + 55/8*\log(2*x^2 - x + 3)$

Fricas [A] time = 1.02616, size = 234, normalized size = 3.71

$$\frac{52900x^3 - 3718\sqrt{23}(2x^2 - x + 3)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 26450x^2 + 29095(2x^2 - x + 3)\log(2x^2 - x + 3) + 592877}{4232(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")`

[Out] $1/4232*(52900*x^3 - 3718*\sqrt{23}*(2*x^2 - x + 3)*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*\log(2*x^2 - x + 3) + 592877)/(2*x^2 - x + 3)$

Sympy [A] time = 0.170856, size = 61, normalized size = 0.97

$$\frac{25x}{4} - \frac{847x - 2299}{368x^2 - 184x + 552} + \frac{55\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{8} - \frac{1859\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)`

[Out] $25x/4 - (847x - 2299)/(368x^2 - 184x + 552) + 55\log(x^2 - x/2 + 3/2)/8 - 1859\sqrt{23}\operatorname{atan}(4\sqrt{23}x/23 - \sqrt{23}/23)/2116$

Giac [A] time = 1.18982, size = 70, normalized size = 1.11

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{25}{4}x - \frac{121(7x-19)}{184(2x^2-x+3)} + \frac{55}{8}\log(2x^2-x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")`

[Out] $-1859/2116\sqrt{23}\arctan(1/23\sqrt{23}(4x-1)) + 25/4x - 121/184(7x-19)/(2x^2-x+3) + 55/8\log(2x^2-x+3)$

$$3.47 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

[Out] $(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*Sqrt[23])$

Rubi [A] time = 0.0260438, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46(2x^2-x+3)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2, x]

[Out] $(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*Sqrt[23])$

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{2+3x+5x^2}{(3-x+2x^2)^2} dx &= -\frac{11(5+3x)}{46(3-x+2x^2)} + \frac{1}{23} \int \frac{41}{3-x+2x^2} dx \\
 &= -\frac{11(5+3x)}{46(3-x+2x^2)} + \frac{41}{23} \int \frac{1}{3-x+2x^2} dx \\
 &= -\frac{11(5+3x)}{46(3-x+2x^2)} - \frac{82}{23} \text{Subst}\left(\int \frac{1}{-23-x^2} dx, x, -1+4x\right) \\
 &= -\frac{11(5+3x)}{46(3-x+2x^2)} - \frac{82 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}
 \end{aligned}$$

Mathematica [A] time = 0.0143532, size = 43, normalized size = 1.

$$\frac{82 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}} - \frac{11(3x+5)}{46(2x^2-x+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2, x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23*Sqrt[23])
```

Maple [A] time = 0.047, size = 34, normalized size = 0.8

$$\left(-\frac{33x}{92} - \frac{55}{92}\right)\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)^{-1} + \frac{82\sqrt{23}}{529} \arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)

[Out] (-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.4319, size = 49, normalized size = 1.14

$$\frac{82}{529} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) - \frac{11(3x+5)}{46(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")

[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 - x + 3)

Fricas [A] time = 0.988498, size = 138, normalized size = 3.21

$$\frac{164\sqrt{23}(2x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - 759x - 1265}{1058(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")

[Out] 1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759*x - 1265)/(2*x^2 - x + 3)

Sympy [A] time = 0.205915, size = 41, normalized size = 0.95

$$-\frac{33x + 55}{92x^2 - 46x + 138} + \frac{82\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)

[Out] -(33*x + 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/529

Giac [A] time = 1.13908, size = 49, normalized size = 1.14

$$\frac{82}{529} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{11(3x + 5)}{46(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")

[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 - x + 3)

$$3.48 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx$$

Optimal. Leaf size=94

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968

Rubi [A] time = 0.0884198, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)), x]

[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(

$2*p + 2*q + 5)*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(\text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1]) \&\& !\text{IGtQ}[q, 0]$

Rule 1072

$\text{Int}[\frac{(A_.) + (B_.)*(x_) + (C_.)*(x_)^2}{((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)}, x_Symbol] \rightarrow \text{With}[\{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2\}, \text{Dist}[1/q, \text{Int}[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + \text{Dist}[1/q, \text{Int}[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; \text{NeQ}[q, 0]] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_)}{(a_) + (b_.)*(x_) + (c_.)*(x_)^2}, x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[\frac{((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}}{x_Symbol}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{((a_) + (b_.)*(x_)^2)^{-1}}{x_Symbol}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_) + (e_.)*(x_)}{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2}, x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)} dx &= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-1892-1067x+330x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{5566} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{\int \frac{-3509+72358x}{3-x+2x^2} dx}{1346972} - \frac{\int \frac{-150282-180895x}{2+3x+5x^2} dx}{1346972} \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{241 \int \frac{1}{3-x+2x^2} dx}{22264} - \frac{13}{968} \int \frac{-1+4x}{3-x+2x^2} dx + \frac{13}{968} \int \frac{3+10x}{2+3x+5x^2} dx \\
&= \frac{13-6x}{506(3-x+2x^2)} - \frac{13}{968} \log(3-x+2x^2) + \frac{13}{968} \log(2+3x+5x^2) + \frac{241 \operatorname{Subst}(\int \frac{1}{3-x+2x^2} dx, x, \frac{1-\sqrt{23}x}{2})}{11132\sqrt{23}} \\
&= \frac{13-6x}{506(3-x+2x^2)} + \frac{241 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968} \log(3-x+2x^2) + \frac{13}{968} \log(2+3x+5x^2)
\end{aligned}$$

Mathematica [A] time = 0.0552043, size = 94, normalized size = 1.

$$\frac{13-6x}{506(2x^2-x+3)} - \frac{13}{968} \log(2x^2-x+3) + \frac{13}{968} \log(5x^2+3x+2) - \frac{241 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69 \tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)), x]

[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(11132*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968

Maple [A] time = 0.049, size = 77, normalized size = 0.8

$$\frac{13 \ln(5x^2 + 3x + 2)}{968} + \frac{69 \sqrt{31}}{15004} \arctan\left(\frac{(3 + 10x) \sqrt{31}}{31}\right) - \frac{1}{484} \left(\frac{66x}{23} - \frac{143}{23}\right) \left(x^2 - \frac{x}{2} + \frac{3}{2}\right)^{-1} - \frac{13 \ln(2x^2 - x + 3)}{968}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2), x)

[Out] $13/968*\ln(5*x^2+3*x+2)+69/15004*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}-1/4$
 $84*(66/23*x-143/23)/(x^2-1/2*x+3/2)-13/968*\ln(2*x^2-x+3)-241/256036*23^{(1/2)}$
 $)*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.44483, size = 105, normalized size = 1.12

$$\frac{69}{15004} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) - \frac{241}{256036} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) - \frac{6x-13}{506(2x^2-x+3)} + \frac{13}{968} \log(5x^2 + 3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] $69/15004*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 241/256036*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*\log(5*x^2 + 3*x + 2) - 13/968*\log(2*x^2 - x + 3)$

Fricas [A] time = 1.00196, size = 363, normalized size = 3.86

$$\frac{73002 \sqrt{31}(2x^2 - x + 3) \arctan\left(\frac{1}{31} \sqrt{31}(10x+3)\right) - 14942 \sqrt{23}(2x^2 - x + 3) \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + 213187(2x^2 - x + 3) \log(5x^2 + 3x + 2) - 213187(2x^2 - x + 3) \log(2x^2 - x + 3) - 188232x + 407836}{15874232(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")`

[Out] $1/15874232*(73002*\sqrt{31}*(2*x^2 - x + 3)*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 14942*\sqrt{23}*(2*x^2 - x + 3)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 213187*(2*x^2 - x + 3)*\log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*\log(2*x^2 - x + 3) - 188232*x + 407836)/(2*x^2 - x + 3)$

Sympy [A] time = 0.306932, size = 102, normalized size = 1.09

$$-\frac{6x-13}{1012x^2-506x+1518} - \frac{13 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{968} + \frac{13 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{968} - \frac{241 \sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{256036} + \frac{69 \sqrt{31} \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{\sqrt{31}}{31}\right)}{15004}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)

[Out] $-(6x - 13)/(1012x^2 - 506x + 1518) - 13\log(x^2 - x/2 + 3/2)/968 + 13\log(x^2 + 3x/5 + 2/5)/968 - 241\sqrt{23}\operatorname{atan}(4\sqrt{23}x/23 - \sqrt{23}/23)/256036 + 69\sqrt{31}\operatorname{atan}(10\sqrt{31}x/31 + 3\sqrt{31}/31)/15004$

Giac [A] time = 1.13347, size = 105, normalized size = 1.12

$$\frac{69}{15004} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{241}{256036} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{6x - 13}{506(2x^2 - x + 3)} + \frac{13}{968} \log(5x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")

[Out] $69/15004\sqrt{31}\operatorname{arctan}(1/31\sqrt{31}(10x + 3)) - 241/256036\sqrt{23}\operatorname{arctan}(1/23\sqrt{23}(4x - 1)) - 1/506(6x - 13)/(2x^2 - x + 3) + 13/968\log(5x^2 - x + 3)$

$$3.49 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=127

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}} + \frac{12643 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{165044\sqrt{31}} - \frac{19 \log(3-x+2x^2)}{10648} + \frac{19 \log(2+3x+5x^2)}{10648}$$

[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[23]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648

Rubi [A] time = 0.122755, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)} + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} + \frac{19 \log(2x^2-x+3)}{10648} - \frac{19 \log(5x^2+3x+2)}{10648} + \frac{2769 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}} + \frac{12643 \tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{165044\sqrt{31}} - \frac{19 \log(3-x+2x^2)}{10648} + \frac{19 \log(2+3x+5x^2)}{10648}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2), x]

[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[23]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +

```

b*c*(c*d - 3*a*f)*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-2321-2299x+990x^2}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{5566} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{-3034196+46548x}{(3-x+2x^2)(2+3x+5x^2)^2} dx}{41750} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{132282766-7212x}{3-x+2x^2} dx}{101049839} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \int \frac{-1+4x}{3-x+2x^2} dx}{10648} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{19 \log(3-x+2x^2)}{10648} \\
&= -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{2769 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.057922, size = 106, normalized size = 0.83

$$\frac{31372(6850x^3-9275x^2+11154x-4342)}{10x^4+x^3+16x^2+7x+6} + 9659011 \log(2x^2-x+3) - 9659011 \log(5x^2+3x+2) - 5322018\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right) + \frac{5413113112}{5413113112}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^2*(2+3*x+5*x^2)^2),x]

[Out] ((31372*(-4342+11154*x-9275*x^2+6850*x^3))/(6+7*x+16*x^2+x^3+10*x^4) - 5322018*sqrt(23)*ArcTan[(-1+4*x)/sqrt(23)] + 13376294*sqrt(31)*ArcTan[(3+10*x)/sqrt(31)] + 9659011*Log[3-x+2*x^2] - 9659011*Log[2+3*x+5*x^2])/5413113112

Maple [A] time = 0.052, size = 94, normalized size = 0.7

$$-\frac{1}{5324} \left(-\frac{759x}{31} + \frac{1078}{155} \right) \left(x^2 + \frac{3x}{5} + \frac{2}{5} \right)^{-1} - \frac{19 \ln(5x^2+3x+2)}{10648} + \frac{12643\sqrt{31}}{5116364} \arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right) + \frac{1}{5324} \left(-\frac{7}{155} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)`

[Out]
$$-1/5324*(-759/31*x+1078/155)/(x^2+3/5*x+2/5)-19/10648*\ln(5*x^2+3*x+2)+12643/5116364*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}+1/5324*(-77/23*x-341/46)/(x^2-1/2*x+3/2)+19/10648*\ln(2*x^2-x+3)-2769/2816396*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$$

Maxima [A] time = 1.44659, size = 130, normalized size = 1.02

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)} - 19/10648 \log(5x^2 + 3x + 2) + 19/10648 \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out]
$$12643/5116364*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 2769/2816396*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*\log(5*x^2 + 3*x + 2) + 19/10648*\log(2*x^2 - x + 3)$$

Fricas [A] time = 0.991225, size = 548, normalized size = 4.31

$$214898200x^3 + 13376294\sqrt{31}(10x^4 + x^3 + 16x^2 + 7x + 6)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) - 5322018\sqrt{23}(10x^4 + x^3 + 16x^2 + 7x + 6)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 290975300x^2 - 9659011(10x^4 + x^3 + 16x^2 + 7x + 6)\log(5x^2 + 3x + 2) + 9659011(10x^4 + x^3 + 16x^2 + 7x + 6)\log(2x^2 - x + 3) + 349923288x - 136217224)/(10x^4 + x^3 + 16x^2 + 7x + 6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")`

[Out]
$$1/5413113112*(214898200*x^3 + 13376294*\sqrt{31}*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 5322018*\sqrt{23}*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 290975300*x^2 - 9659011*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*\log(5*x^2 + 3*x + 2) + 9659011*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*\log(2*x^2 - x + 3) + 349923288*x - 136217224)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)$$

$$x^4 + x^3 + 16x^2 + 7x + 6)$$

Sympy [A] time = 0.449151, size = 122, normalized size = 0.96

$$\frac{6850x^3 - 9275x^2 + 11154x - 4342}{1725460x^4 + 172546x^3 + 2760736x^2 + 1207822x + 1035276} + \frac{19 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{10648} - \frac{19 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{10648} - \frac{2769\sqrt{23}}{2816396}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)

[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364

Giac [A] time = 1.1847, size = 130, normalized size = 1.02

$$\frac{12643}{5116364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{2769}{2816396} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{6850x^3 - 9275x^2 + 11154x - 4342}{172546(10x^4 + x^3 + 16x^2 + 7x + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x + 2) + 19/10648*log(2*x^2 - x + 3)

$$3.50 \quad \int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=148

$$-\frac{9446 - 5765x}{690184(5x^2 + 3x + 2)^2} + \frac{3996965x + 1765599}{235352744(5x^2 + 3x + 2)} + \frac{13 - 6x}{506(2x^2 - x + 3)(5x^2 + 3x + 2)^2} + \frac{97 \log(2x^2 - x + 3)}{468512} - \frac{97 \log(2x^2 - x + 3)}{468512}$$

[Out] $-(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x + 5*x^2])/468512$

Rubi [A] time = 0.161458, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446 - 5765x}{690184(5x^2 + 3x + 2)^2} + \frac{3996965x + 1765599}{235352744(5x^2 + 3x + 2)} + \frac{13 - 6x}{506(2x^2 - x + 3)(5x^2 + 3x + 2)^2} + \frac{97 \log(2x^2 - x + 3)}{468512} - \frac{97 \log(2x^2 - x + 3)}{468512}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3), x]

[Out] $-(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x + 5*x^2])/468512$

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim


```
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1072

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx &= \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{(3-x+2x^2)(2+3x+5x^2)^3} dx}{5566} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-8251111+129}{(3-x+2x^2)^3} dx}{83} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599}{235352744(2+3x+5x^2)} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599}{235352744(2+3x+5x^2)} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599}{235352744(2+3x+5x^2)} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599}{235352744(2+3x+5x^2)} \\
&= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599}{235352744(2+3x+5x^2)}
\end{aligned}$$

Mathematica [A] time = 0.0642083, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904(2x^2-x+3)} + \frac{164380x+67573}{10232728(5x^2+3x+2)} + \frac{345x-98}{30008(5x^2+3x+2)^2} + \frac{97 \log(2x^2-x+3)}{468512} - \frac{97 \log(5x^2+3x+2)}{468512}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^2*(2+3*x+5*x^2)^3),x]

[Out] (-11+90*x)/(244904*(3-x+2*x^2)) + (-98+345*x)/(30008*(2+3*x+5*x^2)^2) + (67573+164380*x)/(10232728*(2+3*x+5*x^2)) + (25557*ArcTan[(-1+4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3+10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3-x+2*x^2])/468512 - (97*Log[2+3*x+5*x^2])/468512

Maple [A] time = 0.053, size = 106, normalized size = 0.7

$$-\frac{25}{234256 (5x^2 + 3x + 2)^2} \left(-\frac{723272x^3}{961} - \frac{3656422x^2}{4805} - \frac{14280728x}{24025} - \frac{2238016}{24025} \right) - \frac{97 \ln(5x^2 + 3x + 2)}{468512} + \frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12)} - \frac{97}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)

[Out] -25/234256*(-723272/961*x^3-3656422/4805*x^2-14280728/24025*x-2238016/24025)/(5*x^2+3*x+2)^2-97/468512*ln(5*x^2+3*x+2)+4464079/6978720496*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/468512*ln(2*x^2-x+3)+25557/123921424*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.49092, size = 159, normalized size = 1.07

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12)} - \frac{97}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 + 21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(2*x^2 - x + 3)

Fricas [A] time = 1.04071, size = 802, normalized size = 5.42

$$1253927859800x^5 + 679296504260x^4 + 2185021181068x^3 + 4722995582\sqrt{31}(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) - 97 \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] $\frac{1}{7383486284768} \cdot (1253927859800x^5 + 679296504260x^4 + 2185021181068x^3 + 4722995582\sqrt{31}(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 1522737174\sqrt{23}(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 1500218514344x^2 - 1528665583(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) \log(5x^2 + 3x + 2) + 1528665583(50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) \log(2x^2 - x + 3) + 1338609358240x + 218880812656) / (50x^6 + 35x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12)$

Sympy [A] time = 0.466039, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 2824232928}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)

[Out] $(39969650x^{**5} + 21652955x^{**4} + 69648769x^{**3} + 47820302x^{**2} + 42668920x + 6976948) / (11767637200x^{**6} + 8237346040x^{**5} + 24241332632x^{**4} + 20004983240x^{**3} + 19534277752x^{**2} + 7531287808x + 2824232928) + 97 \cdot \log(x^{**2} - x/2 + 3/2) / 468512 - 97 \cdot \log(x^{**2} + 3x/5 + 2/5) / 468512 + 25557 \cdot \sqrt{23} \cdot \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right) / 123921424 + 4464079 \cdot \sqrt{31} \cdot \operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right) / 6978720496$

Giac [A] time = 1.13189, size = 149, normalized size = 1.01

$$\frac{4464079}{6978720496} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{25557}{123921424} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{235352744}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] $4464079/6978720496 \cdot \sqrt{31} \arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) + 25557/123921424 \cdot \sqrt{23} \arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 1/235352744 \cdot (39969650x^5 +$

$$\frac{21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{(5x^2 + 3x + 2)^2(2x^2 - x + 3)} - \frac{97}{468512} \log(5x^2 + 3x + 2) + \frac{97}{468512} \log(2x^2 - x + 3)$$

$$3.51 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=98

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}}{16928\sqrt{2}}$$

[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (63799791*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2])/64

Rubi [A] time = 0.11253, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x + 5229)}{135424(2x^2 - x + 3)} - \frac{14641(79x + 101)}{5888(2x^2 - x + 3)^2} - \frac{13915}{64} \log(2x^2 - x + 3) + \frac{2725x}{8} + \frac{63799791 \tan^{-1}}{16928\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]

[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (63799791*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2])/64

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2

- 4*a*c, 0] && LtQ[p, -1]

Rule 1657

Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
x] && IGtQ[p, -2]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} dx &= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1}{46} \int \frac{\frac{2173869}{128} - \frac{661181x}{32} - \frac{488267x^2}{16} + \frac{143635x^3}{8} + \frac{213325x^4}{4} + \frac{83375x^5}{2} + 14}{(3-x+2x^2)^2} dx \\
&= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{\int \frac{-\frac{5460539}{8} - \frac{626865x}{2} + \frac{5170975x^2}{8} + \frac{1124125x^3}{2} + \frac{330625x^4}{2}}{3-x+2x^2} dx}{1058} \\
&= -\frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{\int \left(\frac{1441525}{4} + \frac{2578875x}{8} + \frac{330625x^2}{4} - \frac{121(116609+6083x)}{8(3-x+2x^2)} \right)}{1058} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{121 \int \frac{116609+6083x}{3-x+2x^2}}{8464} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{13915}{64} \int \frac{-1+4x}{3-x+2x^2} \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} - \frac{13915}{64} \log(3-x+2x^2) \\
&= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888(3-x+2x^2)^2} + \frac{1331(5229+76420x)}{135424(3-x+2x^2)} + \frac{63799791 \tan^{-1} \left(\frac{-1+4x}{\sqrt{23}} \right)}{16928\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.0370471, size = 98, normalized size = 1.

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x+5229)}{135424(2x^2-x+3)} - \frac{14641(79x+101)}{5888(2x^2-x+3)^2} - \frac{13915}{64} \log(2x^2-x+3) + \frac{2725x}{8} - \frac{63799791 \tan^{-1} \left(\frac{-1+4x}{\sqrt{23}} \right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]

[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (63799791*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2])/64

Maple [A] time = 0.054, size = 73, normalized size = 0.7

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} - \frac{121}{4(2x^2 - x + 3)^2} \left(-\frac{210155x^3}{4232} + \frac{362791x^2}{16928} - \frac{561121x}{8464} + \frac{54263}{16928} \right) - \frac{13915 \ln(2x^2 - x + 3)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x)

[Out] 625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-561121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/389344*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.45374, size = 111, normalized size = 1.13

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")

[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)

Fricas [A] time = 0.905483, size = 447, normalized size = 4.56

$$486680000x^7 + 2360398000x^6 + 5100406400x^5 + 2157209100x^4 + 24531516180x^3 - 765597492\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)$$

467

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^4 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9))*

$$\arctan(1/23*\sqrt{23}*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$$

Sympy [A] time = 0.277565, size = 95, normalized size = 0.97

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{2725x}{8} + \frac{101715020x^3 - 43897711x^2 + 135791282x - 6565823}{270848x^4 - 270848x^3 + 880256x^2 - 406272x + 609408} - \frac{13915 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{64} - \frac{63799791\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}}{23}\right)}{389344}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)

[Out] 625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 135791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x + 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/389344

Giac [A] time = 1.14796, size = 97, normalized size = 0.99

$$\frac{625}{24}x^3 + \frac{4875}{32}x^2 - \frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331(76420x^3 - 32981x^2 + 102022x - 4933)}{67712(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")

[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)

$$3.52 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=84

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 12828*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32

Rubi [A] time = 0.0865576, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193 - 12828x)}{33856(2x^2 - x + 3)} - \frac{1331(17 - 45x)}{1472(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3) + \frac{125x}{8} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]

[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 12828*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx &= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{1}{46} \int \frac{-\frac{40885}{32} - \frac{19067x}{8} + \frac{22195x^2}{4} + \frac{13225x^3}{2} + 2875x^4}{(3-x+2x^2)^2} dx \\
&= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \frac{\frac{23997}{2} + 92575x + \frac{66125x^2}{2}}{3-x+2x^2} dx}{1058} \\
&= -\frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{\int \left(\frac{66125}{4} - \frac{33(4557-13225x)}{4(3-x+2x^2)} \right) dx}{1058} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{33 \int \frac{4557-13225x}{3-x+2x^2} dx}{4232} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} - \frac{165099 \int \frac{1}{3-x+2x^2} dx}{16928} + \frac{825}{32} \int \frac{-1+4x}{3-x+2x^2} dx \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{825}{32} \log(3-x+2x^2) + \frac{165099 \operatorname{Subst}\left(\frac{1}{3-x+2x^2}, x, \frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} \\
&= \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{165099 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32} \log(3-x+2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0362954, size = 84, normalized size = 1.

$$-\frac{121(12828x-21193)}{33856(2x^2-x+3)} + \frac{1331(45x-17)}{1472(2x^2-x+3)^2} + \frac{825}{32} \log(2x^2-x+3) + \frac{125x}{8} - \frac{165099 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]

[Out] (125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 + 12828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32

Maple [A] time = 0.05, size = 63, normalized size = 0.8

$$\frac{125x}{8} + \frac{11}{2(2x^2 - x + 3)^2} \left(-\frac{35277x^3}{2116} + \frac{303677x^2}{8464} - \frac{132803x}{4232} + \frac{326029}{8464} \right) + \frac{825 \ln(2x^2 - x + 3)}{32} - \frac{165099\sqrt{23}}{194672} \arctan\left(\frac{1}{23}(-1+4x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x)

[Out] 125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(2*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(-1+4*x))*23^(1/2))

Maxima [A] time = 1.45442, size = 97, normalized size = 1.15

$$-\frac{165099}{194672} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{125}{8} x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(4x^4 - 4x^3 + 13x^2 - 6x + 9)} + \frac{825}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")

[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 825/32*log(2*x^2 - x + 3)

Fricas [A] time = 0.985266, size = 377, normalized size = 4.49

$$\frac{24334000x^5 - 24334000x^4 + 43385176x^3 - 330198\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9) \arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + 40329281x}{389344(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281*x)

$$\frac{125x^2 + 10037775(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(2x^2 - x + 3) - 12446818x + 82485337}{(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Sympy [A] time = 0.300081, size = 82, normalized size = 0.98

$$\frac{125x}{8} - \frac{1552188x^3 - 3340447x^2 + 2921666x - 3586319}{67712x^4 - 67712x^3 + 220064x^2 - 101568x + 152352} + \frac{825 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{165099\sqrt{23} \operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)

[Out] 125*x/8 - (1552188*x**3 - 3340447*x**2 + 2921666*x - 3586319)/(67712*x**4 - 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/2)/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672

Giac [A] time = 1.13062, size = 84, normalized size = 1.

$$-\frac{165099}{194672} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{125}{8} x - \frac{121(12828x^3 - 27607x^2 + 24146x - 29639)}{16928(2x^2 - x + 3)^2} + \frac{825}{32} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")

[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*log(2*x^2 - x + 3)

$$3.53 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x + 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])

Rubi [A] time = 0.0529804, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368(2x^2-x+3)^2} - \frac{55(332x+975)}{8464(2x^2-x+3)} - \frac{4330 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]

[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x + 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^3} dx &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{-\frac{195}{8} + \frac{1955x}{2} + 575x^2}{(3 - x + 2x^2)^2} dx \\
 &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} + \frac{\int \frac{4330}{3-x+2x^2} dx}{1058} \\
 &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} + \frac{2165}{529} \int \frac{1}{3 - x + 2x^2} dx \\
 &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} - \frac{4330}{529} \text{Subst} \left(\int \frac{1}{-23 - x^2} dx, x, -1 + 4x \right) \\
 &= \frac{121(19 - 7x)}{368(3 - x + 2x^2)^2} - \frac{55(975 + 332x)}{8464(3 - x + 2x^2)} - \frac{4330 \tan^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{529\sqrt{23}}
 \end{aligned}$$

Mathematica [A] time = 0.0290323, size = 51, normalized size = 0.8

$$\frac{4330 \tan^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{529\sqrt{23}} - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(-2x^2 + x - 3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3, x]
```

[Out] $(-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (4330*\text{ArcTan}[(-1 + 4*x)/\text{Sqrt}[23]])/(529*\text{Sqrt}[23])$

Maple [A] time = 0.047, size = 47, normalized size = 0.7

$$4 \frac{1}{(2x^2 - x + 3)^2} \left(-\frac{4565x^3}{4232} - \frac{44495x^2}{16928} - \frac{5159x}{8464} - \frac{53999}{16928} \right) + \frac{4330\sqrt{23}}{12167} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x)`

[Out] $4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+4330/12167*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.43885, size = 76, normalized size = 1.19

$$\frac{4330}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")`

[Out] $4330/12167*\text{sqrt}(23)*\arctan(1/23*\text{sqrt}(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$

Fricas [A] time = 1.00681, size = 239, normalized size = 3.73

$$\frac{419980x^3 - 34640\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + 1023385x^2 + 237314x + 1241977}{97336(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")`

[Out] $-1/97336*(419980*x^3 - 34640*\sqrt{23}*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$

Sympy [A] time = 0.275541, size = 61, normalized size = 0.95

$$-\frac{18260x^3 + 44495x^2 + 10318x + 53999}{16928x^4 - 16928x^3 + 55016x^2 - 25392x + 38088} + \frac{4330\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)`

[Out] $-(18260*x^3 + 44495*x^2 + 10318*x + 53999)/(16928*x^4 - 16928*x^3 + 55016*x^2 - 25392*x + 38088) + 4330*\sqrt{23}*\operatorname{atan}(4*\sqrt{23}*x/23 - \sqrt{23}/23)/12167$

Giac [A] time = 1.29135, size = 62, normalized size = 0.97

$$\frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11(1660x^3 + 4045x^2 + 938x + 4909)}{4232(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")`

[Out] $4330/12167*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 11/4232*(1660*x^3 + 4045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2$

$$3.54 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^3} dx$$

Optimal. Leaf size=64

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])

Rubi [A] time = 0.0331539, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116(2x^2-x+3)} - \frac{11(3x+5)}{92(2x^2-x+3)^2} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3, x]

[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2 + 3x + 5x^2}{(3 - x + 2x^2)^3} dx &= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} + \frac{1}{46} \int \frac{131}{2(3 - x + 2x^2)^2} dx \\
&= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} + \frac{131}{92} \int \frac{1}{(3 - x + 2x^2)^2} dx \\
&= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} + \frac{131}{529} \int \frac{1}{3 - x + 2x^2} dx \\
&= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} - \frac{262}{529} \operatorname{Subst}\left(\int \frac{1}{-23 - x^2} dx, x, -1 + 4x\right) \\
&= -\frac{11(5 + 3x)}{92(3 - x + 2x^2)^2} - \frac{131(1 - 4x)}{2116(3 - x + 2x^2)} - \frac{262 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.0280116, size = 51, normalized size = 0.8

$$\frac{\frac{46(524x^3 - 393x^2 + 472x - 829)}{(-2x^2 + x - 3)^2} + 1048\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{48668}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3, x]

[Out] ((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*sqrt[23]*ArcTan[(-1 + 4*x)/sqrt[23]])/48668

Maple [A] time = 0.046, size = 47, normalized size = 0.7

$$4 \frac{1}{(2x^2 - x + 3)^2} \left(\frac{131x^3}{1058} - \frac{393x^2}{4232} + \frac{59x}{529} - \frac{829}{4232} \right) + \frac{262\sqrt{23}}{12167} \arctan\left(\frac{(-1 + 4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^3, x)

[Out] 4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*sqrt(23)*arctan(1/23*(-1+4*x)*sqrt(23))

Maxima [A] time = 1.45189, size = 76, normalized size = 1.19

$$\frac{262}{12167} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x-1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3, x, algorithm="maxima")

[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

Fricas [A] time = 1.05006, size = 225, normalized size = 3.52

$$\frac{12052x^3 + 524\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 9039x^2 + 10856x - 19067}{24334(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")

[Out] 1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

Sympy [A] time = 0.19223, size = 61, normalized size = 0.95

$$\frac{524x^3 - 393x^2 + 472x - 829}{4232x^4 - 4232x^3 + 13754x^2 - 6348x + 9522} + \frac{262\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)

[Out] (524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 - 6348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167

Giac [A] time = 1.20159, size = 62, normalized size = 0.97

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) + \frac{524x^3 - 393x^2 + 472x - 829}{1058(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")

[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*x^2 + 472*x - 829)/(2*x^2 - x + 3)^2

$$3.55 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx$$

Optimal. Leaf size=115

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} +$$

[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (119*Log[2 + 3*x + 5*x^2])/21296

Rubi [A] time = 0.1235, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625 - 746x}{256036(2x^2 - x + 3)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2} - \frac{119 \log(2x^2 - x + 3)}{21296} + \frac{119 \log(5x^2 + 3x + 2)}{21296} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} +$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)), x]

[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (119*Log[2 + 3*x + 5*x^2])/21296

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b

```

^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a
*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A
*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)} dx &= \frac{13-6x}{1012(3-x+2x^2)^2} - \frac{\int \frac{-3652-1936x+990x^2}{(3-x+2x^2)^2(2+3x+5x^2)} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-6551908-7779574x+902660x^2}{(3-x+2x^2)(2+3x+5x^2)} dx}{61960712} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2} dx}{14994492304} - \frac{\int \frac{-425}{3-x+2x^2} dx}{14994492304} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} + \frac{53403 \int \frac{1}{3-x+2x^2} dx}{11265584} - \frac{119 \int \frac{-1+4x}{3-x+2x^2} dx}{21296} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{119 \log(3-x+2x^2)}{21296} + \frac{119 \log(2+3x+5x^2)}{21296} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2} + \frac{3625-746x}{256036(3-x+2x^2)} - \frac{53403 \tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + \frac{247 \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{10648\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.154692, size = 99, normalized size = 0.86

$$\frac{713 \left(-\frac{44(1492x^3-7996x^2+7381x-14164)}{(-2x^2+x-3)^2} - 62951 \log(2x^2-x+3) + 62951 \log(5x^2+3x+2) \right) + 3310986\sqrt{23} \tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8032361392}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^3*(2+3*x+5*x^2)),x]

[Out] (3310986*Sqrt[23]*ArcTan[(-1+4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3+10*x)/Sqrt[31]] + 713*((-44*(-14164+7381*x-7996*x^2+1492*x^3))/(-3+x-2*x^2)^2 - 62951*Log[3-x+2*x^2] + 62951*Log[2+3*x+5*x^2]))/8032361392

Maple [A] time = 0.053, size = 89, normalized size = 0.8

$$\frac{119 \ln(5x^2+3x+2)}{21296} + \frac{247\sqrt{31}}{330088} \arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right) - \frac{1}{2662(2x^2-x+3)^2} \left(\frac{8206x^3}{529} - \frac{43978x^2}{529} + \frac{81191x}{1058} - \frac{713}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x)`

[Out] $119/21296*\ln(5*x^2+3*x+2)+247/330088*\arctan(1/31*(3+10*x)*31^{(1/2)})*31^{(1/2)}$
 $-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-$
 $119/21296*\ln(2*x^2-x+3)+53403/129554216*23^{(1/2)}*\arctan(1/23*(-1+4*x)*23^{(1/2)})$

Maxima [A] time = 1.43151, size = 132, normalized size = 1.15

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] $247/330088*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 53403/129554216*\sqrt{23}*$
 $\arctan(1/23*\sqrt{23}*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381$
 $*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*\log(5*x^2 + 3*x$
 $+ 2) - 119/21296*\log(2*x^2 - x + 3)$

Fricas [A] time = 1.02491, size = 556, normalized size = 4.83

$$46807024x^3 - 6010498\sqrt{31}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{31}\sqrt{31}(10x + 3)\right) - 3310986\sqrt{23}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4x - 1)\right) - 250850512x^2 - 44884063(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(5x^2 + 3x + 2) + 44884063(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(2x^2 - x + 3) + 231556732x - 444353008$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")`

[Out] $-1/8032361392*(46807024*x^3 - 6010498*\sqrt{31}*(4*x^4 - 4*x^3 + 13*x^2 - 6*$
 $x + 9)*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 3310986*\sqrt{23}*(4*x^4 - 4*x^3 +$
 $13*x^2 - 6*x + 9)*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 250850512*x^2 - 448840$
 $63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(5*x^2 + 3*x + 2) + 44884063*(4*x^$
 $4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(2*x^2 - x + 3) + 231556732*x - 444353008)$

$$/(4x^4 - 4x^3 + 13x^2 - 6x + 9)$$

Sympy [A] time = 0.396684, size = 122, normalized size = 1.06

$$-\frac{1492x^3 - 7996x^2 + 7381x - 14164}{1024144x^4 - 1024144x^3 + 3328468x^2 - 1536216x + 2304324} - \frac{119 \log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{21296} + \frac{119 \log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{21296} + \frac{53403}{129554216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)

[Out] -(1492*x**3 - 7996*x**2 + 7381*x - 14164)/(1024144*x**4 - 1024144*x**3 + 3328468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*log(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/330088

Giac [A] time = 1.15984, size = 119, normalized size = 1.03

$$\frac{247}{330088} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{53403}{129554216} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{1492x^3 - 7996x^2 + 7381x - 14164}{256036(2x^2 - x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")

[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*log(2*x^2 - x + 3)

$$3.56 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=160

$$\frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} - \frac{252815x + 2328909}{174616552(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log(2x^2 - x + 3)}{468512}$$

[Out] $-(2328909 + 252815x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512$

Rubi [A] time = 0.160139, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665 - 1446x}{512072(2x^2 - x + 3)(5x^2 + 3x + 2)} - \frac{252815x + 2328909}{174616552(5x^2 + 3x + 2)} + \frac{13 - 6x}{1012(2x^2 - x + 3)^2(5x^2 + 3x + 2)} + \frac{181 \log(2x^2 - x + 3)}{468512}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2), x]

[Out] $-(2328909 + 252815*x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512$

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim

```
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]
```

Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)
^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```


Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} - \frac{\int \frac{-4081-3168x+1650x^2}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132} \\
&= -\frac{2328909+252815x}{174616552(2+3x+5x^2)} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} + \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} - \frac{\int \frac{9665-1446x}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx}{11132}
\end{aligned}$$

Mathematica [A] time = 0.101724, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198(2x^2-x+3)} + \frac{1235x-1474}{330088(5x^2+3x+2)} + \frac{-14x-31}{22264(2x^2-x+3)^2} + \frac{181 \log(2x^2-x+3)}{468512} - \frac{181 \log(5x^2+3x+2)}{468512}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^3*(2+3*x+5*x^2)^2),x]

[Out] (-31-14*x)/(22264*(3-x+2*x^2)^2) + (-1782-2923*x)/(1408198*(3-x+2*x^2)) + (-1474+1235*x)/(330088*(2+3*x+5*x^2)) - (2038497*ArcTan[(-1+4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3+10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3-x+2*x^2])/468512 - (181*Log[2+3*x+5*x^2])/468512

Maple [A] time = 0.056, size = 106, normalized size = 0.7

$$-\frac{1}{234256} \left(-\frac{5434x}{31} + \frac{32428}{155} \right) \left(x^2 + \frac{3x}{5} + \frac{2}{5} \right)^{-1} - \frac{181 \ln(5x^2 + 3x + 2)}{468512} + \frac{246757\sqrt{31}}{225120016} \arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right) + \frac{1}{58}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)

[Out] -1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5*x^2+3*x+2)+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/58564*(-128612/529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512*ln(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.46593, size = 157, normalized size = 0.98

$$\frac{246757}{225120016} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{2038497}{2850192752} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{1011260x^5 + 8304376x^4 + 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{174616552(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18)} - \frac{181}{468512} \log(5x^2 + 3x + 2) + \frac{181}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/2850192752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 + 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x + 2) + 181/468512*log(2*x^2 - x + 3)

Fricas [A] time = 1.03044, size = 763, normalized size = 4.77

$$31725248720x^5 + 260524883872x^4 - 158204886268x^3 - 6004584838\sqrt{31}(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18) - \frac{181}{468512} \log(5x^2 + 3x + 2) + \frac{181}{468512} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out]
$$-1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6004584838*\sqrt{31}*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\arctan(1/31*\sqrt{31}*(10*x + 3)) + 3917991234*\sqrt{23}*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\arctan(1/23*\sqrt{23}*(4*x - 1)) + 679966484692*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\log(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*\log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)$$

Sympy [A] time = 0.534726, size = 143, normalized size = 0.89

$$\frac{1011260x^5 + 8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 3143097936} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)

[Out]
$$-(1011260*x**5 + 8304376*x**4 - 5042869*x**3 + 21674311*x**2 - 5887820*x + 8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*\log(x**2 - x/2 + 3/2)/468512 - 181*\log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*\sqrt{23}*atan(4*\sqrt{23}*x/23 - \sqrt{23}/23)/2850192752 + 246757*\sqrt{31}*atan(10*\sqrt{31}*x/31 + 3*\sqrt{31}/31)/225120016$$

Giac [A] time = 1.1564, size = 149, normalized size = 0.93

$$\frac{246757}{225120016} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) - \frac{2038497}{2850192752} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{1011260x^5 + 8304376x^4 - \dots}{174616552}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out]
$$246757/225120016*\sqrt{31}*\arctan(1/31*\sqrt{31}*(10*x + 3)) - 2038497/2850192752*\sqrt{23}*\arctan(1/23*\sqrt{23}*(4*x - 1)) - 1/174616552*(1011260*x^5 + \dots)$$

$$\frac{8304376x^4 - 5042869x^3 + 21674311x^2 - 5887820x + 8829788}{(5x^2 + 3x + 2)(2x^2 - x + 3)^2} - \frac{181}{468512} \log(5x^2 + 3x + 2) + \frac{181}{468512} \log(2x^2 - x + 3)$$

$$3.57 \quad \int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=181

$$\frac{5(302-35x)}{64009(2x^2-x+3)(5x^2+3x+2)^2} + \frac{15(7140435x+2618306)}{14886061058(5x^2+3x+2)} - \frac{5(77020x+223707)}{87308276(5x^2+3x+2)^2} + \frac{13-6x}{1012(2x^2-x+3)^2}$$

[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408

Rubi [A] time = 0.204182, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302-35x)}{64009(2x^2-x+3)(5x^2+3x+2)^2} + \frac{15(7140435x+2618306)}{14886061058(5x^2+3x+2)} - \frac{5(77020x+223707)}{87308276(5x^2+3x+2)^2} + \frac{13-6x}{1012(2x^2-x+3)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3), x]

[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e

```

- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rule 1072

```

Int[((A_) + (B_)*(x_) + (C_)*(x_)^2)/(((a_) + (b_)*(x_) + (c_)*(x_)^2)
*((d_) + (e_)*(x_) + (f_)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
+ c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x]/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B

```

$*f)*x)/(d + e*x + f*x^2), x], x] /; \text{NeQ}[q, 0]] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Dist}[\frac{2*c*d - b*e}{2*c}, \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[\frac{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(a_.) + (b_.)*(x_.)^2}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} dx &= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx}{11132} \\
&= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots \\
&= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - \dots
\end{aligned}$$

Mathematica [A] time = 0.0852828, size = 151, normalized size = 0.83

$$\frac{6850x^3 - 9275x^2 + 11154x - 4342}{345092(10x^4 + x^3 + 16x^2 + 7x + 6)^2} + \frac{5(42842610x^3 - 5711469x^2 + 51156233x + 14085977)}{14886061058(10x^4 + x^3 + 16x^2 + 7x + 6)} + \frac{405 \log(2x^2 - x + 3)}{1288408}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3), x]

[Out] (-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(14886061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)^2) + (405*log(2*x^2 - x + 3))/1288408

061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4) + (880575*ArcTan[(-1 + 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408

Maple [A] time = 0.053, size = 118, normalized size = 0.7

$$-\frac{25}{2576816 (5x^2 + 3x + 2)^2} \left(-\frac{3013197x^3}{961} - \frac{14516062x^2}{4805} - \frac{51193868x}{24025} - \frac{5423968}{24025} \right) - \frac{405 \ln(5x^2 + 3x + 2)}{1288408} + \frac{2768835}{19191481364}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)

[Out] -25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/24025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/19191481364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/644204*(302907/529*x^3-368291/529*x^2+501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7838030068*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))

Maxima [A] time = 1.44651, size = 186, normalized size = 1.03

$$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)} - 405/1288408*\log(5*x^2 + 3*x + 2) + 405/1288408*\log(2*x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 + 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)

Fricas [A] time = 1.17243, size = 1035, normalized size = 5.72

$$67202918046000x^7 - 2238718468800x^6 + 186872434930060x^5 + 62827256425340x^4 + 173919793526820x^3 + 67376830890\sqrt{31}(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)\arctan(1/31\sqrt{31}(10x + 3)) + 52466419650\sqrt{23}(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)\arctan(1/23\sqrt{23}(4x - 1)) + 73595926401690x^2 - 146799174285(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)\log(5x^2 + 3x + 2) + 146799174285(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)\log(2x^2 - x + 3) + 78707350628632x + 7381223830244)/(100x^8 + 20x^7 + 321x^6 + 172x^5 + 390x^4 + 236x^3 + 241x^2 + 84x + 36)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 186872434930060*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/23*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5*x^2 + 3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*x + 7381223830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)

Sympy [A] time = 0.539186, size = 163, normalized size = 0.9

$$\frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 469182977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176}{(2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176) + 405\log(x^2 - x/2 + 3/2)/1288408 - 405\log(x^2 + 3x/5 + 2/5)/1288408 + 880575\sqrt{23}\operatorname{atan}(4\sqrt{23}x/23 - \sqrt{23}/23)/7838030068 + 2768835\sqrt{31}\operatorname{atan}(10\sqrt{31}x/31 + 3\sqrt{31}/31)/19191481364}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)

[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11611127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x + 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/7838030068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364

Giac [A] time = 1.16417, size = 157, normalized size = 0.87

$$\frac{2768835}{19191481364} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31}(10x + 3)\right) + \frac{880575}{7838030068} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) + \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{(10x^4 + x^3 + 16x^2 + 7x + 6)^2} - \frac{405}{1288408} \log(5x^2 + 3x + 2) + \frac{405}{1288408} \log(2x^2 - x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 + 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)

$$3.58 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=208

$$\frac{125}{4} (2x^2 - x + 3)^{3/2} x^7 + \frac{14125}{144} (2x^2 - x + 3)^{3/2} x^6 + \frac{233225 (2x^2 - x + 3)^{3/2} x^5}{1536} + \frac{4796405 (2x^2 - x + 3)^{3/2} x^4}{43008} + \frac{8325631 (2x^2 - x + 3)^{3/2} x^3}{1032192} + \frac{4796405 x^4 (3 - x + 2x^2)^{3/2}}{43008} + \frac{233225 x^5 (3 - x + 2x^2)^{3/2}}{1536} + \frac{14125 x^6 (3 - x + 2x^2)^{3/2}}{144} + \frac{125 x^7 (3 - x + 2x^2)^{3/2}}{4} - \frac{8267844569 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{(134217728 \sqrt{2})}$$

[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x + 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(134217728*Sqrt[2])

Rubi [A] time = 0.309322, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4} (2x^2 - x + 3)^{3/2} x^7 + \frac{14125}{144} (2x^2 - x + 3)^{3/2} x^6 + \frac{233225 (2x^2 - x + 3)^{3/2} x^5}{1536} + \frac{4796405 (2x^2 - x + 3)^{3/2} x^4}{43008} + \frac{8325631 (2x^2 - x + 3)^{3/2} x^3}{1032192} + \frac{4796405 x^4 (3 - x + 2x^2)^{3/2}}{43008} + \frac{233225 x^5 (3 - x + 2x^2)^{3/2}}{1536} + \frac{14125 x^6 (3 - x + 2x^2)^{3/2}}{144} + \frac{125 x^7 (3 - x + 2x^2)^{3/2}}{4} - \frac{8267844569 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{(134217728 \sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]

[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x + 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(134217728*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*

$e^{(q+p)x^{q-1}} - c e^{(q+2p+1)x^q}$, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^4 dx &= \frac{125}{4}x^7 (3-x+2x^2)^{3/2} + \frac{1}{20} \int \sqrt{3-x+2x^2} (320+1920x+7520x^2+18720x^3 \\
&= \frac{14125}{144}x^6 (3-x+2x^2)^{3/2} + \frac{125}{4}x^7 (3-x+2x^2)^{3/2} + \frac{1}{360} \int \sqrt{3-x+2x^2} (5760 \\
&= \frac{233225x^5 (3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6 (3-x+2x^2)^{3/2} + \frac{125}{4}x^7 (3-x+2x^2)^{3/2} \\
&= \frac{4796405x^4 (3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5 (3-x+2x^2)^{3/2}}{1536} + \frac{14125}{144}x^6 (3-x+2x^2)^{3/2} \\
&= \frac{8325631x^3 (3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4 (3-x+2x^2)^{3/2}}{43008} + \frac{233225x^5 (3-x+2x^2)^{3/2}}{1536} \\
&= -\frac{83948353x^2 (3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3 (3-x+2x^2)^{3/2}}{1032192} + \frac{4796405x^4 (3-x+2x^2)^{3/2}}{43008} \\
&= \frac{804243809x (3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2 (3-x+2x^2)^{3/2}}{2293760} + \frac{8325631x^3 (3-x+2x^2)^{3/2}}{1032192} \\
&= \frac{27185733541 (3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x (3-x+2x^2)^{3/2}}{36700160} - \frac{83948353x^2 (3-x+2x^2)^{3/2}}{2293760} \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541 (3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x (3-x+2x^2)^{3/2}}{36700160} \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541 (3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x (3-x+2x^2)^{3/2}}{36700160} \\
&= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864} + \frac{27185733541 (3-x+2x^2)^{3/2}}{440401920} + \frac{804243809x (3-x+2x^2)^{3/2}}{36700160}
\end{aligned}$$

Mathematica [A] time = 0.291108, size = 85, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3-x+2*x^2]*(2+3*x+5*x^2)^4,x]

[Out] $(4\sqrt{3-x+2x^2}*(3801512106459 + 537752185764*x - 174418077792*x^2 + 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 2604371039235*\sqrt{2}*\text{ArcSinh}[(1-4*x)/\sqrt{23}])/84557168640$

Maple [A] time = 0.069, size = 166, normalized size = 0.8

$$\frac{125x^7}{4}(2x^2-x+3)^{\frac{3}{2}} + \frac{14125x^6}{144}(2x^2-x+3)^{\frac{3}{2}} + \frac{233225x^5}{1536}(2x^2-x+3)^{\frac{3}{2}} + \frac{4796405x^4}{43008}(2x^2-x+3)^{\frac{3}{2}} + \frac{8325631}{1032192}(2x^2-x+3)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x)`

[Out] $125/4*x^7*(2*x^2-x+3)^{(3/2)}+14125/144*x^6*(2*x^2-x+3)^{(3/2)}+233225/1536*x^5*(2*x^2-x+3)^{(3/2)}+4796405/43008*x^4*(2*x^2-x+3)^{(3/2)}+8325631/1032192*x^3*(2*x^2-x+3)^{(3/2)}-83948353/2293760*x^2*(2*x^2-x+3)^{(3/2)}+804243809/36700160*x*(2*x^2-x+3)^{(3/2)}+359471503/67108864*(-1+4*x)*(2*x^2-x+3)^{(1/2)}+8267844569/268435456*2^{(1/2)}*\text{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+27185733541/440401920*(2*x^2-x+3)^{(3/2)}$

Maxima [A] time = 1.48403, size = 239, normalized size = 1.15

$$\frac{125}{4}(2x^2-x+3)^{\frac{3}{2}}x^7 + \frac{14125}{144}(2x^2-x+3)^{\frac{3}{2}}x^6 + \frac{233225}{1536}(2x^2-x+3)^{\frac{3}{2}}x^5 + \frac{4796405}{43008}(2x^2-x+3)^{\frac{3}{2}}x^4 + \frac{8325631}{1032192}(2x^2-x+3)^{\frac{3}{2}}x^3 - \frac{83948353}{2293760}(2x^2-x+3)^{\frac{3}{2}}x^2 + \frac{804243809}{36700160}(2x^2-x+3)^{\frac{3}{2}}x + \frac{27185733541}{440401920}(2x^2-x+3)^{\frac{3}{2}} + \frac{359471503}{16777216}\sqrt{2x^2-x+3}x + \frac{8267844569}{268435456}\sqrt{2}\text{arcsinh}(1/23*\sqrt{23}*(4*x-1)) - \frac{359471503}{67108864}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out] $125/4*(2*x^2-x+3)^{(3/2)}*x^7 + 14125/144*(2*x^2-x+3)^{(3/2)}*x^6 + 233225/1536*(2*x^2-x+3)^{(3/2)}*x^5 + 4796405/43008*(2*x^2-x+3)^{(3/2)}*x^4 + 8325631/1032192*(2*x^2-x+3)^{(3/2)}*x^3 - 83948353/2293760*(2*x^2-x+3)^{(3/2)}*x^2 + 804243809/36700160*(2*x^2-x+3)^{(3/2)}*x + 27185733541/440401920*(2*x^2-x+3)^{(3/2)} + 359471503/16777216*\text{sqrt}(2*x^2-x+3)*x + 8267844569/268435456*\text{sqrt}(2)*\text{arcsinh}(1/23*\text{sqrt}(23)*(4*x-1)) - 359471503/67108864*\text{sqrt}(2*x^2-x+3)$

Fricas [A] time = 1.40846, size = 439, normalized size = 2.11

$$\frac{1}{21139292160} (1321205760000x^9 + 3486515200000x^8 + 6327795712000x^7 + 7725962035200x^6 + 7612808028160x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 + 7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x + 3) + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)

[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)

Giac [A] time = 1.13665, size = 126, normalized size = 0.61

$$\frac{1}{21139292160} (4(8(4(16(20(40(140(160(36x+95)x+27587)x+4715553)x+185859571)x+2614620113)x+1727$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 4715553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 134438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/268435456*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.59 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=166

$$\frac{125}{16} (2x^2 - x + 3)^{3/2} x^5 + \frac{8825}{448} (2x^2 - x + 3)^{3/2} x^4 + \frac{247435 (2x^2 - x + 3)^{3/2} x^3}{10752} + \frac{531681 (2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393}{10752}$$

[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 - (155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])

Rubi [A] time = 0.181374, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16} (2x^2 - x + 3)^{3/2} x^5 + \frac{8825}{448} (2x^2 - x + 3)^{3/2} x^4 + \frac{247435 (2x^2 - x + 3)^{3/2} x^3}{10752} + \frac{531681 (2x^2 - x + 3)^{3/2} x^2}{71680} - \frac{9627393}{10752}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]

[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 - (155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
  *e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
  *(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
  *p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
  eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
  *c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
  + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
  t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^3 dx &= \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{16} \int \sqrt{3-x+2x^2} (128+576x+1824x^2+3312x^3+2048x^4) dx \\
&= \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{125}{16} x^5 (3-x+2x^2)^{3/2} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{8825}{448} x^4 (3-x+2x^2)^{3/2} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= -\frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{247435x^3 (3-x+2x^2)^{3/2}}{10752} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= -\frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{531681x^2 (3-x+2x^2)^{3/2}}{71680} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119 (3-x+2x^2)^{3/2}}{4587520} - \frac{9627393x (3-x+2x^2)^{3/2}}{1146880} + \frac{1}{224} \int \sqrt{3-x+2x^2} (1792+10800x+247435x^2+247435x^3+9627393x^4+9627393x^5+531681x^6+9627393x^7+6766097(1-4x)\sqrt{3-x+2x^2}) dx
\end{aligned}$$

Mathematica [A] time = 0.172468, size = 75, normalized size = 0.45

$$4\sqrt{2x^2-x+3} (3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2 -$$

880803840

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 9872163456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000*x^7) - 16340124255*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/880803840

Maple [A] time = 0.059, size = 132, normalized size = 0.8

$$\frac{125x^5}{16}(2x^2-x+3)^{\frac{3}{2}} + \frac{8825x^4}{448}(2x^2-x+3)^{\frac{3}{2}} + \frac{247435x^3}{10752}(2x^2-x+3)^{\frac{3}{2}} + \frac{531681x^2}{71680}(2x^2-x+3)^{\frac{3}{2}} - \frac{9627393}{1146880}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x)

[Out] 125/16*x^5*(2*x^2-x+3)^(3/2)+8825/448*x^4*(2*x^2-x+3)^(3/2)+247435/10752*x^3*(2*x^2-x+3)^(3/2)+531681/71680*x^2*(2*x^2-x+3)^(3/2)-9627393/1146880*x*(2*x^2-x+3)^(3/2)+6766097/2097152*(-1+4*x)*(2*x^2-x+3)^(1/2)+155620231/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-22548119/4587520*(2*x^2-x+3)^(3/2)

Maxima [A] time = 1.47232, size = 193, normalized size = 1.16

$$\frac{125}{16}(2x^2-x+3)^{\frac{3}{2}}x^5 + \frac{8825}{448}(2x^2-x+3)^{\frac{3}{2}}x^4 + \frac{247435}{10752}(2x^2-x+3)^{\frac{3}{2}}x^3 + \frac{531681}{71680}(2x^2-x+3)^{\frac{3}{2}}x^2 - \frac{9627393}{1146880}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 247435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*x^2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 - x + 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.52335, size = 352, normalized size = 2.12

$$\frac{1}{220200960}(3440640000x^7 + 6955008000x^6 + 10958233600x^5 + 11212171264x^4 + 9872163456x^3 + 4583812128x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 11212171264*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt(2

$*x^2 - x + 3) + 155620231/16777216*\sqrt{2}*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)

[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)

Giac [A] time = 1.32715, size = 112, normalized size = 0.67

$\frac{1}{220200960} (4 (8 (4 (16 (100 (120 (140x + 283)x + 53507)x + 5474693)x + 77126277)x + 143244129)x - 405326979)x -$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] $1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x + 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*\sqrt{2*x^2 - x + 3} - 155620231/8388608*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})) + 1)$

$$3.60 \quad \int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=124

$$\frac{25}{12} (2x^2 - x + 3)^{3/2} x^3 + \frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384}$$

[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/16 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32768*Sqrt[2])

Rubi [A] time = 0.0990913, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12} (2x^2 - x + 3)^{3/2} x^3 + \frac{63}{16} (2x^2 - x + 3)^{3/2} x^2 + \frac{769}{256} (2x^2 - x + 3)^{3/2} x - \frac{2107 (2x^2 - x + 3)^{3/2}}{3072} + \frac{12371(1 - 4x)\sqrt{2x^2 - x + 3}}{16384}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]

[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/16 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32768*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2} (2+3x+5x^2)^2 dx &= \frac{25}{12} x^3 (3-x+2x^2)^{3/2} + \frac{1}{12} \int \sqrt{3-x+2x^2} \left(48 + 144x + 123x^2 + \frac{945x^3}{2} \right) dx \\
&= \frac{63}{16} x^2 (3-x+2x^2)^{3/2} + \frac{25}{12} x^3 (3-x+2x^2)^{3/2} + \frac{1}{120} \int \sqrt{3-x+2x^2} (480 - 139x) dx \\
&= \frac{769}{256} x (3-x+2x^2)^{3/2} + \frac{63}{16} x^2 (3-x+2x^2)^{3/2} + \frac{25}{12} x^3 (3-x+2x^2)^{3/2} + \frac{1}{960} \int \sqrt{3-x+2x^2} dx \\
&= -\frac{2107 (3-x+2x^2)^{3/2}}{3072} + \frac{769}{256} x (3-x+2x^2)^{3/2} + \frac{63}{16} x^2 (3-x+2x^2)^{3/2} + \frac{25}{12} x^3 (3-x+2x^2)^{3/2} \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107 (3-x+2x^2)^{3/2}}{3072} + \frac{769}{256} x (3-x+2x^2)^{3/2} + \frac{63}{16} x^2 (3-x+2x^2)^{3/2} \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107 (3-x+2x^2)^{3/2}}{3072} + \frac{769}{256} x (3-x+2x^2)^{3/2} + \frac{63}{16} x^2 (3-x+2x^2)^{3/2} \\
&= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107 (3-x+2x^2)^{3/2}}{3072} + \frac{769}{256} x (3-x+2x^2)^{3/2} + \frac{63}{16} x^2 (3-x+2x^2)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.10554, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023) + 853599\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 284672*x^4 + 204800*x^5) + 853599*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/196608

Maple [A] time = 0.053, size = 98, normalized size = 0.8

$$\frac{25x^3}{12} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{63x^2}{16} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{769x}{256} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{2107}{3072} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{-12371 + 49484x}{16384} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x)`

[Out] $25/12*x^3*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+3)^(3/2)-2107/3072*(2*x^2-x+3)^(3/2)-12371/16384*(-1+4*x)*(2*x^2-x+3)^(1/2)-284533/65536*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))$

Maxima [A] time = 1.53431, size = 147, normalized size = 1.19

$$\frac{25}{12} (2x^2 - x + 3)^{\frac{3}{2}} x^3 + \frac{63}{16} (2x^2 - x + 3)^{\frac{3}{2}} x^2 + \frac{769}{256} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{2107}{3072} (2x^2 - x + 3)^{\frac{3}{2}} - \frac{12371}{4096} \sqrt{2x^2 - x + 3} - \frac{284533}{65536} 2^{\frac{1}{2}} \operatorname{arcsinh}\left(\frac{4}{23} \sqrt{2} (x - \frac{1}{4})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out] $25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/256*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*\operatorname{sqrt}(2*x^2 - x + 3)*x - 284533/65536*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) + 12371/16384*\operatorname{sqrt}(2*x^2 - x + 3)$

Fricas [A] time = 1.52922, size = 257, normalized size = 2.07

$$\frac{1}{49152} (204800x^5 + 284672x^4 + 408960x^3 + 365536x^2 + 328204x - 64023) \sqrt{2x^2 - x + 3} + \frac{284533}{131072} \sqrt{2} \log\left(4\sqrt{2}\sqrt{2x^2 - x + 3} + 23\sqrt{2x^2 - x + 3} + 284533/131072\sqrt{2}\right) - 32x^2 + 16x - 25$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out] $1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 64023)*\operatorname{sqrt}(2*x^2 - x + 3) + 284533/131072*\operatorname{sqrt}(2)*\log(4*\operatorname{sqrt}(2)*\operatorname{sqrt}(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2), x)

[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)

Giac [A] time = 1.19608, size = 99, normalized size = 0.8

$$\frac{1}{49152} (4 (8 (4 (16 (100x + 139)x + 3195)x + 11423)x + 82051)x - 64023) \sqrt{2x^2 - x + 3} + \frac{284533}{65536} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 64023)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

3.61 $\int \sqrt{3 - x + 2x^2} (2 + 3x + 5x^2) dx$

Optimal. Leaf size=82

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2])

Rubi [A] time = 0.039791, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x(2x^2 - x + 3)^{3/2} + \frac{73}{96}(2x^2 - x + 3)^{3/2} - \frac{81}{512}(1 - 4x)\sqrt{2x^2 - x + 3} - \frac{1863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]

[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \sqrt{3-x+2x^2} (2+3x+5x^2) dx &= \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{1}{8} \int \left(1 + \frac{73x}{2}\right) \sqrt{3-x+2x^2} dx \\
 &= \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{81}{64} \int \sqrt{3-x+2x^2} dx \\
 &= -\frac{81}{512} (1-4x)\sqrt{3-x+2x^2} + \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{1863}{10} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
 &= -\frac{81}{512} (1-4x)\sqrt{3-x+2x^2} + \frac{73}{96} (3-x+2x^2)^{3/2} + \frac{5}{8}x(3-x+2x^2)^{3/2} + \frac{1863}{10} \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.0534298, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2 - x + 3} (1920x^3 + 1376x^2 + 2684x + 3261) - 5589\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/6144

Maple [A] time = 0.048, size = 64, normalized size = 0.8

$$\frac{5x}{8} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{73}{96} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{-81 + 324x}{512} \sqrt{2x^2 - x + 3} + \frac{1863\sqrt{2}}{2048} \operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x)

[Out] 5/8*x*(2*x^2-x+3)^(3/2)+73/96*(2*x^2-x+3)^(3/2)+81/512*(-1+4*x)*(2*x^2-x+3)^(1/2)+1863/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))

Maxima [A] time = 1.50358, size = 101, normalized size = 1.23

$$\frac{5}{8} (2x^2 - x + 3)^{\frac{3}{2}} x + \frac{73}{96} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{81}{128} \sqrt{2x^2 - x + 3} + \frac{1863}{2048} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{81}{512} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*x^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/512*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.64869, size = 207, normalized size = 2.52

$$\frac{1}{1536} (1920x^3 + 1376x^2 + 2684x + 3261) \sqrt{2x^2 - x + 3} + \frac{1863}{4096} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/409
6*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2 - x + 3} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2), x)
```

Giac [A] time = 1.16374, size = 85, normalized size = 1.04

$$\frac{1}{1536} (4(8(60x + 43)x + 671)x + 3261)\sqrt{2x^2 - x + 3} - \frac{1863}{2048} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/2048
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

$$3.62 \quad \int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=174

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left(\frac{\sqrt{\frac{11}{62(10\sqrt{2}-13)}} ((20 - 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right)$$

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))]/31)
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]]/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))]/31)*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]]/5
```

Rubi [A] time = 0.441296, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5} \sqrt{\frac{11}{31} (13 + 10\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} ((20 + 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right) - \frac{1}{5} \sqrt{\frac{11}{31} (10\sqrt{2} - 13)} \tanh^{-1} \left(\frac{\sqrt{\frac{11}{62(10\sqrt{2}-13)}} ((20 - 13\sqrt{2})x + 7\sqrt{2} + 6)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2), x]
```

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))]/31)
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]]/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))]/31)*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]]/5
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
```


4*d*f, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1035

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2} dx &= -\left(\frac{1}{5} \int \frac{-11+11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx\right) + \frac{2}{5} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
 &= \frac{1}{5} \sqrt{\frac{2}{23}} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x\right) + \frac{\int \frac{121(2+\sqrt{2})-121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}} - \frac{\int \frac{121(2-\sqrt{2})+121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{110\sqrt{2}} \\
 &= -\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) - \frac{1}{5} (1331(20-13\sqrt{2})) \operatorname{Subst}\left(\int \frac{1}{-907742(13-10\sqrt{2})-11x^2} dx, x, \frac{121(2+\sqrt{2})-121\sqrt{2}x}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\right) \\
 &= -\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{1}{5} \sqrt{\frac{11}{31}} (13+10\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(13+10\sqrt{2})}} (6+7\sqrt{2}+(20+13\sqrt{2})x)}{\sqrt{3-x+2x^2}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.463022, size = 185, normalized size = 1.06

$$-\frac{1}{5} \sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) - \frac{1}{5} i \sqrt{\frac{11}{62}} \left(\sqrt{13+i\sqrt{31}} \tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}\sqrt{2x^2-x+3}}}\right) - \sqrt{13-i\sqrt{31}} \tanh^{-1}\left(\frac{4i\sqrt{31}}{2\sqrt{286}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2), x]

[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 - (I/5)*Sqrt[11/62]*(Sqrt[13 + I*Sqrt[31]]*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - Sqrt[13 - I*Sqrt[31]]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])

Maple [B] time = 0.276, size = 2065, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^2-x+3)^{(1/2)}/(5*x^2+3*x+2), x)$

[Out] $\frac{1}{5}2^{(1/2)}*\text{arcsinh}(4/23*23^{(1/2)}*(x-1/4))-1/52855*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(285*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\text{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+386*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\text{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))-274846*\text{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-1543366*\text{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)}))/((1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))^2)^{(1/2)}/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+1/21142*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(151*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\text{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+218*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\text{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+401698*\text{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)})*2^{(1/2)}-63426*\text{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)}))/((1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))^2)^{(1/2)}/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+3/21142*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(369*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\text{arctan}(1/11692487*(-775687+549$

$$\begin{aligned}
& 362 \cdot 2^{(1/2)} \cdot (-23 \cdot (8+3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + \\
& 24 \cdot 2^{(1/2)} - 41))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 \cdot 2^{(1/2)} + 10368 \cdot \\
& 2^{(1/2)} - 1+x)^2 / (2^{(1/2)}+1-x)^2 + 22379 \cdot 2^{(1/2)} + 32016) / (23 \cdot (2^{(1/2)}-1+x)^4 / (2^{(1/2)}+1-x)^4 + \\
& 82 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + 23) \cdot (8+3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)}-1+x) / (2^{(1/2)}+1-x) \\
& + 520 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan(1/11692487 \cdot \\
& (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot (-23 \cdot (8+3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + \\
& 24 \cdot 2^{(1/2)} - 41))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 \cdot 2^{(1/2)} + 10368 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + \\
& 22379 \cdot 2^{(1/2)} + 32016) / (23 \cdot (2^{(1/2)}-1+x)^4 / (2^{(1/2)}+1-x)^4 + 82 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + 23) \cdot \\
& (8+3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)}-1+x) / (2^{(1/2)}+1-x) + 465124 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + 3 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 \cdot 2^{(1/2)} + \\
& 8 - 3 \cdot 2^{(1/2)})^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}) \cdot 2^{(1/2)} - 866822 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + 3 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 \cdot 2^{(1/2)} + \\
& 8 - 3 \cdot 2^{(1/2)})^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}) / ((8 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 + 3 \cdot (2^{(1/2)}-1+x)^2 / (2^{(1/2)}+1-x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) / (1 + (2^{(1/2)}-1+x) / (2^{(1/2)}+1-x))^2)^{(1/2)} / (1 + (2^{(1/2)}-1+x) / (2^{(1/2)}+1-x)) / (8+3 \cdot 2^{(1/2)}) / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")

[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)

Fricas [B] time = 4.39329, size = 7056, normalized size = 40.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] $1/1550 \cdot 6050^{(1/4)} \cdot \sqrt{31} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{13 \cdot \sqrt{2} + 20} \cdot \arctan(1/8 \cdot 9125 \cdot (460 \cdot \sqrt{5}) \cdot (4 \cdot 6050^{(3/4)} \cdot \sqrt{31}) \cdot (4702 \cdot x^7 - 19541 \cdot x^6 + 40352 \cdot x^5 - 68777 \cdot x^4 + 35480 \cdot x^3 - 19080 \cdot x^2 - \sqrt{2}) \cdot (4028 \cdot x^7 - 14488 \cdot x^6 + 30919$

$$\begin{aligned}
& *x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764 \\
& 8) + 5*6050^{(1/4)}*\sqrt{31}*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096* \\
& x^4 + 5840640*x^3 - 4011840*x^2 - \sqrt{2}*(18463*x^7 - 280047*x^6 + 1453472 \\
& *x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398 \\
& 1312*x + 3068928))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} + 20} + 253000*\sqrt{(\\
& 31)*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^ \\
& 4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - \\
& 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1 \\
& 154304*x - 456192) - 2*\sqrt{10}*(\sqrt{5}*(4*6050^{(3/4)}*\sqrt{31}*(15454*x^7 \\
& - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - \sqrt{2}*(1543 \\
& 8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x \\
&) - 13824*x) + 5*6050^{(1/4)}*\sqrt{31}*(77254*x^7 - 1000024*x^6 + 3868360*x^5 \\
& - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - \sqrt{2}*(69479*x^7 - 898236*x^ \\
& 6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24 \\
& 05376*x))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} + 20} + 550*\sqrt{31}*\sqrt{2}* \\
& (123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336* \\
& x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x \\
& ^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25 \\
& *\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819 \\
& 20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517 \\
& *x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))* \\
& \sqrt{-(6050^{(1/4)}*\sqrt{5}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(3*x + 5) - 8*x + 2) \\
& *\sqrt{13*\sqrt{2} + 20} - 245*x^2 - 220*\sqrt{2}*(2*x^2 - x + 3) + 755*x - 10 \\
& 00)/x^2) + 2875*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835 \\
& 344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348 \\
& *x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + \\
& 4320*x - 5184) + 223064064*x - 94887936))/ (2585191*x^8 - 4661200*x^7 + 1419 \\
& 1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726 \\
& 08*x + 18579456)) + 1/1550*6050^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{2}*\sqrt{13*\sqrt{2} \\
& (2) + 20)*\arctan(1/89125*(460*\sqrt{5}*(4*6050^{(3/4)}*\sqrt{31}*(4702*x^7 - 19 \\
& 541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - \sqrt{2}*(4028*x^7 \\
& - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172 \\
& 80) - 34560*x + 27648) + 5*6050^{(1/4)}*\sqrt{31}*(22836*x^7 - 355266*x^6 + 19 \\
& 14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - \sqrt{2}*(18463*x^7 - \\
& 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892 \\
& 8*x + 1990656) - 3981312*x + 3068928))*\sqrt{2*x^2 - x + 3}*\sqrt{13*\sqrt{2} \\
& + 20} - 253000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385 \\
& 256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335 \\
& *x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54 \\
& 6048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{10}*(\sqrt{5}*(4*6050^{(3/4)}* \\
& \sqrt{31}*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824 \\
& *x^2 - \sqrt{2}*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + \\
& 44928*x^2 - 44928*x) - 13824*x) + 5*6050^{(1/4)}*\sqrt{31}*(77254*x^7 - 10000 \\
& 24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - \sqrt{2}*(6 \\
& 9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
\end{aligned}$$

```

^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
))*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
+ 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)

[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.63 \quad \int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2 - x + 3}(10x + 3)}{31(5x^2 + 3x + 2)} + \frac{1}{62} \sqrt{\frac{1}{682} (70517 + 49942\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}} ((973 + 696\sqrt{2})x + 277\sqrt{2} + 419)}{\sqrt{2x^2 - x + 3}} \right)$$

[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))])*(419 + 277*Sqrt[2] + (973 + 696*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517 + 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))])*(419 - 277*Sqrt[2] + (973 - 696*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/62

Rubi [A] time = 0.393492, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2 - x + 3}(10x + 3)}{31(5x^2 + 3x + 2)} + \frac{1}{62} \sqrt{\frac{1}{682} (70517 + 49942\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}} ((973 + 696\sqrt{2})x + 277\sqrt{2} + 419)}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]

[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))])*(419 + 277*Sqrt[2] + (973 + 696*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517 + 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))])*(419 - 277*Sqrt[2] + (973 - 696*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/62

Rule 971

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +

3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1035

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{-\frac{63}{2} + 11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{\int \frac{\frac{11}{2}(85-63\sqrt{2}) - \frac{11}{2}(41-22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}(85+63\sqrt{2}) - \frac{11}{2}(41+22\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{682\sqrt{2}} \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} - \frac{1}{248} \left(11(99884 - 70517\sqrt{2}) \right) \text{Subst} \left(\int \frac{1}{-\frac{3751}{4}(70517 - 49942\sqrt{2}) - \dots} \right) \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} + \frac{1}{62} \sqrt{\frac{1}{682} (70517 + 49942\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(70517+49942\sqrt{2})}} (419 + 277\dots)}{\sqrt{3-x+2\dots}} \right)
\end{aligned}$$

Mathematica [C] time = 1.08304, size = 214, normalized size = 1.14

$$\frac{27280\sqrt{2x^2-x+3}(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}(973\sqrt{31}+1271i) \tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}(973\sqrt{31}-1271i) \tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)$$

845680

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2, x]

[Out] ((27280*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) + I*Sqrt[286 - (2*I)*Sqrt[31]]*(1271*I + 973*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - I*Sqrt[286 + (22*I)*Sqrt[31]]*(-1271*I + 973*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/845680

Maple [B] time = 0.292, size = 16357, normalized size = 87.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2, x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)

Fricas [B] time = 5.82357, size = 8142, normalized size = 43.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/186703822445536*(88412*4988406728^{(1/4)}*\sqrt{24971}*\sqrt{341}*\sqrt{2}*(5 \\ & *x^2 + 3*x + 2)*\sqrt{70517*\sqrt{2} + 99884}*\arctan(1/10668926457462302923*(\\ & 3096404*\sqrt{24971}*(11*4988406728^{(3/4)}*\sqrt{341}*(537184*x^7 - 2047820*x^ \\ & 6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - \sqrt{2}*(370014 \\ & *x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 \\ & - 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^{(1/4)}*\sqrt{ \\ & t(341)*(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - \\ & 5992992*x^2 - \sqrt{2}*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + \\ & 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*\sqrt{ \\ & \sqrt{2*x^2 - x + 3}*\sqrt{70517*\sqrt{2} + 99884} + 30285984782473634104*\sqrt{ \\ & 31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^ \\ & 4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - \\ & 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1 \\ & 154304*x - 456192) - 2*\sqrt{49942}*(\sqrt{24971}*(11*4988406728^{(3/4)}*\sqrt{3 \\ & 41}*(84604*x^7 - 121310*x^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624 \\ & *x^2 - \sqrt{2}*(57082*x^7 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^ \\ & 3 + 110592*x^2 - 110592*x) - 186624*x) + 774101*4988406728^{(1/4)}*\sqrt{341}*(\\ & 6379*x^7 - 82508*x^6 + 318020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - \end{aligned}$$

$$\begin{aligned}
& \sqrt{2}*(4365*x^7 - 56468*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207 \\
& 360*x^2 - 207360*x) - 331776*x))*\sqrt{2*x^2 - x + 3}*\sqrt{70517*\sqrt{2} + 9 \\
& 9884) + 425261673562*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 \\
& - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 \\
& - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209 \\
& 600*x^2 - 1036800*x) + 3276288*x) + 19330076071*\sqrt{31}*(254591*x^8 - 4815 \\
& 126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689 \\
& 56928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - \\
& 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(4988406728^{(1/4)}*\sqrt{24971})*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(10*x + 3) - 13*x - \\
& 7)*\sqrt{70517*\sqrt{2} + 99884) - 1175859419*x^2 - 1055873764*\sqrt{2}*(2*x^2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*\sqrt{31}*(\\
& (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - \\
& 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 \\
& - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406 \\
& 4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1 \\
& 3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412 \\
& *4988406728^{(1/4)}*\sqrt{24971}*\sqrt{341}*\sqrt{2}*(5*x^2 + 3*x + 2)*\sqrt{7051 \\
& 7*\sqrt{2} + 99884)*\arctan(1/10668926457462302923*(3096404*\sqrt{24971})*(11*4 \\
& 988406728^{(3/4)}*\sqrt{341}*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210 \\
& *x^4 + 3421536*x^3 - 1589328*x^2 - \sqrt{2}*(370014*x^7 - 1438653*x^6 + 3014 \\
& 868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) - \\
& 3787776*x + 2633472) + 774101*4988406728^{(1/4)}*\sqrt{341}*(40625*x^7 - 62250 \\
& 9*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - \sqrt{2}*(28 \\
& 204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^2 \\
& - 4810752*x + 3442176) - 6884352*x + 4810752))*\sqrt{2*x^2 - x + 3}*\sqrt{7 \\
& 0517*\sqrt{2} + 99884) - 30285984782473634104*\sqrt{31}*\sqrt{2}*(28180*x^8 - \\
& 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 \\
& - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 \\
& - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{2} \\
& *(49942)*(\sqrt{24971}*(11*4988406728^{(3/4)}*\sqrt{341}*(84604*x^7 - 121310*x^6 \\
& + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - \sqrt{2}*(57082*x^7 \\
& - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x \\
&) - 186624*x) + 774101*4988406728^{(1/4)}*\sqrt{341}*(6379*x^7 - 82508*x^6 + 3 \\
& 18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - \sqrt{2}*(4365*x^7 - 5646 \\
& 8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331 \\
& 776*x))*\sqrt{2*x^2 - x + 3}*\sqrt{70517*\sqrt{2} + 99884) - 425261673562*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480* \\
& x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 \\
& - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32 \\
& 76288*x) - 19330076071*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - \\
& 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2} \\
& *(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19 \\
& 44*x) + 144820224*x))*\sqrt{((4988406728^{(1/4)}*\sqrt{24971})*\sqrt{341}*\sqrt{31} \\
& *\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(10*x + 3) - 13*x - 7)*\sqrt{70517*\sqrt{2} + 9
\end{aligned}$$

```

9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
+ 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
- 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
+ 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.64 \quad \int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62(5x^2+3x+2)^2} + \frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(112285869463+79399380740\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(112285869463+79399380740\sqrt{2})}}{169136}\right)}{169136}$$

[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 79399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sqrt[2]))])*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))])*(509587 - 362788*Sqrt[2] + (1235163 - 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/169136

Rubi [A] time = 0.45864, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {971, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62(5x^2+3x+2)^2} + \frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(112285869463+79399380740\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(112285869463+79399380740\sqrt{2})}}{169136}\right)}{169136}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]

[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 79399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sqrt[2]))])*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))])*(509587 - 362788*Sqrt[2] + (1235163 - 872375*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/169136

Rule 971

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

```

Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{-\frac{183}{2} + 31x - 40x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{-213004 + \frac{358655x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{465124} \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{4}(110061-77456\sqrt{2}) - \frac{121}{4}(44851-32605\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{10232728\sqrt{2}} \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} - \frac{(11(158798761480 - 11228586946\sqrt{2}))}{10232728\sqrt{2}} \\
&= \frac{(3+10x)\sqrt{3-x+2x^2}}{62(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}(112285869463 + 79399380\sqrt{2})}}{10232728\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 2.06866, size = 299, normalized size = 1.34

$$5 \left(\frac{i\sqrt{286+22i\sqrt{31}}(258253\sqrt{31}+1004586i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{(\sqrt{31}-13i)^2} + \frac{2000\left(1364(\sqrt{31}+13i)\sqrt{2x^2-x+3}(68325x^3+58315x^2+51362x+11020)-5\sqrt{286-22i\sqrt{31}}\right)}{(\sqrt{31}-13i)(\sqrt{31}+13i)^2(-10\sqrt{31}+13)} \right)$$

14418844

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]

[Out] (5*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1004586*I + 258253*Sqrt[31]))*ArcTanh[(6 + 3 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(-13*I + Sqrt[31])^2 + (2000*(1364*(13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3) - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-202151*I + 174475*Sqrt[31]))*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/((-13*I + Sqrt[31])*(13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10*I)*x)^2*(3*I + Sqrt[31] + (10*I)*x)^2))/14418844

Maple [B] time = 0.428, size = 44343, normalized size = 198.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)`

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] `integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)`

Fricas [B] time = 6.15615, size = 10298, normalized size = 46.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")`

[Out] `1/65052151896952926425996714240*(14205421276*788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(112285869463*sqrt(2) + 158798761480)*arctan(1/861047662213971287591057659551879544939625*(2461380802940*sqrt(39699690370))*(22*788032707736935368450^(3/4)*sqrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 8671430212*x^4 + 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195871*x^6 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2 - 3352976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*`

$$\begin{aligned}
& 88032707736935368450^{(1/4)} \sqrt{341} (50730703x^7 - 778833417x^6 + 411636 \\
& 7112x^5 - 9392273180x^4 + 12133646496x^3 - 7660912032x^2 - \sqrt{2} (359 \\
& 38543x^7 - 551546778x^6 + 2913578540x^5 - 6643469608x^4 + 8580088800x^ \\
& 3 - 5403919680x^2 - 6107913216x + 4313793024) - 8627586048x + 6107913216 \\
&) \sqrt{2x^2 - x + 3} \sqrt{112285869463\sqrt{2} + 158798761480} + 24442643 \\
& 31446112042193970130340819353377000\sqrt{31}\sqrt{2} (28180x^8 - 254666x^ \\
& 7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} (\\
& 2) (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x \\
& x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2\sqrt{2} (396996 \\
& 90370/160673) (\sqrt{2} (39699690370) (22*788032707736935368450^{(3/4)} \sqrt{341} * \\
& (104024992x^7 - 149335248x^6 + 480784368x^5 - 188730368x^4 + 223535232x \\
& x^3 + 214417152x^2 - \sqrt{2} (73906058x^7 - 106073653x^6 + 341348823x^5 \\
& - 133050960x^4 + 156704760x^3 + 154338048x^2 - 154338048x) - 214417152 \\
& *x) + 615345200735*788032707736935368450^{(1/4)} \sqrt{341} (7903323x^7 - 102 \\
& 233612x^6 + 394216580x^5 - 510585408x^4 + 657060192x^3 + 391744512x^2 \\
& - 4\sqrt{2} (1401761x^7 - 18132196x^6 + 69912940x^5 - 90501120x^4 + 116 \\
& 274240x^3 + 70118784x^2 - 70118784x) - 391744512x) \sqrt{2x^2 - x + 3} \\
& \sqrt{112285869463\sqrt{2} + 158798761480} + 43175912524323866211143695850* \\
& \sqrt{31}\sqrt{2} (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396 \\
& 480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} (15550x^8 - 118051x^7 + 2440 \\
& 47x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) \\
& + 3276288x) + 1962541478378357555051986175\sqrt{31} (254591x^8 - 4815126x \\
& x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 16895692 \\
& 8x^2 - 15488\sqrt{2} (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 361 \\
& 8x^3 + 2268x^2 - 1944x) + 144820224x) \sqrt{-(788032707736935368450^{(1/ \\
& 4)} \sqrt{39699690370} \sqrt{341} \sqrt{31} \sqrt{2x^2 - x + 3} (\sqrt{2} (12053 \\
& *x + 5138) - 17191x - 6915) \sqrt{112285869463\sqrt{2} + 158798761480} - 15 \\
& 0182556985858180945x^2 - 134857806273015509420\sqrt{2} (2x^2 - x + 3) + 4 \\
& 62807471527848680055x - 612990028513706861000)/x^2) + 27775731039160364115 \\
& 840569662963856288375\sqrt{31} (2828123x^8 - 9696916x^7 + 53385560x^6 - \\
& 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2} (2) \\
& * (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x \\
& x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 \\
& + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - \\
& 24772608x + 18579456) + 14205421276*788032707736935368450^{(1/4)} \sqrt{3969 \\
& 9690370} \sqrt{341} \sqrt{2} (25x^4 + 30x^3 + 29x^2 + 12x + 4) \sqrt{11228 \\
& 5869463\sqrt{2} + 158798761480} \arctan(1/8610476622139712875910576595518795 \\
& 44939625 (2461380802940\sqrt{39699690370} (22*788032707736935368450^{(3/4)} *s \\
& qrt(341) (667937076x^7 - 2573871186x^6 + 5404850058x^5 - 8671430212x^4 \\
& + 4348809776x^3 - 2064441888x^2 - \sqrt{2} (473555282x^7 - 1821195871x^6 \\
& + 3826055542x^5 - 6128133137x^4 + 3070797960x^3 - 1452037320x^2 - 3352 \\
& 976640x + 2366869248) - 4733738496x + 3352976640) + 615345200735*78803270 \\
& 7736935368450^{(1/4)} \sqrt{341} (50730703x^7 - 778833417x^6 + 4116367112x^ \\
& 5 - 9392273180x^4 + 12133646496x^3 - 7660912032x^2 - \sqrt{2} (35938543x \\
& ^7 - 551546778x^6 + 2913578540x^5 - 6643469608x^4 + 8580088800x^3 - 540
\end{aligned}$$

$$\begin{aligned}
& 3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216)) * \text{sqrt} \\
& (2*x^2 - x + 3) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) - 244426433144611 \\
& 2042193970130340819353377000 * \text{sqrt}(31) * \text{sqrt}(2) * (28180*x^8 - 254666*x^7 + 704 \\
& 270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2) * (874 \\
& 6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3 \\
& 96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2 * \text{sqrt}(39699690370/1 \\
& 60673) * (\text{sqrt}(39699690370) * (22*788032707736935368450^{(3/4)} * \text{sqrt}(341) * (104024 \\
& 992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2 \\
& 14417152*x^2 - \text{sqrt}(2) * (73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330 \\
& 50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6 \\
& 15345200735*788032707736935368450^{(1/4)} * \text{sqrt}(341) * (7903323*x^7 - 102233612* \\
& x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4 * \text{sqrt} \\
& t(2) * (1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240* \\
& x^3 + 70118784*x^2 - 70118784*x) - 391744512*x) * \text{sqrt}(2*x^2 - x + 3) * \text{sqrt}(1 \\
& 12285869463 * \text{sqrt}(2) + 158798761480) - 43175912524323866211143695850 * \text{sqrt}(31 \\
&) * \text{sqrt}(2) * (123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 \\
& + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2) * (15550*x^8 - 118051*x^7 + 244047*x^6 \\
& - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762 \\
& 88*x) - 1962541478378357555051986175 * \text{sqrt}(31) * (254591*x^8 - 4815126*x^7 + 3 \\
& 2303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - \\
& 15488 * \text{sqrt}(2) * (4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + \\
& 2268*x^2 - 1944*x) + 144820224*x) * \text{sqrt}((788032707736935368450^{(1/4)} * \text{sqrt}(\\
& 39699690370) * \text{sqrt}(341) * \text{sqrt}(31) * \text{sqrt}(2*x^2 - x + 3) * (\text{sqrt}(2) * (12053*x + 513 \\
& 8) - 17191*x - 6915) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) + 1501825569 \\
& 85858180945*x^2 + 134857806273015509420 * \text{sqrt}(2) * (2*x^2 - x + 3) - 462807471 \\
& 527848680055*x + 612990028513706861000)/x^2) - 2777573103916036411584056966 \\
& 2963856288375 * \text{sqrt}(31) * (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534 \\
& 4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744 * \text{sqrt}(2) * (1348*x \\
& ^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43 \\
& 20*x - 5184) + 223064064*x - 94887936) / (2585191*x^8 - 4661200*x^7 + 141919 \\
& 20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608 \\
& *x + 18579456) + 788032707736935368450^{(1/4)} * \text{sqrt}(39699690370) * \text{sqrt}(341) * \text{s} \\
& \text{qrt}(31) * (3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285 \\
& 869463 * \text{sqrt}(2) * (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63 \\
& 5195045920) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) * \log(635195045920/1606 \\
& 73 * (788032707736935368450^{(1/4)} * \text{sqrt}(39699690370) * \text{sqrt}(341) * \text{sqrt}(31) * \text{sqrt}(2 \\
& *x^2 - x + 3) * (\text{sqrt}(2) * (12053*x + 5138) - 17191*x - 6915) * \text{sqrt}(112285869463 \\
& * \text{sqrt}(2) + 158798761480) + 150182556985858180945*x^2 + 13485780627301550942 \\
& 0 * \text{sqrt}(2) * (2*x^2 - x + 3) - 462807471527848680055*x + 612990028513706861000 \\
&) / x^2) - 788032707736935368450^{(1/4)} * \text{sqrt}(39699690370) * \text{sqrt}(341) * \text{sqrt}(31) * (\\
& 3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285869463 * \text{s} \\
& \text{qrt}(2) * (25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63519504592 \\
& 0) * \text{sqrt}(112285869463 * \text{sqrt}(2) + 158798761480) * \log(-635195045920/160673 * (7880 \\
& 32707736935368450^{(1/4)} * \text{sqrt}(39699690370) * \text{sqrt}(341) * \text{sqrt}(31) * \text{sqrt}(2*x^2 - x \\
& + 3) * (\text{sqrt}(2) * (12053*x + 5138) - 17191*x - 6915) * \text{sqrt}(112285869463 * \text{sqrt}(2)
\end{aligned}$$

+ 158798761480) - 150182556985858180945*x^2 - 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) + 462807471527848680055*x - 612990028513706861000)/x^2) + 769228926981280465731680*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*(x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)

[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.65 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=231

$$\frac{625}{24} (2x^2 - x + 3)^{5/2} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{5/2} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{5/2} x^5 + \frac{941905 (2x^2 - x + 3)^{5/2} x^4}{9216} + \frac{10444117}{9216}$$

[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/146800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 - x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 + (941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2))/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])

Rubi [A] time = 0.342448, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24} (2x^2 - x + 3)^{5/2} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{5/2} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{5/2} x^5 + \frac{941905 (2x^2 - x + 3)^{5/2} x^4}{9216} + \frac{10444117}{9216}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]

[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/146800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 - x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 + (941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2))/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*

$e^{(q+p)x^{q-1}} - c e^{(q+2p+1)x^q}$, x , x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 dx &= \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{24} \int (3-x+2x^2)^{3/2} (384+2304x+9024x^2+22464x^3+3840x^4+1920x^5) dx \\
&= \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{7625}{96} x^6 (3-x+2x^2)^{5/2} + \frac{625}{24} x^7 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= -\frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= -\frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx \\
&= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x) (3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283 (3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x (3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2 (3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3 (3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4 (3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768} x^5 (3-x+2x^2)^{5/2} + \frac{625}{24} x^6 (3-x+2x^2)^{5/2} + \frac{1}{528} \int (3-x+2x^2)^{3/2} (8400x^4+50400x^3+100800x^2+100800x+38400) dx
\end{aligned}$$

Mathematica [A] time = 0.366561, size = 95, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 745133220000x^7 + 534038708224000x^6 + 144451829760000x^5 + 70464307200000x^4 + 144451829760000x^3 + 70464307200000x^2 + 144451829760000x + 70464307200000)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]


```
[Out] (4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*
x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 765
087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651174
400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/2705829396480
```

Maple [A] time = 0.072, size = 185, normalized size = 0.8

$$\frac{7625 x^6}{96} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{95165 x^5}{768} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{941905 x^4}{9216} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{10444117 x^3}{294912} (2x^2 - x + 3)^{\frac{5}{2}} - \frac{56422489}{8257536} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{48669967}{22020096} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{26366414481}{2147483648} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{606427533063}{8589934592} (2x^2 - x + 3)^{\frac{5}{2}} \operatorname{arcsinh}\left(\frac{4x - 1}{23\sqrt{23}}\right) + \frac{382121949}{134217728} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{2124689283}{146800640} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{625}{24} (2x^2 - x + 3)^{\frac{5}{2}} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{\frac{5}{2}} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{\frac{5}{2}} x^5 + \frac{941905}{9216} (2x^2 - x + 3)^{\frac{5}{2}} x^4 + \frac{10444117}{294912} (2x^2 - x + 3)^{\frac{5}{2}} x^3 - \frac{56422489}{8257536} (2x^2 - x + 3)^{\frac{5}{2}} x^2 + \frac{48669967}{22020096} (2x^2 - x + 3)^{\frac{5}{2}} x + \frac{2124689283}{146800640} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{382121949}{134217728} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{382121949}{134217728} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{26366414481}{536870912} \sqrt{2x^2 - x + 3} + \frac{606427533063}{8589934592} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23\sqrt{23}}(4x - 1)\right) - \frac{26366414481}{2147483648} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 7625/96*x^6*(2*x^2-x+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+941905/9216*x
^4*(2*x^2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)-56422489/8257536
*x^2*(2*x^2-x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)+26366414481/21
47483648*(-1+4*x)*(2*x^2-x+3)^(1/2)+606427533063/8589934592*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+382121949/134217728*(-1+4*x)*(2*x^2-x+3)^(3/2)+2124
689283/146800640*(2*x^2-x+3)^(3/2)+625/24*x^7*(2*x^2-x+3)^(5/2)
```

Maxima [A] time = 1.54572, size = 278, normalized size = 1.2

$$\frac{625}{24} (2x^2 - x + 3)^{\frac{5}{2}} x^7 + \frac{7625}{96} (2x^2 - x + 3)^{\frac{5}{2}} x^6 + \frac{95165}{768} (2x^2 - x + 3)^{\frac{5}{2}} x^5 + \frac{941905}{9216} (2x^2 - x + 3)^{\frac{5}{2}} x^4 + \frac{10444117}{294912} (2x^2 - x + 3)^{\frac{5}{2}} x^3 - \frac{56422489}{8257536} (2x^2 - x + 3)^{\frac{5}{2}} x^2 + \frac{48669967}{22020096} (2x^2 - x + 3)^{\frac{5}{2}} x + \frac{2124689283}{146800640} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{382121949}{134217728} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{382121949}{134217728} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{26366414481}{536870912} \sqrt{2x^2 - x + 3} + \frac{606427533063}{8589934592} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23\sqrt{23}}(4x - 1)\right) - \frac{26366414481}{2147483648} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 9516
5/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 + 1
0444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x + 3)
^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/1468006
40*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x - 382
121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2*x^2 -
x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)
) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

Fricas [A] time = 1.58455, size = 532, normalized size = 2.3

$$\frac{1}{676457349120} (70464307200000x^{11} + 144451829760000x^{10} + 349379651174400x^9 + 534038708224000x^8 + 745133229998080x^7 + 765087080448000x^6 + 675479464714240x^5 + 451581382260736x^4 + 239021184223104x^3 + 65151998063712x^2 + 12971175524316x + 74032009514181) \sqrt{2x^2 - x + 3} + 606427533063/17179869184 \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2 - x + 3})(4x - 1) - 32x^2 + 16x - 25$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 349379651174400*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 + 675479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 65151998063712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 606427533063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3))*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)

[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)

Giac [A] time = 1.17427, size = 139, normalized size = 0.6

$$\frac{1}{676457349120} (4(8(4(16(20(8(28(160(12(200(20x+41)x+19833)x+363785)x+81213077)x+2334860475)x+16491197869)x+22049872180$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833)*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 22049872180

$$\begin{aligned} & 7)x + 1867353001743)x + 2035999939491)x + 3242793881079)x + 74032009514 \\ & 181)\sqrt{2x^2 - x + 3} - 606427533063/8589934592\sqrt{2}\log(-2\sqrt{2})(\\ & \sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1) \end{aligned}$$

$$3.66 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=189

$$\frac{25}{4} (2x^2 - x + 3)^{5/2} x^5 + \frac{725}{48} (2x^2 - x + 3)^{5/2} x^4 + \frac{27785 (2x^2 - x + 3)^{5/2} x^3}{1536} + \frac{384739 (2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685 (2x^2 - x + 3)^{5/2}}{114}$$

[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (81685*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/43008 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sqrt[23]])/(67108864*Sqrt[2])

Rubi [A] time = 0.18971, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4} (2x^2 - x + 3)^{5/2} x^5 + \frac{725}{48} (2x^2 - x + 3)^{5/2} x^4 + \frac{27785 (2x^2 - x + 3)^{5/2} x^3}{1536} + \frac{384739 (2x^2 - x + 3)^{5/2} x^2}{43008} - \frac{81685 (2x^2 - x + 3)^{5/2}}{114}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]

[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (81685*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/43008 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sqrt[23]])/(67108864*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c,

$p\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

Rule 640

$\text{Int}[\{(d_.) + (e_.)*(x_.)\} \{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2\}^{(p_.)}, x_Symbol]$
 $:= \text{Simp}[(e*(a + b*x + c*x^2)^{(p + 1)})/(2*c*(p + 1)), x] + \text{Dist}[(2*c*d - b$
 $*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, p\}, x]$
 $\&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

Rule 612

$\text{Int}[\{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2\}^{(p_.)}, x_Symbol] := \text{Simp}[(b + 2*c*x$
 $)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2$
 $*p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p - 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x] \&\& \text{N}$
 $eQ[b^2 - 4*a*c, 0] \&\& \text{GtQ}[p, 0] \&\& \text{IntegerQ}[4*p]$

Rule 619

$\text{Int}[\{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2\}^{(p_.)}, x_Symbol] := \text{Dist}[1/(2*c*((-4$
 $*c)/(b^2 - 4*a*c))^p], \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b$
 $+ 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x] \&\& \text{GtQ}[4*a - b^2/c, 0]$

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_.)^2], x_Symbol] := \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqr}$
 $t[a]]/\text{Rt}[b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{PosQ}[b]$

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 dx &= \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{20} \int (3-x+2x^2)^{3/2} (160+720x+2280x^2+4140x^3 \\
&= \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \frac{1}{360} \int (3-x+2x^2)^{3/2} (2880 \\
&= \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} + \frac{25}{4}x^5(3-x+2x^2)^{5/2} + \int \\
&= \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4(3-x+2x^2)^{5/2} \\
&= -\frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} \\
&= -\frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} \\
&= -\frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907}{2} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907}{2} \\
&= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907}{2}
\end{aligned}$$

Mathematica [A] time = 0.2339, size = 85, normalized size = 0.45

$$4\sqrt{2x^2-x+3}(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+48789140928x^4+111278019825\sqrt{3-x+2x^2})$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2*x^2)^(3/2)*(2+3*x+5*x^2)^3,x]

[Out] (4*Sqrt[3-x+2*x^2]*(-72152399943+53985432012*x+199615064544*x^2+389257196928*x^3+487891884032*x^4+571298324480*x^5+430820229120*x^6+328328806400*x^7+124780544000*x^8+88080384000*x^9)-111278019825*Sqrt[

2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/14092861440

Maple [A] time = 0.059, size = 151, normalized size = 0.8

$$\frac{25x^5}{4}(2x^2-x+3)^{\frac{5}{2}} + \frac{725x^4}{48}(2x^2-x+3)^{\frac{5}{2}} + \frac{27785x^3}{1536}(2x^2-x+3)^{\frac{5}{2}} + \frac{384739x^2}{43008}(2x^2-x+3)^{\frac{5}{2}} - \frac{81685x}{114688}(2x^2-x+3)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x)

[Out] 25/4*x^5*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+27785/1536*x^3*(2*x^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)-81685/114688*x*(2*x^2-x+3)^(5/2)+46077855/33554432*(-1+4*x)*(2*x^2-x+3)^(1/2)+1059790665/134217728*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+667795/2097152*(-1+4*x)*(2*x^2-x+3)^(3/2)-4625907/2293760*(2*x^2-x+3)^(5/2)

Maxima [A] time = 1.50344, size = 232, normalized size = 1.23

$$\frac{25}{4}(2x^2-x+3)^{\frac{5}{2}}x^5 + \frac{725}{48}(2x^2-x+3)^{\frac{5}{2}}x^4 + \frac{27785}{1536}(2x^2-x+3)^{\frac{5}{2}}x^3 + \frac{384739}{43008}(2x^2-x+3)^{\frac{5}{2}}x^2 - \frac{81685}{114688}(2x^2-x+3)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] 25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/1536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 - 81685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/2) + 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3)^(3/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.59326, size = 423, normalized size = 2.24

$$\frac{1}{3523215360}(88080384000x^9 + 124780544000x^8 + 328328806400x^7 + 430820229120x^6 + 571298324480x^5 + 487808000000x^4 + 286720000000x^3 + 105979066500x^2 + 10597906650x + 460778550)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 430820229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 199615064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 1059790665/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

Giac [A] time = 1.16042, size = 126, normalized size = 0.67

$$\frac{1}{3523215360} (4 (8 (4 (16 (20 (8 (140 (160 (12x + 17)x + 7157)x + 1314759)x + 13947713)x + 238228459)x + 3041071851$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 1314759)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 13496358003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```


$$3.67 \quad \int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=147

$$\frac{25}{16} (2x^2 - x + 3)^{5/2} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x)}{196608}$$

[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 - x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])

Rubi [A] time = 0.122099, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16} (2x^2 - x + 3)^{5/2} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{24499 (2x^2 - x + 3)^{5/2} x}{10752} + \frac{73861 (2x^2 - x + 3)^{5/2}}{215040} + \frac{24293(1 - 4x)}{196608}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 - x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
  *e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
  )*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
  *p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
  eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
  *c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
  + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
  t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 dx &= \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{16} \int (3-x+2x^2)^{3/2} \left(64+192x+239x^2 + \frac{1235x^3}{2}\right) \\
&= \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \frac{1}{224} \int (3-x+2x^2)^{3/2} (89 \\
&= \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \frac{25}{16}x^3(3-x+2x^2)^{5/2} + \\
&= \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2(3-x+2x^2)^{5/2} + \\
&= \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} \\
&= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)(3-x+2x^2)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040}
\end{aligned}$$

Mathematica [A] time = 0.14373, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+128312x+1349354685)\text{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]+440401920}{440401920}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7) + 1349354685*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/440401920

Maple [A] time = 0.055, size = 117, normalized size = 0.8

$$\frac{25x^3}{16}(2x^2-x+3)^{\frac{5}{2}} + \frac{1235x^2}{448}(2x^2-x+3)^{\frac{5}{2}} + \frac{24499x}{10752}(2x^2-x+3)^{\frac{5}{2}} - \frac{-558739+2234956x}{1048576}\sqrt{2x^2-x+3} - \frac{128312}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)`

[Out] $25/16*x^3*(2*x^2-x+3)^{(5/2)}+1235/448*x^2*(2*x^2-x+3)^{(5/2)}+24499/10752*x*(2*x^2-x+3)^{(5/2)}-558739/1048576*(-1+4*x)*(2*x^2-x+3)^{(1/2)}-12850997/4194304*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))-24293/196608*(-1+4*x)*(2*x^2-x+3)^{(3/2)}+73861/215040*(2*x^2-x+3)^{(5/2)}$

Maxima [A] time = 1.49987, size = 186, normalized size = 1.27

$$\frac{25}{16} (2x^2 - x + 3)^{\frac{5}{2}} x^3 + \frac{1235}{448} (2x^2 - x + 3)^{\frac{5}{2}} x^2 + \frac{24499}{10752} (2x^2 - x + 3)^{\frac{5}{2}} x + \frac{73861}{215040} (2x^2 - x + 3)^{\frac{5}{2}} - \frac{24293}{49152} (2x^2 - x + 3)^{\frac{3}{2}} x + \frac{24293}{196608} (2x^2 - x + 3)^{\frac{3}{2}} - 558739/262144*\sqrt{2*x^2 - x + 3}*x - 12850997/4194304*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + 558739/1048576*\sqrt{2*x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] $25/16*(2*x^2 - x + 3)^{(5/2)}*x^3 + 1235/448*(2*x^2 - x + 3)^{(5/2)}*x^2 + 24499/10752*(2*x^2 - x + 3)^{(5/2)}*x + 73861/215040*(2*x^2 - x + 3)^{(5/2)} - 24293/49152*(2*x^2 - x + 3)^{(3/2)}*x + 24293/196608*(2*x^2 - x + 3)^{(3/2)} - 558739/262144*\sqrt{2*x^2 - x + 3}*x - 12850997/4194304*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + 558739/1048576*\sqrt{2*x^2 - x + 3}$

Fricas [A] time = 1.51727, size = 342, normalized size = 2.33

$$\frac{1}{110100480} (688128000 x^7 + 525926400 x^6 + 2025840640 x^5 + 2061273088 x^4 + 2728413312 x^3 + 1799647136 x^2 + 1619403428 x + 439831323) \sqrt{2x^2 - x + 3} + 12850997/8388608*\sqrt{2}*\log(4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")`

[Out] $1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*\sqrt{2*x^2 - x + 3} + 12850997/8388608*\sqrt{2}*\log(4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)

[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)

Giac [A] time = 1.15408, size = 112, normalized size = 0.76

$$\frac{1}{110100480} (4 (8 (4 (16 (20 (120 (140x + 107)x + 49459)x + 1006481)x + 21315729)x + 56238973)x + 404850857)x + 439831323) \sqrt{2x^2 - x + 3} + 12850997/4194304 \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x + 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3) + 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

3.68 $\int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) dx$

Optimal. Leaf size=105

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691 \sinh^{-1}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2))/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])

Rubi [A] time = 0.0504904, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x(2x^2 - x + 3)^{5/2} + \frac{107}{240}(2x^2 - x + 3)^{5/2} - \frac{179(1 - 4x)(2x^2 - x + 3)^{3/2}}{1536} - \frac{4117(1 - 4x)\sqrt{2x^2 - x + 3}}{8192} - \frac{94691 \sinh^{-1}\left(\frac{1 - 4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2), x]

[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2))/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]

$\&\& \text{NeQ}[2*c*d - b*e, 0] \ \&\& \text{NeQ}[p, -1]$

Rule 612

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_.)}, x_Symbol] \text{ :> } \text{Simp}[(b + 2*c*x) * (a + b*x + c*x^2)^p / (2*c*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c)) / (2*c*(2*p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p - 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x] \ \&\& \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \text{GtQ}[p, 0] \ \&\& \text{IntegerQ}[4*p]$

Rule 619

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_.)}, x_Symbol] \text{ :> } \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c)], x]^p, x], x, b + 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x] \ \&\& \text{GtQ}[4*a - b^2/c, 0]$

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \text{ :> } \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x] \ \&\& \text{GtQ}[a, 0] \ \&\& \text{PosQ}[b]$

Rubi steps

$$\begin{aligned} \int (3-x+2x^2)^{3/2} (2+3x+5x^2) dx &= \frac{5}{12}x(3-x+2x^2)^{5/2} + \frac{1}{12} \int \left(9 + \frac{107x}{2}\right) (3-x+2x^2)^{3/2} dx \\ &= \frac{107}{240} (3-x+2x^2)^{5/2} + \frac{5}{12}x(3-x+2x^2)^{5/2} + \frac{179}{96} \int (3-x+2x^2)^{3/2} dx \\ &= -\frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} + \frac{5}{12}x(3-x+2x^2)^{5/2} + \\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} \\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} \\ &= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)(3-x+2x^2)^{3/2}}{1536} + \frac{107}{240} (3-x+2x^2)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.0837233, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2 - x + 3} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341) - 1420365\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{491520}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2), x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336*x^4 + 204800*x^5) - 1420365*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/491520

Maple [A] time = 0.058, size = 83, normalized size = 0.8

$$\frac{5x}{12} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{107}{240} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{-179 + 716x}{1536} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{-4117 + 16468x}{8192} \sqrt{2x^2 - x + 3} + \frac{94691\sqrt{2}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2), x)

[Out] 5/12*x*(2*x^2-x+3)^(5/2)+107/240*(2*x^2-x+3)^(5/2)+179/1536*(-1+4*x)*(2*x^2-x+3)^(3/2)+4117/8192*(-1+4*x)*(2*x^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))

Maxima [A] time = 1.49874, size = 140, normalized size = 1.33

$$\frac{5}{12} (2x^2 - x + 3)^{\frac{5}{2}} x + \frac{107}{240} (2x^2 - x + 3)^{\frac{5}{2}} + \frac{179}{384} (2x^2 - x + 3)^{\frac{3}{2}} x - \frac{179}{1536} (2x^2 - x + 3)^{\frac{3}{2}} + \frac{4117}{2048} \sqrt{2x^2 - x + 3} x + \frac{94691}{32768} \sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (4x - 1)\right) - \frac{4117}{192} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2), x, algorithm="maxima")

[Out] 5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*x^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x^2 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4117/192*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.60156, size = 257, normalized size = 2.45

$$\frac{1}{122880} (204800x^5 + 14336x^4 + 561024x^3 + 319072x^2 + 565276x + 388341) \sqrt{2x^2 - x + 3} + \frac{94691}{65536} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 388341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)

[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2), x)

Giac [A] time = 1.24798, size = 99, normalized size = 0.94

$$\frac{1}{122880} (4(8(4(16(100x + 7)x + 4383)x + 9971)x + 141319)x + 388341) \sqrt{2x^2 - x + 3} - \frac{94691}{32768} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")

[Out] 1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 388341)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.69 \quad \int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=197

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x) + \frac{11}{125}\sqrt{\frac{11}{31}(247+500\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}}((130+69\sqrt{2})x+61\sqrt{2}+8)}{\sqrt{2x^2-x+3}} \right) - \frac{11}{125}\sqrt{2x^2-x+3}$$

[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(247 + 500*Sqrt[2]))])*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-247 + 500*Sqrt[2]))])*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/125

Rubi [A] time = 0.487911, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x) + \frac{11}{125}\sqrt{\frac{11}{31}(247+500\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{62(247+500\sqrt{2})}}((130+69\sqrt{2})x+61\sqrt{2}+8)}{\sqrt{2x^2-x+3}} \right) - \frac{11}{125}\sqrt{2x^2-x+3}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2), x]

[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(247 + 500*Sqrt[2]))])*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-247 + 500*Sqrt[2]))])*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/125

Rule 977

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p

+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q) * (2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p) * (2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))]*x + ((c*e - b*f)^2*(1 - p)*p + c*(p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2) * Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1035

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2} dx &= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50} \int \frac{-\frac{731}{2} + \frac{1195x}{4} - \frac{2203x^2}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250} \int \frac{-726+3146x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx + \frac{2203}{1000} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662(16+3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} - \frac{\int \frac{2662(16-3\sqrt{2})+2662(10+13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{5500\sqrt{2}} \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125} \left(322102(1000-247\sqrt{2})\right) \text{Subst}\left(\int \frac{1}{\sqrt{62(247-x+2x^2)}} dx\right) \\
&= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125} \sqrt{\frac{11}{31}} (247+500\sqrt{2}) \tan^{-1}\left(\frac{\sqrt{62(247-x+2x^2)}}{247-x+2x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.695284, size = 310, normalized size = 1.57

$$400\sqrt{31}\sqrt{2x^2-x+3}x - 980\sqrt{31}\sqrt{2x^2-x+3} + 44\sqrt{286+22i\sqrt{31}}(\sqrt{31}-13i)\tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + 572i$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2), x]

[Out] (-980*Sqrt[31]*Sqrt[3 - x + 2*x^2] + 400*Sqrt[31]*x*Sqrt[3 - x + 2*x^2] - 2203*Sqrt[62]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 44*Sqrt[286 + (22*I)*Sqrt[31]]*(-13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 44*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (572*I)*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(2000*Sqrt[31])

Maple [B] time = 0.19, size = 3460, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2), x)

[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+6154*(-775687+549362*2^(1/2))^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+123

$$\begin{aligned}
& /(-8866+6820*2^{(1/2)})^{(1/2)}+3/10571*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(151*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+218*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+401698*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-63426*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))^{(1/2)})/((1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}+9/21142*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(369*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+520*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\arctan(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+465124*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-866822*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}))/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))^{(1/2)})/((1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)

Fricas [B] time = 4.32353, size = 7468, normalized size = 37.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
+ 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^
5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 595

$$\begin{aligned}
& 0*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781 \\
& 920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 51 \\
& 7*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x)) \\
& *\sqrt{((24200^{(1/4)}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(x - 75) + 74*x - \\
& 76)*\sqrt{247*\sqrt{2} + 1000}) + 58310*x^2 + 52360*\sqrt{2}*(2*x^2 - x + 3) - \\
& 179690*x + 238000)/x^2) + 342125*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 5338 \\
& 5560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7 \\
& 744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568* \\
& x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4 \\
& 661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3461 \\
& 5296*x^2 - 24772608*x + 18579456)) + 11/77500*24200^{(1/4)}*\sqrt{31}*\sqrt{10} \\
& *\sqrt{2}*\sqrt{247*\sqrt{2} + 1000}*\arctan(1/10605875*(230*\sqrt{10}*(2*24200^{(3/4)} \\
& *\sqrt{31}*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x \\
& ^3 - 156600*x^2 - \sqrt{2}*(28854*x^7 - 90639*x^6 + 200187*x^5 - 262838*x^4 \\
& + 117544*x^3 - 23472*x^2 - 186624*x + 86400) - 172800*x + 186624) + 5*24200 \\
& ^{(1/4)}*\sqrt{31}*(112238*x^7 - 1817988*x^6 + 10351960*x^5 - 25791248*x^4 + 3 \\
& 4522560*x^3 - 28368000*x^2 - \sqrt{2}*(125839*x^7 - 1864281*x^6 + 9323336*x^5 \\
& - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - 19989504*x + 10533888) - 2 \\
& 1067776*x + 19989504))*\sqrt{2*x^2 - x + 3}*\sqrt{247*\sqrt{2} + 1000}) - 30107 \\
& 000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1 \\
& 549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 3961 \\
& 04*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53 \\
& 9136) + 1154304*x - 456192) - \sqrt{5/119}*(\sqrt{10}*(2*24200^{(3/4)}*\sqrt{31} \\
& *(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 \\
& - 16*\sqrt{2}*(7714*x^7 - 10881*x^6 + 33771*x^5 - 5576*x^4 - 576*x^3 + 3218 \\
& 4*x^2 - 32184*x) + 269568*x) + 5*24200^{(1/4)}*\sqrt{31}*(309512*x^7 - 4017952 \\
& *x^6 + 15741280*x^5 - 22625280*x^4 + 37693440*x^3 - 13519872*x^2 - \sqrt{2}*(\\
& 516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46 \\
& 199808*x^2 - 46199808*x) + 13519872*x))*\sqrt{2*x^2 - x + 3}*\sqrt{247*\sqrt{2} \\
&) + 1000) - 130900*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 \\
& - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 \\
& - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 120960 \\
& 0*x^2 - 1036800*x) + 3276288*x) - 5950*\sqrt{31}*(254591*x^8 - 4815126*x^7 + \\
& 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 \\
& - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 \\
& + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(24200^{(1/4)}*\sqrt{10}*\sqrt{2*x^2 \\
& 2 - x + 3}*(\sqrt{2}*(x - 75) + 74*x - 76)*\sqrt{247*\sqrt{2} + 1000}) - 58310* \\
& x^2 - 52360*\sqrt{2}*(2*x^2 - x + 3) + 179690*x - 238000)/x^2) - 342125*\sqrt{ \\
& 31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592* \\
& x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 97 \\
& 89*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223 \\
& 064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^ \\
& 5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + \\
& 11/36890000*24200^{(1/4)}*\sqrt{10}*\sqrt{247*\sqrt{2} + 1000}*(247*\sqrt{2} - 10 \\
& 00)*\log(1512500/119*(24200^{(1/4)}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(x -
\end{aligned}$$

$$\begin{aligned}
 & (75 + 74x - 76)\sqrt{247\sqrt{2} + 1000} + 58310x^2 + 52360\sqrt{2}(2x^2 - x + 3) - 179690x + 238000)/x^2) - 11/36890000 \cdot 24200^{1/4} \cdot \sqrt{10} \cdot \sqrt{247\sqrt{2} + 1000} \cdot (247\sqrt{2} - 1000) \cdot \log(-1512500/119 \cdot (24200^{1/4}) \cdot \sqrt{10} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2}(x - 75) + 74x - 76) \cdot \sqrt{247\sqrt{2} + 1000}) - 58310x^2 - 52360\sqrt{2}(2x^2 - x + 3) + 179690x - 238000)/x^2) + 1/100 \cdot \sqrt{2x^2 - x + 3} \cdot (20x - 49) + 2203/4000 \cdot \sqrt{2} \cdot \log(-4\sqrt{2}) \cdot \sqrt{2x^2 - x + 3} \cdot (4x - 1) - 32x^2 + 16x - 25)
 \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)

[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.70 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=232

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}((9440+6477\sqrt{2})x)}{\sqrt{2x^2-x+3}}\right)}{1550}$$

[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 2265350*Sqrt[2])))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-3169333 + 2265350*Sqrt[2])))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550

Rubi [A] time = 0.575455, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)\sqrt{2x^2-x+3} + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}((9440+6477\sqrt{2})x)}{\sqrt{2x^2-x+3}}\right)}{1550}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]

[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 2265350*Sqrt[2])))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-3169333 + 2265350*Sqrt[2])))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550

Rule 971

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), x, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

```

Rule 1066

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/((2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/((2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x, Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 619

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

```

Rule 215

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{\sqrt{3-x+2x^2} \left(-\frac{69}{2} + 13x + 40x^2\right)}{2+3x+5x^2} dx \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{3100} \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{15500} + \frac{4}{25} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} + \frac{1}{25} \left(2\sqrt{\frac{2}{23}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, \right. \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2} \sinh^{-1} \left(\frac{1-4x}{\sqrt{23}} \right) + \frac{1}{155} \left(1331 \left(4 \right. \right. \\
&= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2} \sinh^{-1} \left(\frac{1-4x}{\sqrt{23}} \right) + \frac{\sqrt{\frac{11}{31}}(316933)}{155}
\end{aligned}$$

Mathematica [C] time = 2.58377, size = 530, normalized size = 2.28

$$\frac{62000\sqrt{2x^2-x+3x^2}}{10x-i\sqrt{31}+3} + \frac{62000\sqrt{2x^2-x+3x^2}}{10x+i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3x^2}}{10x-i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3x^2}}{10x+i\sqrt{31}+3} - 12400\sqrt{2x^2-x+3x^2} + \frac{93000\sqrt{2x^2-x+3x^2}}{10x-i\sqrt{31}+3} + \frac{93000\sqrt{2x^2-x+3x^2}}{10x+i\sqrt{31}+3}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]

[Out] (9920*Sqrt[3 - x + 2*x^2] - 12400*x*Sqrt[3 - x + 2*x^2] + (93000*Sqrt[3 - x + 2*x^2]))/(3 - I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) + (93000*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - 7688*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - (Sqrt[286 + (22*I)*Sqrt[31]]*(10199*I + 6477*Sqrt[31]))*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]/(-

$$\frac{13\sqrt{31} - (6477\sqrt{682(13 - \sqrt{31})})\operatorname{ArcTanh}\left(\frac{63 - \sqrt{31} - 22x + (4\sqrt{31})x}{2\sqrt{286 - (22\sqrt{31})x}}\right)}{13\sqrt{31} + (10199\sqrt{682(13 - \sqrt{31})})\operatorname{ArcTanh}\left(\frac{63 - \sqrt{31} - 22x + (4\sqrt{31})x}{2\sqrt{286 - (22\sqrt{31})x}}\right)} \sqrt{3 - x + 2x^2}}{96100}$$

Maple [B] time = 0.355, size = 28185, normalized size = 121.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)`

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")`

[Out] `integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)`

Fricas [B] time = 5.10758, size = 8892, normalized size = 38.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")`

```
[Out] 1/90746855745853600*(10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(2)*(5*x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/172758074198
807633719789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118
*x^7 - 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x
^2 - sqrt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 113
67152*x^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) +
1404517*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x
^5 - 70476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 423
2733*x^6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 4669
7472*x + 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(31693
33*sqrt(2) + 4530700) + 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^
8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 9849
6*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
sqrt(45307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7
- 12210811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2
- sqrt(2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 121140
72*x^3 + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(
1/4)*sqrt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 1
12374720*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 5039126
0*x^5 - 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304
*x))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) + 7590571938849196
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
+ 3276288*x) + 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 323035
80*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 1548
8*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268
*x^2 - 1944*x) + 144820224*x))*sqrt(-(1987037073032^(1/4)*sqrt(45307)*sqrt(
62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*s
qrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2
)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187
343023219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^
5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 -
2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x
- 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x
^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x +
18579456)) + 10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(2)*(5*
x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/1727580741988076337
19789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118*x^7 -
9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq
rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x
^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517
*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70
476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 4232733*x^
```


$$\begin{aligned}
& 6 + 22218448x^5 - 50249260x^4 + 64668384x^3 - 39479328x^2 - 46697472x \\
& + 32016384) - 64032768x + 46697472))\sqrt{2x^2 - x + 3})\sqrt{(3169333\sqrt{2} \\
& (2) + 4530700) - 490410017080486186043272\sqrt{31})\sqrt{2}*(28180x^8 - 254 \\
& 666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \\
& \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 7 \\
& 52088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{45 \\
& 307/2711}*(\sqrt{45307}*(2*1987037073032^{(3/4)}\sqrt{62}*(8480726x^7 - 12210 \\
& 811x^6 + 39548601x^5 - 16962480x^4 + 21434760x^3 + 14432256x^2 - \sqrt{2} \\
& (2)*(6779042x^7 - 9704193x^6 + 31062363x^5 - 11094928x^4 + 12114072x^3 \\
& + 16301952x^2 - 16301952x) - 14432256x) + 1404517*1987037073032^{(1/4)}\sqrt{2} \\
& \sqrt{62}*(1312966x^7 - 16987736x^6 + 65572040x^5 - 85530240x^4 + 11237472 \\
& 0x^3 + 57314304x^2 - \sqrt{2}*(1011501x^7 - 13081364x^6 + 50391260x^5 - \\
& 64806336x^4 + 81634464x^3 + 56070144x^2 - 56070144x) - 57314304x))\sqrt{2} \\
& \sqrt{2x^2 - x + 3})\sqrt{(3169333\sqrt{2} + 4530700) - 7590571938849196\sqrt{3} \\
& 1)\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 \\
& + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 \\
& - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276 \\
& 288x) - 345025997220418\sqrt{31}*(254591x^8 - 4815126x^7 + 32303580x^6 \\
& - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488\sqrt{2} \\
& (2)*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - \\
& 1944x) + 144820224x))\sqrt{(1987037073032^{(1/4)}\sqrt{45307})\sqrt{62})\sqrt{31} \\
& \sqrt{2x^2 - x + 3})\sqrt{(2) * (1867x + 1425) - 3292x - 442)\sqrt{(3169 \\
& 333\sqrt{2} + 4530700) + 11567627293306x^2 + 10387257161336\sqrt{2} * (2x^2 \\
& - x + 3) - 35647177985494x + 47214805278800)/x^2) - 557284110318734302321 \\
& 9\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 2541 \\
& 46592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2}*(1348x^8 - 2692x^7 \\
& + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) \\
& + 223064064x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490 \\
& 880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 1857945 \\
& 6) + 1987037073032^{(1/4)}\sqrt{45307})\sqrt{62})\sqrt{31}*(22653500x^2 - 316 \\
& 9333\sqrt{2}*(5x^2 + 3x + 2) + 13592100x + 9061400)\sqrt{(3169333\sqrt{2} \\
& + 4530700)*\log(113267500/2711*(1987037073032^{(1/4)}\sqrt{45307})\sqrt{62})\sqrt{31} \\
& \sqrt{2x^2 - x + 3})\sqrt{(2) * (1867x + 1425) - 3292x - 442)\sqrt{(31 \\
& 69333\sqrt{2} + 4530700) + 11567627293306x^2 + 10387257161336\sqrt{2} * (2x \\
& ^2 - x + 3) - 35647177985494x + 47214805278800)/x^2) - 1987037073032^{(1/4)} \\
& *\sqrt{45307})\sqrt{62})\sqrt{31}*(22653500x^2 - 3169333\sqrt{2}*(5x^2 + 3x \\
& + 2) + 13592100x + 9061400)\sqrt{(3169333\sqrt{2} + 4530700)*\log(-11326750 \\
& 0/2711*(1987037073032^{(1/4)}\sqrt{45307})\sqrt{62})\sqrt{31})\sqrt{2x^2 - x + \\
& 3})\sqrt{(2) * (1867x + 1425) - 3292x - 442)\sqrt{(3169333\sqrt{2} + 4530700) \\
& - 11567627293306x^2 - 10387257161336\sqrt{2} * (2x^2 - x + 3) + 3564717798 \\
& 5494x - 47214805278800)/x^2) + 3629874229834144\sqrt{2}*(5x^2 + 3x + 2)* \\
& \log(-4\sqrt{2})\sqrt{2x^2 - x + 3}*(4x - 1) - 32x^2 + 16x - 25) + 644009 \\
& 9440028320\sqrt{2x^2 - x + 3}*(13x + 7))/(5x^2 + 3x + 2)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)

[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.71 \quad \int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=223

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(366990269+259509026\sqrt{2})}}}{\dots}\right)}{7688}$$

[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 259509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))])]*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x)/Sqrt[3 - x + 2*x^2]])/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-366990269 + 259509026*Sqrt[2]))])]*(29367 - 20575*Sqrt[2] + (70517 - 49942*Sqrt[2])*x)/Sqrt[3 - x + 2*x^2]])/7688

Rubi [A] time = 0.432945, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{3/2}}{62(5x^2+3x+2)^2} + \frac{3(696x+277)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)} + \frac{3\sqrt{\frac{1}{682}(366990269+259509026\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(366990269+259509026\sqrt{2})}}}{\dots}\right)}{7688}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]

[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 259509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))])]*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x)/Sqrt[3 - x + 2*x^2]])/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-366990269 + 259509026*Sqrt[2]))])]*(29367 - 20575*Sqrt[2] + (70517 - 49942*Sqrt[2])*x)/Sqrt[3 - x + 2*x^2]])/7688

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), x, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((g*b - 2*a*h - (b*h - 2*g*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), x, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) + (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(2*p + 2*q + 3)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{\left(-\frac{189}{2} + 33x\right) \sqrt{3-x+2x^2}}{(2+3x+5x^2)^2} dx \\
&= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\frac{13359}{4} - 1353x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{1922} \\
&= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{\int \frac{-\frac{33}{4}(6257-4453\sqrt{2}) + \frac{33}{4}(2649-1804\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{42284\sqrt{2}} \\
&= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{(99(519018052 - 366990269\sqrt{2}))}{42284\sqrt{2}} \\
&= \frac{(3+10x)(3-x+2x^2)^{3/2}}{62(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844(2+3x+5x^2)} + \frac{3\sqrt{\frac{1}{682}(366990269 + 259509026\sqrt{2})}}{42284\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 5.38845, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (((248000*I)*Sqrt[31]*(3 - x + 2*x^2)^(3/2))/(3*I + Sqrt[31] + (10*I)*x)^2
+ (744000*(3 - x + 2*x^2)^(3/2))/(3 - I*Sqrt[31] + 10*x) + ((248000*I)*Sqrt
[31]*(3 - x + 2*x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x)^2 + (744000*(3 - x + 2*
x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x) + (3*I)*Sqrt[31]*(20*(1199 + (98*I)*Sqr
t[31] - 20*(11 + (2*I)*Sqrt[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(13453 +
(4406*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (352*Sqrt[286 + (22*I)*Sqr
t[31]]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[3
1])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(-13*I + Sqrt[
31])) + 558*(20*(27 + (4*I)*Sqrt[31] - 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*
(569 + (88*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 + (22*I)*
Sqrt[31]]*(-81*I + 37*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(-13*I + Sq
rt[31])) + (744*Sqrt[31]*(220*(-439 + (497*I)*Sqrt[31] + 20*(69 + (13*I)*Sq
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 - (398*I)*Sqrt[31] + 5*(4
7 - (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 + (22*I)*S
qrt[31]]*(19548 - (4904*I)*Sqrt[31] + (-23345 - (8565*I)*Sqrt[31])*x)*ArcTa
nh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])))/(11*(-13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10
*I)*x)) + (744*Sqrt[31]*(220*(-439 - (497*I)*Sqrt[31] + 20*(69 - (13*I)*Sq
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 + (398*I)*Sqrt[31] + 5*(47
+ (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 - (22*I)*Sq
rt[31]]*(-19548 - (4904*I)*Sqrt[31] + 5*(4669 - (1713*I)*Sqrt[31])*x)*ArcTa
nh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])))/(11*(13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I
)*x)) + (3*Sqrt[31]*(20*(12549 - (2473*I)*Sqrt[31] + (20*I)*(81*I + 37*Sqrt
[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(38303 - (70731*I)*Sqrt[31])*ArcSinh
[(1 - 4*x)/Sqrt[23]] + (352*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(69*I + 13*Sqrt[
31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(13*I + Sqrt[31]) + 558*(-20*(-27 + (
4*I)*Sqrt[31] + 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(569 - (88*I)*Sqrt[31])
*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 - (22*I)*Sqrt[31]]*(81*I + 37*Sq
rt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])))/(13*I + Sqrt[31])))/4766560
```

Maple [B] time = 0.607, size = 81552, normalized size = 365.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)

Fricas [B] time = 4.94277, size = 9586, normalized size = 42.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/85773071417697924109696*(189113268*134689869150937352^{(1/4)}*\sqrt{129754513}*\sqrt{341}*\sqrt{2}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\sqrt{366990269*\sqrt{2} + 519018052}*\arctan(1/1067259092343193675559267622545473*(160895596 \\ & 12*\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^2 - \sqrt{2}*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 191303424) + 4022389903*134689869150937352^{(1/4)}*\sqrt{341}*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 - \sqrt{2}*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696576))*\sqrt{2*x^2 - x + 3}*\sqrt{366990269*\sqrt{2} + 519018052} + 3029638713748420756426308089806504*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{259509026/713}*(\sqrt{129754513}*(11*134689869150937352^{(3/4)}*\sqrt{341}*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - \sqrt{2})* \end{aligned}$$

$$\begin{aligned}
& (4201650x^7 - 6032009x^6 + 19421619x^5 - 7633552x^4 + 9050328x^3 + 8640000x^2 - 8640000x) - 12517632x) + 4022389903 \cdot 134689869150937352^{(1/4)} \cdot \sqrt[3]{341} \cdot (453599x^7 - 5867420x^6 + 22622900x^5 - 29282112x^4 + 37610208x^3 + 22726656x^2 - \sqrt{2} \cdot (319303x^7 - 4130364x^6 + 15927060x^5 - 20630592x^4 + 26556768x^3 + 15800832x^2 - 15800832x) - 22726656x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} + 8186887989068712800954 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 372131272230396036407 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{-(134689869150937352^{(1/4)} \cdot \sqrt{29754513} \cdot \sqrt{341} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (696x + 277) - 973x - 419) \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} - 4356437317274441x^2 - 3911902897144396 \cdot \sqrt{2} \cdot (2x^2 - x + 3) + 13424939487927359x - 17781376805201800) / x^2) + 34427712656232054050298955565983 \cdot \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 189113268 \cdot 134689869150937352^{(1/4)} \cdot \sqrt{129754513} \cdot \sqrt{341} \cdot \sqrt{2} \cdot (25x^4 + 30x^3 + 29x^2 + 12x + 4) \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} \cdot \arctan(1/1067259092343193675559267622545473 \cdot (16089559612 \cdot \sqrt{129754513} \cdot (11 \cdot 134689869150937352^{(3/4)} \cdot \sqrt{341} \cdot (38305160x^7 - 147261352x^6 + 309398878x^5 - 495410374x^4 + 248212864x^3 - 117285552x^2 - \sqrt{2} \cdot (26988622x^7 - 104036813x^6 + 218448200x^5 - 350579241x^4 + 175844824x^3 - 83534472x^2 - 191303424x + 135585792) - 271171584x + 191303424) + 4022389903 \cdot 134689869150937352^{(1/4)} \cdot \sqrt{341} \cdot (2906601x^7 - 44604657x^6 + 235604928x^5 - 537156764x^4 + 693706464x^3 - 436717728x^2 - \sqrt{2} \cdot (2050114x^7 - 31475955x^6 + 166375268x^5 - 379661892x^4 + 490500864x^3 - 309827808x^2 - 348696576x + 246965760) - 493931520x + 348696576)) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} - 3029638713748420756426308089806504 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2 \cdot \sqrt{259509026/713} \cdot (\sqrt{129754513} \cdot (11 \cdot 134689869150937352^{(3/4)} \cdot \sqrt{341} \cdot (5980372x^7 - 8582986x^6 + 27618126x^5 - 10751392x^4 + 12649968x^3 + 12517632x^2 - \sqrt{2} \cdot (4201650x^7 - 6032009x^6 + 19421619x^5 - 7633552x^4 + 9050328x^3 + 8640000x^2 - 8640000x) - 12517632x) + 4022389903 \cdot 134689869150937352^{(1/4)} \cdot \sqrt{341} \cdot (453599x^7 - 5867420x^6 + 22622900x^5 - 29282112x^4 + 37610208x^3 + 22726656x^2 - \sqrt{2} \cdot (319303x^7 - 4130364x^6 + 15927060x^5 - 20630592x^4 + 26556768x^3 + 15800832x^2 - 15800832x) - 22726656x) \cdot \sqrt{2x^2 - x + 3} \cdot \sqrt{366990269 \cdot \sqrt{2} + 519018052} -
\end{aligned}$$


```

8186887989068712800954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 372131272230396036407*sqrt(31)*(2545
91*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742193
28*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5
+ 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((134689869
150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqr
t(2)*(696*x + 277) - 973*x - 419)*sqrt(366990269*sqrt(2) + 519018052) + 435
6437317274441*x^2 + 3911902897144396*sqrt(2)*(2*x^2 - x + 3) - 134249394879
27359*x + 17781376805201800)/x^2) - 34427712656232054050298955565983*sqrt(3
1)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^
4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789
*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306
4064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5
- 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 3*
134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300*x^
4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2) +
519018052)*log(9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)*sqr
t(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 419)*s
qrt(366990269*sqrt(2) + 519018052) + 4356437317274441*x^2 + 391190289714439
6*sqrt(2)*(2*x^2 - x + 3) - 13424939487927359*x + 17781376805201800)/x^2) +
3*134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300
*x^4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x
^3 + 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2)
+ 519018052)*log(-9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)
*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 41
9)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39119028971
44396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^
2) - 22313494125311634784*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt(2*x^
2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.72 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^4 dx$$

Optimal. Leaf size=254

$$\frac{625}{28} (2x^2 - x + 3)^{7/2} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{7/2} x^6 + \frac{1046225 (2x^2 - x + 3)^{7/2} x^5}{9984} + \frac{3684995 (2x^2 - x + 3)^{7/2} x^4}{39936} + \frac{2344125 (2x^2 - x + 3)^{7/2} x^3}{122683392} + \frac{122595067 (2x^2 - x + 3)^{7/2} x^2}{19169280} + \frac{23460839 (2x^2 - x + 3)^{7/2} x}{532480} + \frac{3684995 (2x^2 - x + 3)^{7/2}}{39936} + \frac{1046225 (2x^2 - x + 3)^{7/2}}{9984} + \frac{13875 (2x^2 - x + 3)^{7/2}}{208} + \frac{625 (2x^2 - x + 3)^{7/2}}{28} - \frac{14641852251147 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{(68719476736 \sqrt{2})}$$

[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])

Rubi [A] time = 0.372556, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{28} (2x^2 - x + 3)^{7/2} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{7/2} x^6 + \frac{1046225 (2x^2 - x + 3)^{7/2} x^5}{9984} + \frac{3684995 (2x^2 - x + 3)^{7/2} x^4}{39936} + \frac{2344125 (2x^2 - x + 3)^{7/2} x^3}{122683392} + \frac{122595067 (2x^2 - x + 3)^{7/2} x^2}{19169280} + \frac{23460839 (2x^2 - x + 3)^{7/2} x}{532480} + \frac{3684995 (2x^2 - x + 3)^{7/2}}{39936} + \frac{1046225 (2x^2 - x + 3)^{7/2}}{9984} + \frac{13875 (2x^2 - x + 3)^{7/2}}{208} + \frac{625 (2x^2 - x + 3)^{7/2}}{28} - \frac{14641852251147 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{(68719476736 \sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]

[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +

```

c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

Rule 640

```

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

```

Rule 612

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

```

Rule 619

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

```

Rule 215

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

```

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 dx &= \frac{625}{28} x^7 (3-x+2x^2)^{7/2} + \frac{1}{28} \int (3-x+2x^2)^{5/2} (448+2688x+10528x^2+26880x^3+13875x^4) dx \\
&= \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} + \frac{625}{28} x^7 (3-x+2x^2)^{7/2} + \frac{1}{728} \int (3-x+2x^2)^{5/2} (13875x^4+26880x^3+10528x^2+26880x+448) dx \\
&= \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} + \frac{625}{28} x^7 (3-x+2x^2)^{7/2} \\
&= \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208} x^6 (3-x+2x^2)^{7/2} \\
&= \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5 (3-x+2x^2)^{7/2}}{9984} \\
&= \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4 (3-x+2x^2)^{7/2}}{39936} \\
&= \frac{112244125x (3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3 (3-x+2x^2)^{7/2}}{532480} \\
&= \frac{25250178739 (3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x (3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2 (3-x+2x^2)^{7/2}}{19169280} \\
&= -\frac{401135647(1-4x)(3-x+2x^2)^{5/2}}{335544320} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x(3-x+2x^2)^{7/2}}{122683392} \\
&= -\frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} - \frac{401135647(1-4x)(3-x+2x^2)^{5/2}}{335544320} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} \\
&= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648}
\end{aligned}$$

Mathematica [A] time = 0.453735, size = 105, normalized size = 0.41

$$4\sqrt{2x^2-x+3} (25125558681600000x^{13} + 37398427729920000x^{12} + 137233466130432000x^{11} + 20493241166069760x^{10} + 137233466130432000x^9 + 37398427729920000x^8 + 25125558681600000x^7 + 137233466130432000x^6 + 37398427729920000x^5 + 25125558681600000x^4 + 137233466130432000x^3 + 37398427729920000x^2 + 25125558681600000x + 137233466130432000)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]

[Out] (4*sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 50064174038215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 405468382284161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 439064558846345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 59958384968446965*sqrt[2]*ArcSinh[(1 - 4*x)/sqrt[23]])/562812514467840

Maple [A] time = 0.08, size = 204, normalized size = 0.8

$$\frac{1046225 x^5}{9984} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{3684995 x^4}{39936} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{23460839 x^3}{532480} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{122595067 x^2}{19169280} (2x^2 - x + 3)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)

[Out] 1046225/9984*x^5*(2*x^2-x+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+23460839/532480*x^3*(2*x^2-x+3)^(7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+112244125/122683392*x*(2*x^2-x+3)^(7/2)+636602271789/34359738368*(-1+4*x)*(2*x^2-x+3)^(1/2)+14641852251147/137438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+401135647/335544320*(-1+4*x)*(2*x^2-x+3)^(5/2)+9226119881/2147483648*(-1+4*x)*(2*x^2-x+3)^(3/2)+25250178739/5725224960*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2-x+3)^(7/2)

Maxima [A] time = 1.53405, size = 317, normalized size = 1.25

$$\frac{625}{28} (2x^2 - x + 3)^{\frac{7}{2}} x^7 + \frac{13875}{208} (2x^2 - x + 3)^{\frac{7}{2}} x^6 + \frac{1046225}{9984} (2x^2 - x + 3)^{\frac{7}{2}} x^5 + \frac{3684995}{39936} (2x^2 - x + 3)^{\frac{7}{2}} x^4 + \frac{23460839}{532480} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{122595067}{19169280} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{112244125}{122683392} (2x^2 - x + 3)^{\frac{7}{2}} x + 2525017$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")

[Out] 625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 1046225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2)*x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*x^2 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 2525017

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387)
*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 20237081761
67)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113210
402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 1082056749856866
9)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```


$$3.73 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^3 dx$$

Optimal. Leaf size=212

$$\frac{125}{24} (2x^2 - x + 3)^{7/2} x^5 + \frac{1175}{96} (2x^2 - x + 3)^{7/2} x^4 + \frac{3823}{256} (2x^2 - x + 3)^{7/2} x^3 + \frac{80483 (2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257 (2x^2 - x + 3)^{7/2} x}{9216} + \frac{10569777075 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{2147483648 \sqrt{2}}$$

[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 - x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2147483648*Sqrt[2])

Rubi [A] time = 0.219527, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24} (2x^2 - x + 3)^{7/2} x^5 + \frac{1175}{96} (2x^2 - x + 3)^{7/2} x^4 + \frac{3823}{256} (2x^2 - x + 3)^{7/2} x^3 + \frac{80483 (2x^2 - x + 3)^{7/2} x^2}{9216} + \frac{509257 (2x^2 - x + 3)^{7/2} x}{9216} + \frac{10569777075 \operatorname{ArcSinh}[(1 - 4x)/\sqrt{23}]}{2147483648 \sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]

[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 - x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2147483648*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*

$e^{(q+p)x^{q-1}} - c e^{(q+2p+1)x^q}$, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 dx &= \frac{125}{24}x^5(3-x+2x^2)^{7/2} + \frac{1}{24} \int (3-x+2x^2)^{5/2} (192+864x+2736x^2+4968x^3+2736x^4+576x^5) dx \\
&= \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{125}{24}x^5(3-x+2x^2)^{7/2} + \frac{1}{528} \int (3-x+2x^2)^{5/2} (48+216x+576x^2+864x^3+576x^4+144x^5) dx \\
&= \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{125}{24}x^5(3-x+2x^2)^{7/2} + \frac{1}{138} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{1}{3456} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1}{864} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{1}{216} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} - \frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{1}{54} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} - \frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{1}{13} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} + \frac{1}{3} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} + \frac{1}{3} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} + \frac{1}{3} \int (3-x+2x^2)^{5/2} (12+54x+144x^2+216x^3+144x^4+36x^5) dx
\end{aligned}$$

Mathematica [A] time = 0.287949, size = 95, normalized size = 0.45

$$4\sqrt{2x^2-x+3}(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^9+14341894045696x^8+2783556114840x^7+20384824684416x^6+26186527209472x^5+34378613923840x^4+2834753600x^3+141737680x^2+141737680x+57915)$$

Antiderivative was successfully verified.

[In] Integrate[(3-x+2*x^2)^(5/2)*(2+3*x+5*x^2)^3,x]

[Out] (4*Sqrt[3-x+2*x^2]*(-1191399152715+4560943728924*x+10060731582048*x^2+20384824684416*x^3+26186527209472*x^4+34378613923840*x^5+2834753600*x^6+141737680*x^7+141737680*x^8+57915*x^9))

8538496*x⁶ + 27835561148416*x⁷ + 14341894045696*x⁸ + 12943588589568*x⁹
 + 2395786444800*x¹⁰ + 2818572288000*x¹¹) - 665895955725*sqrt[2]*ArcSinh[(
 1 - 4*x)/sqrt[23]])/270582939648

Maple [A] time = 0.061, size = 170, normalized size = 0.8

$$\frac{125x^5}{24}(2x^2 - x + 3)^{\frac{7}{2}} + \frac{1175x^4}{96}(2x^2 - x + 3)^{\frac{7}{2}} + \frac{3823x^3}{256}(2x^2 - x + 3)^{\frac{7}{2}} + \frac{80483x^2}{9216}(2x^2 - x + 3)^{\frac{7}{2}} + \frac{509257x}{294912}(2x^2 - x + 3)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x²-x+3)^(5/2)*(5*x²+3*x+2)³,x)

[Out] 125/24*x⁵*(2*x²-x+3)^(7/2)+1175/96*x⁴*(2*x²-x+3)^(7/2)+3823/256*x³*(2*x²-x+3)^(7/2)+80483/9216*x²*(2*x²-x+3)^(7/2)+509257/294912*x*(2*x²-x+3)^(7/2)+459555525/1073741824*(-1+4*x)*(2*x²-x+3)^(1/2)+10569777075/4294967296*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+57915/2097152*(-1+4*x)*(2*x²-x+3)^(5/2)+6660225/67108864*(-1+4*x)*(2*x²-x+3)^(3/2)-1696165/2752512*(2*x²-x+3)^(7/2)

Maxima [A] time = 1.48024, size = 271, normalized size = 1.28

$$\frac{125}{24}(2x^2 - x + 3)^{\frac{7}{2}}x^5 + \frac{1175}{96}(2x^2 - x + 3)^{\frac{7}{2}}x^4 + \frac{3823}{256}(2x^2 - x + 3)^{\frac{7}{2}}x^3 + \frac{80483}{9216}(2x^2 - x + 3)^{\frac{7}{2}}x^2 + \frac{509257}{294912}(2x^2 - x + 3)^{\frac{7}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x²-x+3)^(5/2)*(5*x²+3*x+2)³,x, algorithm="maxima")

[Out] 125/24*(2*x² - x + 3)^(7/2)*x⁵ + 1175/96*(2*x² - x + 3)^(7/2)*x⁴ + 3823/256*(2*x² - x + 3)^(7/2)*x³ + 80483/9216*(2*x² - x + 3)^(7/2)*x² + 509257/294912*(2*x² - x + 3)^(7/2)*x - 1696165/2752512*(2*x² - x + 3)^(7/2) + 57915/524288*(2*x² - x + 3)^(5/2)*x - 57915/2097152*(2*x² - x + 3)^(5/2) + 6660225/16777216*(2*x² - x + 3)^(3/2)*x - 6660225/67108864*(2*x² - x + 3)^(3/2) + 459555525/268435456*sqrt(2*x² - x + 3)*x + 10569777075/4294967296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824*sqrt(2*x² - x + 3)

Fricas [A] time = 1.35757, size = 512, normalized size = 2.42

$$\frac{1}{67645734912} (2818572288000 x^{11} + 2395786444800 x^{10} + 12943588589568 x^9 + 14341894045696 x^8 + 27835561148$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^9 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 34378613923840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2 + 4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/8589934592*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)

[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)

Giac [A] time = 1.15239, size = 139, normalized size = 0.66

$$\frac{1}{67645734912} (4(8(4(16(4(8(28(32(12(200(20x+17)x+18369)x+244241)x+15169177)x+432549111)x+4196$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")

[Out] 1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x + 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x + 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*sqrt(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.74 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^2 dx$$

Optimal. Leaf size=170

$$\frac{5}{4}x^3(2x^2 - x + 3)^{7/2} + \frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 2x^2)^{3/2}}{98304}$$

[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 + (23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 + (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (94111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])

Rubi [A] time = 0.13038, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3(2x^2 - x + 3)^{7/2} + \frac{305}{144}x^2(2x^2 - x + 3)^{7/2} + \frac{8467x(2x^2 - x + 3)^{7/2}}{4608} + \frac{23225(2x^2 - x + 3)^{7/2}}{43008} - \frac{1547(1 - 4x)(2x^2 - x + 2x^2)^{3/2}}{98304}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 - x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 + (23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 + (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (94111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
  *e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
  *(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
  *p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
  eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
  *c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
  + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
  t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 dx &= \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{20} \int (3-x+2x^2)^{5/2} \left(80+240x+355x^2 + \frac{1525x^3}{2}\right) dx \\
&= \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{1}{360} \int (3-x+2x^2)^{5/2} \left(1440- \right. \\
&= \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} + \frac{\int(-}{ \\
&= \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3 \\
&= -\frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} \\
&= -\frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304} \\
&= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)(3-x+2x^2)^{5/2}}{98304}
\end{aligned}$$

Mathematica [A] time = 0.190448, size = 85, normalized size = 0.5

$$\frac{4\sqrt{2x^2-x+3}(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+4227858432x^3+57147467776x^2+75389820928x+26401898496+44163137536x+2055208960x+10569646080x+5929039935\sqrt{2}\operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right])}{4227858432}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 77872272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 44163137536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/4227858432

Maple [A] time = 0.053, size = 136, normalized size = 0.8

$$\frac{5x^3}{4} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{305x^2}{144} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{8467x}{4608} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{-4091815 + 16367260x}{16777216} \sqrt{2x^2 - x + 3} + \frac{94111745}{67108864} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x)

[Out] 5/4*x^3*(2*x^2-x+3)^(7/2)+305/144*x^2*(2*x^2-x+3)^(7/2)+8467/4608*x*(2*x^2-x+3)^(7/2)+4091815/16777216*(-1+4*x)*(2*x^2-x+3)^(1/2)+94111745/67108864*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+1547/98304*(-1+4*x)*(2*x^2-x+3)^(5/2)+177905/3145728*(-1+4*x)*(2*x^2-x+3)^(3/2)+23225/43008*(2*x^2-x+3)^(7/2)

Maxima [A] time = 1.47394, size = 225, normalized size = 1.32

$$\frac{5}{4} (2x^2 - x + 3)^{\frac{7}{2}} x^3 + \frac{305}{144} (2x^2 - x + 3)^{\frac{7}{2}} x^2 + \frac{8467}{4608} (2x^2 - x + 3)^{\frac{7}{2}} x + \frac{23225}{43008} (2x^2 - x + 3)^{\frac{7}{2}} + \frac{1547}{24576} (2x^2 - x + 3)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] 5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/4608*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/24576*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786432*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 4091815/4194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.37715, size = 409, normalized size = 2.41

$$\frac{1}{1056964608} (10569646080x^9 + 2055208960x^8 + 44163137536x^7 + 26401898496x^6 + 75389820928x^5 + 5714746720x^4 + 10569646080x^3 + 2055208960x^2 + 44163137536x + 26401898496)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")

```
[Out] 1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 26401898
496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 42992644128
*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421772
8*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

Giac [A] time = 1.1628, size = 126, normalized size = 0.74

$$\frac{1}{1056964608} (4 (8 (4 (16 (4 (8 (28 (160 (36x + 7)x + 24067)x + 402861)x + 9202859)x + 27904037)x + 608377125)x + 1343520129)x + 9883312413)x + 14824182519) \sqrt{2x^2 - x + 3} - 94111745/67108864 \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)*
x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)*x
+ 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

$$3.75 \quad \int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2) dx$$

Optimal. Leaf size=128

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665(1 - 4x)^2(2x^2 - x + 3)^{1/2}}{1048576} - \frac{16851295 \operatorname{ArcSin}\left[\frac{1 - 4x}{\sqrt{23}}\right]}{1048576\sqrt{2}}$$

[Out] $(-732665*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^{(5/2)})/3072 + (141*(3 - x + 2*x^2)^{(7/2)})/448 + (5*x*(3 - x + 2*x^2)^{(7/2)})/16 - (16851295*\operatorname{ArcSin}[\frac{1 - 4*x}{\operatorname{Sqrt}[23]}])/(1048576*\operatorname{Sqrt}[2])$

Rubi [A] time = 0.0624409, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x(2x^2 - x + 3)^{7/2} + \frac{141}{448}(2x^2 - x + 3)^{7/2} - \frac{277(1 - 4x)(2x^2 - x + 3)^{5/2}}{3072} - \frac{31855(1 - 4x)(2x^2 - x + 3)^{3/2}}{98304} - \frac{732665(1 - 4x)^2(2x^2 - x + 3)^{1/2}}{1048576} - \frac{16851295 \operatorname{ArcSin}\left[\frac{1 - 4x}{\sqrt{23}}\right]}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(3 - x + 2*x^2)^{(5/2)}*(2 + 3*x + 5*x^2), x]$

[Out] $(-732665*(1 - 4*x)*\operatorname{Sqrt}[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x + 2*x^2)^{(3/2)})/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^{(5/2)})/3072 + (141*(3 - x + 2*x^2)^{(7/2)})/448 + (5*x*(3 - x + 2*x^2)^{(7/2)})/16 - (16851295*\operatorname{ArcSin}[\frac{1 - 4*x}{\operatorname{Sqrt}[23]}])/(1048576*\operatorname{Sqrt}[2])$

Rule 1661

$\operatorname{Int}[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}, x_Symbol] :> \operatorname{With}[\{q = \operatorname{Expon}[Pq, x], e = \operatorname{Coeff}[Pq, x, \operatorname{Expon}[Pq, x]]\}, \operatorname{Simp}[(e*x^{(q - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*(q + 2*p + 1)), x] + \operatorname{Dist}[1/(c*(q + 2*p + 1)), \operatorname{Int}[(a + b*x + c*x^2)^p * \operatorname{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x]] /; \operatorname{FreeQ}[\{a, b, c, p\}, x] \&\& \operatorname{PolyQ}[Pq, x] \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& !\operatorname{LeQ}[p, -1]$

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
  *e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
  )*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
  *p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
  eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
  *c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
  + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
  t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int (3-x+2x^2)^{5/2} (2+3x+5x^2) dx &= \frac{5}{16}x(3-x+2x^2)^{7/2} + \frac{1}{16} \int \left(17 + \frac{141x}{2}\right) (3-x+2x^2)^{5/2} dx \\
&= \frac{141}{448} (3-x+2x^2)^{7/2} + \frac{5}{16}x(3-x+2x^2)^{7/2} + \frac{277}{128} \int (3-x+2x^2)^{5/2} dx \\
&= -\frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} + \frac{141}{448} (3-x+2x^2)^{7/2} + \frac{5}{16}x(3-x+2x^2)^{7/2} + \\
&= -\frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} + \frac{141}{448} (3-x+ \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)}{3} \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)}{3} \\
&= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.107564, size = 75, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3}(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+353877195)-353877195\sqrt{2}\operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{44040192}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2), x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 353877195*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/44040192

Maple [A] time = 0.049, size = 102, normalized size = 0.8

$$\frac{5x}{16} (2x^2 - x + 3)^{7/2} + \frac{141}{448} (2x^2 - x + 3)^{7/2} + \frac{-277 + 1108x}{3072} (2x^2 - x + 3)^{5/2} + \frac{-31855 + 127420x}{98304} (2x^2 - x + 3)^{3/2} + \frac{-7}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)`

[Out] $\frac{5}{16}x(2x^2-x+3)^{7/2} + \frac{141}{448}(2x^2-x+3)^{7/2} + \frac{277}{3072}(-1+4x)(2x^2-x+3)^{5/2} + \frac{31855}{98304}(-1+4x)(2x^2-x+3)^{3/2} + \frac{732665}{524288}(-1+4x)(2x^2-x+3)^{1/2} + \frac{16851295}{2097152}2^{1/2}\operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}\sqrt{2x^2-x+3}\right)$

Maxima [A] time = 1.43838, size = 180, normalized size = 1.41

$$\frac{5}{16}(2x^2-x+3)^{7/2}x + \frac{141}{448}(2x^2-x+3)^{7/2} + \frac{277}{768}(2x^2-x+3)^{5/2}x - \frac{277}{3072}(2x^2-x+3)^{5/2} + \frac{31855}{24576}(2x^2-x+3)^{3/2}x - \frac{31855}{98304}(2x^2-x+3)^{3/2} + \frac{732665}{131072}\sqrt{2x^2-x+3}x + \frac{16851295}{2097152}\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}\sqrt{4x-1}\right) - \frac{732665}{524288}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] $\frac{5}{16}(2x^2-x+3)^{7/2}x + \frac{141}{448}(2x^2-x+3)^{7/2} + \frac{277}{768}(2x^2-x+3)^{5/2}x - \frac{277}{3072}(2x^2-x+3)^{5/2} + \frac{31855}{24576}(2x^2-x+3)^{3/2}x - \frac{31855}{98304}(2x^2-x+3)^{3/2} + \frac{732665}{131072}\sqrt{2x^2-x+3}x + \frac{16851295}{2097152}\sqrt{2}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}\sqrt{4x-1}\right) - \frac{732665}{524288}\sqrt{2x^2-x+3}$

Fricas [A] time = 1.38597, size = 327, normalized size = 2.55

$$\frac{1}{11010048}(27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675)\sqrt{2x^2-x+3} + \frac{16851295}{4194304}\sqrt{2}\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")`

[Out] $\frac{1}{11010048}(27525120x^7 - 13565952x^6 + 118808576x^5 - 1619968x^4 + 172684416x^3 + 67272352x^2 + 148957444x + 58536675)\sqrt{2x^2-x+3} + \frac{16851295}{4194304}\sqrt{2}\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (2x^2 - x + 3)^{5/2} (5x^2 + 3x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2), x)

[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)

Giac [A] time = 1.14499, size = 112, normalized size = 0.88

$$\frac{1}{11010048} (4(8(4(16(4(24(140x - 69)x + 14503)x - 791)x + 1349097)x + 2102261)x + 37239361)x + 58536675)\sqrt{2x^2 - x + 3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2), x, algorithm="giac")

[Out] 1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 1349097)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295/2097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.76 \quad \int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx$$

Optimal. Leaf size=222

$$-\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)}\tan^{-1}\left(\sqrt{\frac{11}{62(25000\sqrt{2}-15457)}}\right)}{3125}$$

[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) - (121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 25000*Sqrt[2])))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(15457 + 25000*Sqrt[2])))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/3125

Rubi [A] time = 0.538982, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{600}(103-60x)(2x^2-x+3)^{3/2} - \frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000} - \frac{121\sqrt{\frac{11}{31}(25000\sqrt{2}-15457)}\tan^{-1}\left(\sqrt{\frac{11}{62(25000\sqrt{2}-15457)}}\right)}{3125}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2), x]

[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) - (121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 25000*Sqrt[2])))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(15457 + 25000*Sqrt[2])))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/3125

Rule 977


```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))]*x + ((c*e - b*f)^2*(1 - p)*p + c*(p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1066

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

```

Rule 1076

```

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

```

Rule 619

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

```

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1035

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2} dx &= -\frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{1}{300} \int \frac{\left(-\frac{4731}{2} + \frac{6135x}{4} - \frac{14943x^2}{4}\right) \sqrt{3-x+2x^2}}{2+3x+5x^2} dx \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{\frac{3205293}{8} - \frac{11339385x}{16} + \frac{21648}{1}}{\sqrt{3-x+2x^2}(2+3x+5x^2)}}{30000} dx \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} + \frac{\int \frac{-702768-7602672x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{150000} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{\int \frac{-702768(108-11\sqrt{2})-702}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{330000} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}} \\
&= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)(3-x+2x^2)^{3/2} - \frac{7216203 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 1.06988, size = 229, normalized size = 1.03

$$620\sqrt{2x^2-x+3}(48000x^3-106400x^2+412060x-802347)+46464\sqrt{286+22i\sqrt{31}}(403-69i\sqrt{31})\tanh^{-1}\left(\frac{-22-4i\sqrt{31}}{2\sqrt{286+22i\sqrt{31}}}\right)$$

14880

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2), x]

[Out] (620*sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) + 671106879*sqrt[2]*ArcSinh[(-1 + 4*x)/sqrt[23]] + 46464*sqrt[286 + (22*I)*sqrt[31]]*(403 - (69*I)*sqrt[31])*ArcTanh[(63 + I*sqrt[31] + (-22 - (4*I)*sqrt[31])*x)/(2*sqrt[286 + (22*I)*sqrt[31]]*sqrt[3 - x + 2*x^2])] - (46464*I)*sqrt[286 - (22*I)*sqrt[31]]*(-403*I + 69*sqrt[31])*ArcTanh[(-63 + I*sqrt[31] + (22 - (4*I)*sqrt[31])*x)/(2*sqrt[286 - (22*I)*sqrt[31]]*sqrt[3 - x + 2*x^2])])/148800000

Maple [B] time = 0.179, size = 4860, normalized size = 21.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^2-x+3)^{(5/2)}/(5*x^2+3*x+2), x)$

[Out] $\frac{1}{5}x^3(2x^2-x+3)^{1/2}-\frac{133}{300}x^2(2x^2-x+3)^{1/2}+\frac{20603}{12000}x(2x^2-x+3)^{1/2}-\frac{267449}{80000}(2x^2-x+3)^{1/2}+\frac{7216203}{1600000}2^{1/2}\operatorname{arcsinh}\left(\frac{4}{23}23^{1/2}(x-1/4)\right)+\frac{4}{33034375}(8(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))^{1/2}*2^{1/2}*(75195*(-775687+549362*2^{1/2}))^{1/2}*2^{1/2}*(-8866+6820*2^{1/2}))^{1/2}\operatorname{arctan}(1/11692487*(-775687+549362*2^{1/2}))^{1/2}*(-23*(8+3*2^{1/2}))*(-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2}*(6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})*2^{1/2}*(2^{1/2}-1+x)/(2^{1/2}+1-x)+106294*(-775687+549362*2^{1/2}))^{1/2}*(-8866+6820*2^{1/2}))^{1/2}\operatorname{arctan}(1/11692487*(-775687+549362*2^{1/2}))^{1/2}*(-23*(8+3*2^{1/2}))*(-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2}*(6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})*2^{1/2}*(2^{1/2}-1+x)/(2^{1/2}+1-x)+108099046*\operatorname{arctanh}(31/2*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))^{1/2}/(-8866+6820*2^{1/2}))^{1/2}*2^{1/2}-158290154*\operatorname{arctanh}(31/2*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))^{1/2}/(-8866+6820*2^{1/2}))^{1/2}/((8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))/((1+(2^{1/2}-1+x)/(2^{1/2}+1-x))^2)^{1/2}/(1+(2^{1/2}-1+x)/(2^{1/2}+1-x)))/(8+3*2^{1/2}))/(-8866+6820*2^{1/2}))^{1/2}+6/6606875*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))^{1/2}*2^{1/2}*(10915*(-775687+549362*2^{1/2}))^{1/2}*2^{1/2}*(-8866+6820*2^{1/2}))^{1/2}\operatorname{arctan}(1/11692487*(-775687+549362*2^{1/2}))^{1/2}*(-23*(8+3*2^{1/2}))*(-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2}*(6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})*2^{1/2}*(2^{1/2}-1+x)/(2^{1/2}+1-x)+14918*(-775687+549362*2^{1/2}))^{1/2}*(-8866+6820*2^{1/2}))^{1/2}\operatorname{arctan}(1/11692487*(-775687+549362*2^{1/2}))^{1/2}*(-23*(8+3*2^{1/2}))*(-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2}*(6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})*2^{1/2}*(2^{1/2}-1+x)/(2^{1/2}+1-x))-5052938*\operatorname{arctanh}(31/2*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2*2^{1/2}+8-3*2^{1/2}))^{1/2}/(-8866+6820*2^{1/2}))^{1/2}$

$$\begin{aligned}
& 2)/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-51565338*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))^2)^{(1/2)}/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-21/1321375*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(4245*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+6154*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+12325786*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-359414*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))^2)^{(1/2)}/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-37/528550*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(2365*(-775687+549362*2^{(1/2)})^{(1/2)}*2^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+3338*(-775687+549362*2^{(1/2)})^{(1/2)}*(-8866+6820*2^{(1/2)})^{(1/2)}*\operatorname{arctan}(1/11692487*(-775687+549362*2^{(1/2)})^{(1/2)}*(-23*(8+3*2^{(1/2)})*(-23*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+24*2^{(1/2)}-41))^{(1/2)}*(6485*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+10368*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+22379*2^{(1/2)}+32016)/(23*(2^{(1/2)}-1+x)^4/(2^{(1/2)}+1-x)^4+82*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+23)*(8+3*2^{(1/2)})*(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))+3192442*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}*2^{(1/2)}-5264358*\operatorname{arctanh}(31/2*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)})/((8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2*2^{(1/2)}+8-3*2^{(1/2)})/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x))^2)^{(1/2)}/(1+(2^{(1/2)}-1+x)/(2^{(1/2)}+1-x)))/(8+3*2^{(1/2)})/(-8866+6820*2^{(1/2)})^{(1/2)}-63/105710*(8*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2+3*(2^{(1/2)}-1+x)^2/(2^{(1/2)}+1-x)^2
\end{aligned}$$

$$\begin{aligned}
& 2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \cdot (285 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \\
& \cdot (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)}) \\
& \cdot (-23 \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 24 \cdot 2^{(1/2)} - 4 \\
& 1))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 10368 \cdot (2^{(1/2)} - 1 + x) \\
& ^2 / (2^{(1/2)} + 1 - x)^2 + 22379 \cdot 2^{(1/2)} + 32016) / (23 \cdot (2^{(1/2)} - 1 + x)^4 / (2^{(1/2)} + 1 - x)^4 \\
& + 82 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)} - 1 + x) / (2^{(1/2)} \\
& + 1 - x)) + 386 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan \\
& (1/11692487 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-23 \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)} \\
& - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 24 \cdot 2^{(1/2)} - 41))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} \\
& + 1 - x)^2 \cdot 2^{(1/2)} + 10368 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 22379 \cdot 2^{(1/2)} + 32016 \\
&) / (23 \cdot (2^{(1/2)} - 1 + x)^4 / (2^{(1/2)} + 1 - x)^4 + 82 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 23 \\
&) \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x) - 274846 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)} \\
& - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) \\
&)^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} - 1543366 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)} \\
& - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) \\
&)^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}) / ((8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \\
&)^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x))^2)^{(1/2)} / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x)) / (8 + 3 \cdot 2^{(1/2)}) / (-88 \\
& 66 + 6820 \cdot 2^{(1/2)})^{(1/2)} + 27/21142 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \\
&)^{(1/2)} \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \cdot (151 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \cdot (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan(1/11692487 \cdot \\
& (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-23 \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} \\
& + 1 - x)^2 + 24 \cdot 2^{(1/2)} - 41))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} \\
& + 10368 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 22379 \cdot 2^{(1/2)} + 32016) / (23 \cdot (2^{(1/2)} - 1 + x)^4 / (2^{(1/2)} + 1 - x)^4 + 82 \cdot (2^{(1/2)} - 1 \\
& + x)^2 / (2^{(1/2)} + 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x)) + 401698 \\
& \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} \\
& + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}) \cdot 2^{(1/2)} - 6342 \\
& 6 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} \\
& + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)} / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}) / ((8 \cdot (2^{(1/2)} \\
& - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) \\
&) / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x))^2)^{(1/2)} / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x)) / (8 + 3 \cdot 2^{(1/2)}) / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} + 27/21142 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2 \\
& ^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)} \cdot \\
& 2^{(1/2)} \cdot (369 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)} \cdot 2^{(1/2)} \cdot (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{(1/2)})^{(1/2)}) \cdot (-23 \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (-2 \\
& 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 24 \cdot 2^{(1/2)} - 41))^{(1/2)} \cdot (6485 \cdot (2^{(1/2)} - 1 + x) \\
& ^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 10368 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 22379 \cdot 2^{(1/2)} \\
& + 32016) / (23 \cdot (2^{(1/2)} - 1 + x)^4 / (2^{(1/2)} + 1 - x)^4 + 82 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + \\
& 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x)) + 520 \cdot (-775687 + 549362 \cdot 2
\end{aligned}$$

$$\begin{aligned} &^{(1/2)} \cdot (-8866 + 6820 \cdot 2^{(1/2)}) \cdot \arctan\left(\frac{1}{11692487} \cdot (-775687 + 549362 \cdot 2^{(1/2)}) \cdot (-23 \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (-23 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 24 \cdot 2^{(1/2)} - 41)) \cdot (6485 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 10368 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 22379 \cdot 2^{(1/2)} + 32016) / (23 \cdot (2^{(1/2)} - 1 + x)^4 / (2^{(1/2)} + 1 - x)^4 + 82 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{(1/2)}) \cdot (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x) + 465124 \cdot \operatorname{arctanh}\left(\frac{31/2 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)}}{-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}}\right) \cdot 2^{(1/2)} - 866822 \cdot \operatorname{arctanh}\left(\frac{31/2 \cdot (8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)})^{(1/2)}}{-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)}}\right) / ((8 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 + 3 \cdot (2^{(1/2)} - 1 + x)^2 / (2^{(1/2)} + 1 - x)^2 \cdot 2^{(1/2)} + 8 - 3 \cdot 2^{(1/2)}) / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x))^2)^{(1/2)} / (1 + (2^{(1/2)} - 1 + x) / (2^{(1/2)} + 1 - x)) / (8 + 3 \cdot 2^{(1/2)}) / (-8866 + 6820 \cdot 2^{(1/2)})^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{5/2}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)

Fricas [B] time = 3.90656, size = 7899, normalized size = 35.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")

[Out] $121/96875000 \cdot 6050^{(1/4)} \cdot \sqrt{31} \cdot \sqrt{2} \cdot \sqrt{-772850000 \cdot \sqrt{2} + 25000000} \cdot \arctan\left(\frac{1}{254496437500} \cdot (722441500000 \cdot \sqrt{31} \cdot \sqrt{2}) \cdot (28180 \cdot x^8 - 254666 \cdot x^7 + 704270 \cdot x^6 - 1385256 \cdot x^5 + 1549144 \cdot x^4 - 642048 \cdot x^3 - 98496 \cdot x^2 - \sqrt{2} \cdot (8746 \cdot x^8 - 102335 \cdot x^7 + 396104 \cdot x^6 - 783113 \cdot x^5 + 1320710 \cdot x^4 - 752088 \cdot x^3 + 396144 \cdot x^2 + 546048 \cdot x - 539136) + 1154304 \cdot x - 456192) + 2300 \cdot (4 \cdot 6050^{(3/4)} \cdot \sqrt{31} \cdot (35898 \cdot x^7 - 441939 \cdot x^6 + 782418 \cdot x^5 - 2117233 \cdot x^4 + 1272680 \cdot x^3 - 1081800 \cdot x^2 - \sqrt{2} \cdot (173702 \cdot x^7 - 453907 \cdot x^6 + 1056481 \cdot x^5 - 1$

$$\begin{aligned}
& 083344*x^4 + 393672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 104 \\
& 3712) + 5*6050^{(1/4)}*\sqrt{31}*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 11 \\
& 1531424*x^4 + 156761280*x^3 - 168192000*x^2 - \sqrt{2}*(712757*x^7 - 1023330 \\
& 3*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417 \\
& 152*x + 37407744) - 74815488*x + 106417152))*\sqrt{2*x^2 - x + 3}*\sqrt{-7728 \\
& 50000*\sqrt{2} + 2500000000) - \sqrt{10/5711}*(314105000*\sqrt{31}*\sqrt{2}*(12 \\
& 3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 \\
& - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 \\
& + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - (4*60 \\
& 50^{(3/4)}*\sqrt{31}*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 369 \\
& 3960*x^3 + 4195584*x^2 + 22*\sqrt{2}*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4 \\
& 464*x^4 - 49464*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^{(1/4)}*\sqrt{ \\
& 31}*(160956*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x \\
& x^3 - 296027136*x^2 - \sqrt{2}*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - \\
& 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x \\
&))*\sqrt{2*x^2 - x + 3}*\sqrt{-772850000*\sqrt{2} + 2500000000) + 14277500*\sqrt{ \\
& 31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x \\
& ^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 \\
& - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{ \\
& ((6050^{(1/4)}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(163*x - 725) + 562*x - 888)*\sqrt{ \\
& (-772850000*\sqrt{2} + 2500000000) + 139919500*x^2 + 125642000*\sqrt{2}*(2*x^ \\
& 2 - x + 3) - 431180500*x + 571100000)/x^2) + 8209562500*\sqrt{31}*(2828123*x \\
& ^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096 \\
& *x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070* \\
& x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 9488 \\
& 7936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^ \\
& 4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 121/96875000*60 \\
& 50^{(1/4)}*\sqrt{31}*\sqrt{2}*\sqrt{-772850000*\sqrt{2} + 2500000000)*\arctan(-1/2 \\
& 54496437500*(722441500000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270 \\
& *x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x \\
& ^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961 \\
& 44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^{(3/4)}*\sqrt{ \\
& 31}*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108 \\
& 1800*x^2 - \sqrt{2}*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3 \\
& 93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050 \\
& ^{(1/4)}*\sqrt{31}*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 + \\
& 156761280*x^3 - 168192000*x^2 - \sqrt{2}*(712757*x^7 - 10233303*x^6 + 485297 \\
& 68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077 \\
& 44) - 74815488*x + 106417152))*\sqrt{2*x^2 - x + 3}*\sqrt{-772850000*\sqrt{2} \\
& + 2500000000) - \sqrt{10/5711}*(314105000*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914 \\
& 152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 \\
& - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 \\
& - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^{(3/4)}*\sqrt{ \\
& 31}*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419 \\
& 5584*x^2 + 22*\sqrt{2}*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946
\end{aligned}$$


```

4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sqrt(31)*(160956
*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713
6*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4
+ 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*sqrt(2*x^2
- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*s
qrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*
x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^(1/4)
*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(91506250
00/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 88
8)*sqrt(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2
)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^
(1/4)*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(-91
50625000/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*
x - 888)*sqrt(-772850000*sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*
sqrt(2)*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x
^3 - 106400*x^2 + 412060*x - 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*
sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{5x^2 + 3x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2), x)

[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.77 \quad \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=255

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)(2x^2-x+3)^{3/2} - \frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383+194487500\sqrt{2})}}{31}$$

[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4*799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 + 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]))])*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-224510383 + 194487500*Sqrt[2]))])*(21136 - 33287*Sqrt[2] + (87710 - 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750

Rubi [A] time = 0.659844, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)} + \frac{4}{155}(4-5x)(2x^2-x+3)^{3/2} - \frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750} + \frac{11\sqrt{\frac{11}{31}(224510383+194487500\sqrt{2})}}{31}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2, x]

[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4*799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 + 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]))])*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(62*(-224510383 + 194487500*Sqrt[2]))])*(21136 - 33287*Sqrt[2] + (87710 - 54423*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/38750

- 54423* $\sqrt{2}$)*x))/ $\sqrt{3 - x + 2x^2}$]/38750

Rule 971

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1066

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)* $\sqrt{(d_.) + (e_.)*(x_) + (f_.)*(x_)^2}$), x_Symbol] :> Dist[C/c, Int[1/ $\sqrt{d + e*x + f*x^2}$], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)* $\sqrt{d + e*x + f*x^2}$), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1035

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{1}{31} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{75}{2} + 15x + 80x^2\right)}{2+3x+5x^2} dx \\
&= \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \frac{\int \frac{(87660-54300x-53760x^2)\sqrt{3-x+2x^2}}{2+3x+5x^2} dx}{18600} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{\int}{\dots} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{\int}{\dots} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \frac{\int}{\dots} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{47}{\dots} \\
&= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{47}{\dots}
\end{aligned}$$

Mathematica [C] time = 1.69943, size = 685, normalized size = 2.69

$$-1922000\sqrt{2x^2-x+3x^3} + 7784100\sqrt{2x^2-x+3x^2} - 5759180\sqrt{2x^2-x+3x} - 5577520\sqrt{2x^2-x+3} - 4611839\sqrt{2} (5$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]

[Out] -(-5577520*sqrt[3 - x + 2*x^2] - 5759180*x*sqrt[3 - x + 2*x^2] + 7784100*x^2*sqrt[3 - x + 2*x^2] - 1922000*x^3*sqrt[3 - x + 2*x^2] - 4611839*sqrt[2]*(2 + 3*x + 5*x^2)*ArcSinh[(-1 + 4*x)/sqrt[23]] + (11*I)*sqrt[286 + (22*I)*Sq

```

rt[31]]*(5177*I + 8771*Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 + I*Sqrt[31]
- 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] + (192962*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*Sqrt[31] +
22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2
])] + 113894*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[31] + 22*x -
(4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (2
89443*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (
4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 170
841*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*
Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (482405*
I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)
*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 284735*
Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sq
rt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]/(4805000*(2
+ 3*x + 5*x^2))

```

Maple [B] time = 0.414, size = 40028, normalized size = 157.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)

Fricas [B] time = 5.25698, size = 9335, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/1322759922435707900000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1/296975447063866363819995875*(110935670*sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 579888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 537311232*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 7599242656001778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 - 6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 27528822527278000)/x^2) + 9579853131092463349032125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4

$$\begin{aligned}
& 4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936)) / (258519 \\
& 1x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x \\
& ^3 - 34615296x^2 - 24772608x + 18579456)) + 38925001324*1464599010050^{(1/ \\
& 4)*\sqrt{155590}*\sqrt{62}*\sqrt{2}*(5x^2 + 3x + 2)*\sqrt{224510383*\sqrt{2} + \\
& 388975000)}*\arctan(1/296975447063866363819995875*(110935670*\sqrt{155590})*(4 \\
& *1464599010050^{(3/4)*\sqrt{62}*(18997882x^7 - 82713851x^6 + 169131062x^5 \\
& - 298338397x^4 + 156222120x^3 - 89116200x^2 - \sqrt{2}*(18111018x^7 - 62 \\
& 947113x^6 + 135463929x^5 - 197908246x^4 + 94500248x^3 - 34095024x^2 - \\
& 122404608x + 71452800) - 142905600x + 122404608) + 2411645*1464599010050^{ \\
& (1/4)*\sqrt{62}*(3035566x^7 - 47612316x^6 + 259553720x^5 - 615321136x^4 \\
& + 807721920x^3 - 579888000x^2 - \sqrt{2}*(2643323x^7 - 39854517x^6 + 204 \\
& 950152x^5 - 451004140x^4 + 573424416x^3 - 311722272x^2 - 434377728x + \\
& 268655616) - 537311232x + 434377728))*\sqrt{2x^2 - x + 3}*\sqrt{224510383*s \\
& \sqrt{2} + 388975000) - 843027075536136774714827000*\sqrt{31}*\sqrt{2}*(28180*x \\
& ^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 984 \\
& 96x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710 \\
& *x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \\
& \sqrt{77795/920561}*(\sqrt{155590}*(4*1464599010050^{(3/4)*\sqrt{62}*(58767374 \\
& *x^7 - 85793239x^6 + 285539949x^5 - 168939120x^4 + 253241640x^3 + 60134 \\
& 4*x^2 - 4*\sqrt{2}*(17889302x^7 - 25424283x^6 + 80174553x^5 - 21241168x^ \\
& 4 + 15593832x^3 + 58564512x^2 - 58564512x) - 601344x) + 2411645*1464599 \\
& 010050^{(1/4)*\sqrt{62}*(9891184x^7 - 128099264x^6 + 496592960x^5 - 666984 \\
& 960x^4 + 949582080x^3 + 183223296x^2 - \sqrt{2}*(10181049x^7 - 131588036 \\
& *x^6 + 505509740x^5 - 637596864x^4 + 754818336x^3 + 725677056x^2 - 7256 \\
& 77056x) - 183223296x))*\sqrt{2x^2 - x + 3}*\sqrt{224510383*\sqrt{2} + 38897 \\
& 5000) - 7599242656001778100*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x^7 + 157 \\
& 8888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(1 \\
& 5550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 \\
& + 1209600x^2 - 1036800x) + 3276288x) - 345420120727353550*\sqrt{31}*(254 \\
& 591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219 \\
& 328x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^ \\
& 5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{(14645990 \\
& 10050^{(1/4)*\sqrt{155590}*\sqrt{62}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(97 \\
& 33x + 29025) - 38758x + 19292)*\sqrt{224510383*\sqrt{2} + 388975000) + 6744 \\
& 561519183110x^2 + 6056340956001160*\sqrt{2}*(2x^2 - x + 3) - 2078426100809 \\
& 4890x + 27528822527278000)/x^2) - 9579853131092463349032125*\sqrt{31}*(2828 \\
& 123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 2493 \\
& 00096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 - 1 \\
& 0070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - \\
& 94887936)) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 135629 \\
& 44x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 11*1464599 \\
& 010050^{(1/4)*\sqrt{155590}*\sqrt{62}*\sqrt{31}*(1944875000x^2 - 224510383*sqr \\
& t(2)*(5x^2 + 3x + 2) + 1166925000x + 777950000)*\sqrt{224510383*\sqrt{2} + \\
& 388975000)}*\log(14708117187500/920561*(1464599010050^{(1/4)*\sqrt{155590}*\sqrt{62}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(9733x + 29025) - 38758x + 19
\end{aligned}$$

292)*sqrt(224510383*sqrt(2) + 388975000) + 6744561519183110*x^2 + 6056340956001160*sqrt(2)*(2*x^2 - x + 3) - 20784261008094890*x + 27528822527278000)/x^2) - 11*1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^2 - 224510383*sqrt(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(24510383*sqrt(2) + 388975000)*log(-14708117187500/920561*(1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 674456151918310*x^2 - 6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 27528822527278000)/x^2) + 634792486776896221210*sqrt(2)*(5*x^2 + 3*x + 2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170678699669123600*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*sqrt(2*x^2 - x + 3))/(5*x^2 + 3*x + 2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)

[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.78 \quad \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=281

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}(2937349 + 1978861\sqrt{2})}{29791000}$$

[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sqrt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(3531015707557 + 2498852071250*Sqrt[2])])*(3957722 + 2937349*Sqrt[2] + (9832420 + 6895071*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 1978861*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 + 2498852071250*Sqrt[2])])*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/29791000

Rubi [A] time = 0.654547, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.37$, Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2} + \frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)} + \frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050} + \frac{\sqrt{11(1+4\sqrt{2})}(2937349 + 1978861\sqrt{2})}{29791000}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3, x]

[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sqrt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(3531015707557 + 2498852071250*Sqrt[2])])*(3957722 + 2937349*Sqrt[2] + (9832420 + 6895071*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 1978861*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 + 2498852071250*Sqrt[2])])*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2]])/29791000

2498852071250*sqrt[2]))*(3957722 - 2937349*sqrt[2] + (9832420 - 6895071*sqrt[2])*x))/sqrt[3 - x + 2*x^2]]/29791000

Rule 971

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1054

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2)))]*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

Rule 1066

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

Rule 1076

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1035

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} - \frac{1}{62} \int \frac{(3-x+2x^2)^{3/2} \left(-\frac{195}{2} + 35x + 40x^2\right)}{(2+3x+5x^2)^2} dx \\
 &= \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} + \frac{\int \frac{\left(\frac{66735}{4} - 7375x - 25840x^2\right)\sqrt{3-x+2x^2}}{2+3x+5x^2} dx}{9610} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} \\
 &= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)}
 \end{aligned}$$

Mathematica [C] time = 2.11623, size = 1009, normalized size = 3.59

$$12599950\sqrt{286 - 22i\sqrt{31}} \tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)x^4 - 12290525i\sqrt{682(13 - i\sqrt{31})} \tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]

[Out] (153804640*Sqrt[3 - x + 2*x^2] + 474815220*x*Sqrt[3 - x + 2*x^2] + 640207040*x^2*Sqrt[3 - x + 2*x^2] + 662597100*x^3*Sqrt[3 - x + 2*x^2] + 1906624*Sqrt[2]*(2 + 3*x + 5*x^2)^2*ArcSinh[(-1 + 4*x)/Sqrt[23]] - I*Sqrt[286 + (22*I)*Sqrt[31]]*(-503998*I + 491621*Sqrt[31]))*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (1966484*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 2015992*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (5899452*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 6047976*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (14257009*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 14615942*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (14748630*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^3*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 15119940*Sqrt[286 - (22*I)*Sqrt[31]]*x^3*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (12290525*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^4*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 12599950*Sqrt[286 - (22*I)*Sqrt[31]]*x^4*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]/(59582000*(2 + 3*x + 5*x^2)^2)

Maple [B] time = 0.69, size = 119458, normalized size = 425.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)`

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")`

[Out] `integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)`

Fricas [B] time = 5.49537, size = 10927, normalized size = 38.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")`

[Out] `1/758714159921174808909075728000*(3184949732636*3868444992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/4535484880629403103991789624695893204150231*(2850690442882*sqrt(1999081657))*(2*3868444992270541948232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000*x^2 - sqrt(2)*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 12874924822431853972621854418491567805329688*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4`

$$\begin{aligned}
& - 642048x^3 - 98496x^2 - \sqrt{2}(8746x^8 - 102335x^7 + 396104x^6 - 78 \\
& 3113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 115 \\
& 4304x - 456192) - \sqrt{1999081657/828550919}(\sqrt{1999081657}(2*38684449 \\
& 92270541948232^{3/4})\sqrt{62}(9351066298x^7 - 13433496653x^6 + 433103458 \\
& 23x^5 - 17374572240x^4 + 20927636280x^3 + 18483199488x^2 - \sqrt{2}(683 \\
& 9273266x^7 - 9809465289x^6 + 31524099699x^5 - 12024617744x^4 + 13914887 \\
& 256x^3 + 14839341696x^2 - 14839341696x) - 18483199488x) + 61971531367*3 \\
& 868444992270541948232^{1/4})\sqrt{62}(1427210918x^7 - 18462714328x^6 + 71 \\
& 210222920x^5 - 92387041920x^4 + 119489780160x^3 + 68726817792x^2 - \sqrt{2} \\
& (2)(1033310523x^7 - 13365477772x^6 + 51521534980x^5 - 66583614528x^4 + \\
& 85122955872x^3 + 53108877312x^2 - 53108877312x) - 68726817792x))\sqrt{2} \\
& (2x^2 - x + 3)\sqrt{3531015707557\sqrt{2} + 4997704142500} + 45164233298567 \\
& 21284677540671884\sqrt{31}\sqrt{2}(123408x^8 - 914152x^7 + 1578888x^6 - \\
& 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}(15550x^8 - \\
& 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600 \\
& x^2 - 1036800x) + 3276288x) + 205291969538941876576251848722\sqrt{31}(2 \\
& 54591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 742 \\
& 19328x^3 - 168956928x^2 - 15488\sqrt{2}(4x^8 - 76x^7 + 517x^6 - 1536x \\
& x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))\sqrt{-(38684 \\
& 44992270541948232^{1/4})\sqrt{1999081657})\sqrt{62})\sqrt{31})\sqrt{2x^2 - x + \\
& 3})(\sqrt{2}(2141441x + 1076175) - 3217616x - 1065266)\sqrt{353101570755 \\
& 7\sqrt{2} + 4997704142500} - 155990877430002205517374x^2 - 140073440957553 \\
& 000872744\sqrt{2}(2x^2 - x + 3) + 480706581467965980267826x - 6366974588 \\
& 97968185785200)/x^2) + 146305963891271067870702891119222361424201\sqrt{31} \\
& (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - \\
& 249300096x^3 + 37981440x^2 - 7744\sqrt{2}(1348x^8 - 2692x^7 + 9789x^6 \\
& 6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 22306406 \\
& 4x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 1 \\
& 3562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 31849 \\
& 49732636*3868444992270541948232^{1/4})\sqrt{1999081657})\sqrt{62})\sqrt{2}(25 \\
& x^4 + 30x^3 + 29x^2 + 12x + 4)\sqrt{3531015707557\sqrt{2} + 49977041425 \\
& 00})\arctan(1/4535484880629403103991789624695893204150231*(2850690442882\sqrt{1999081657} \\
& (2*3868444992270541948232^{3/4})\sqrt{62}(2627559914x^7 - 10 \\
& 187615527x^6 + 21362956024x^5 - 34451465819x^4 + 17321103240x^3 - 83207 \\
& 57400x^2 - \sqrt{2}(1893366636x^7 - 7237484076x^6 + 15226003533x^5 - 24 \\
& 262105817x^4 + 12127036096x^3 - 5664787848x^2 - 13367586816x + 93380256 \\
& 00) - 18676051200x + 13367586816) + 61971531367*3868444992270541948232^{1/4} \\
&)\sqrt{62}(400116332x^7 - 6149336082x^6 + 32552996440x^5 - 74427496472 \\
& x^4 + 96235107840x^3 - 61219656000x^2 - \sqrt{2}(286685371x^7 - 4395067 \\
& 059x^6 + 23180544704x^5 - 52748573780x^4 + 68065744032x^3 - 42544702944 \\
& x^2 - 48625837056x + 34092306432) - 68184612864x + 48625837056))\sqrt{2} \\
& (2x^2 - x + 3)\sqrt{3531015707557\sqrt{2} + 4997704142500} - 1287492482243185 \\
& 3972621854418491567805329688\sqrt{31}\sqrt{2}(28180x^8 - 254666x^7 + 704 \\
& 270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}(874 \\
& 6x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 3
\end{aligned}$$

$$\begin{aligned}
& 96144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{1999081657/8285} \\
& 50919) * (\sqrt{1999081657}) * (2 * 3868444992270541948232^{(3/4)} * \sqrt{62}) * (93510662 \\
& 98x^7 - 13433496653x^6 + 43310345823x^5 - 17374572240x^4 + 20927636280x \\
& x^3 + 18483199488x^2 - \sqrt{2}) * (6839273266x^7 - 9809465289x^6 + 31524099 \\
& 699x^5 - 12024617744x^4 + 13914887256x^3 + 14839341696x^2 - 14839341696 \\
& *x) - 18483199488*x) + 61971531367 * 3868444992270541948232^{(1/4)} * \sqrt{62} * (1 \\
& 427210918x^7 - 18462714328x^6 + 71210222920x^5 - 92387041920x^4 + 11948 \\
& 9780160x^3 + 68726817792x^2 - \sqrt{2}) * (1033310523x^7 - 13365477772x^6 + \\
& 51521534980x^5 - 66583614528x^4 + 85122955872x^3 + 53108877312x^2 - 53 \\
& 108877312*x) - 68726817792*x) * \sqrt{2x^2 - x + 3} * \sqrt{3531015707557 * \sqrt{2} \\
& (2) + 4997704142500) - 4516423329856721284677540671884 * \sqrt{31} * \sqrt{2} * (123 \\
& 408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 \\
& - 3822336x^2 - \sqrt{2}) * (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + \\
& 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800*x) + 3276288*x) - 205291 \\
& 969538941876576251848722 * \sqrt{31} * (254591x^8 - 4815126x^7 + 32303580x^6 \\
& - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 * \sqrt{2} \\
& (2) * (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - \\
& 1944*x) + 144820224*x) * \sqrt{(3868444992270541948232^{(1/4)} * \sqrt{1999081657}) \\
& * \sqrt{62} * \sqrt{31} * \sqrt{2x^2 - x + 3} * (\sqrt{2}) * (2141441x + 1076175) - 321 \\
& 7616*x - 1065266) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500) + 15599087743 \\
& 0002205517374x^2 + 140073440957553000872744 * \sqrt{2} * (2x^2 - x + 3) - 4807 \\
& 06581467965980267826*x + 636697458897968185785200)/x^2) - 14630596389127106 \\
& 7870702891119222361424201 * \sqrt{31} * (2828123x^8 - 9696916x^7 + 53385560x^6 \\
& - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 * \sqrt{2} \\
& (2) * (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1 \\
& 080x^2 + 4320*x - 5184) + 223064064*x - 94887936)) / (2585191x^8 - 4661200x \\
& x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 \\
& - 24772608*x + 18579456)) + 3868444992270541948232^{(1/4)} * \sqrt{1999081657} \\
& * \sqrt{62} * \sqrt{31} * (124942603562500x^4 + 149931124275000x^3 + 14493342013 \\
& 2500x^2 - 3531015707557 * \sqrt{2}) * (25x^4 + 30x^3 + 29x^2 + 12x + 4) + 59 \\
& 972449710000*x + 19990816570000) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500} \\
&) * \log(3123565089062500/828550919 * (3868444992270541948232^{(1/4)} * \sqrt{1999081} \\
& 657) * \sqrt{62} * \sqrt{31} * \sqrt{2x^2 - x + 3} * (\sqrt{2}) * (2141441x + 1076175) - \\
& 3217616*x - 1065266) * \sqrt{3531015707557 * \sqrt{2} + 4997704142500) + 1559908 \\
& 77430002205517374x^2 + 140073440957553000872744 * \sqrt{2} * (2x^2 - x + 3) - \\
& 480706581467965980267826*x + 636697458897968185785200)/x^2) - 3868444992270 \\
& 541948232^{(1/4)} * \sqrt{1999081657} * \sqrt{62} * \sqrt{31} * (124942603562500x^4 + 1 \\
& 49931124275000x^3 + 144933420132500x^2 - 3531015707557 * \sqrt{2}) * (25x^4 + \\
& 30x^3 + 29x^2 + 12x + 4) + 59972449710000*x + 19990816570000) * \sqrt{35310} \\
& 15707557 * \sqrt{2} + 4997704142500) * \log(-3123565089062500/828550919 * (38684449 \\
& 92270541948232^{(1/4)} * \sqrt{1999081657} * \sqrt{62} * \sqrt{31} * \sqrt{2x^2 - x + 3} \\
& * (\sqrt{2}) * (2141441x + 1076175) - 3217616*x - 1065266) * \sqrt{3531015707557 * \sqrt{2} \\
& (2) + 4997704142500) - 155990877430002205517374x^2 - 140073440957553000 \\
& 872744 * \sqrt{2} * (2x^2 - x + 3) + 480706581467965980267826*x - 6366974588979 \\
& 68185785200)/x^2) + 12139426558738796942545211648 * \sqrt{2} * (25x^4 + 30x^3
\end{aligned}$$

```
+ 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 86845533393682860541101280*(97155*x^3 + 93872*x^2 + 69621*x
+ 22552)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.79 \quad \int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=185

$$\frac{625}{16}\sqrt{2x^2-x+3x^7} + \frac{57375}{448}\sqrt{2x^2-x+3x^6} + \frac{2116475\sqrt{2x^2-x+3x^5}}{10752} + \frac{686531\sqrt{2x^2-x+3x^4}}{6144} - \frac{19750457\sqrt{2x^2-x}}{229376}$$

[Out] (16493087661* $\sqrt{3-x+2x^2}$)/29360128 + (1572007407*x* $\sqrt{3-x+2x^2}$)/7340032 - (15428243*x^2* $\sqrt{3-x+2x^2}$)/131072 - (19750457*x^3* $\sqrt{3-x+2x^2}$)/229376 + (686531*x^4* $\sqrt{3-x+2x^2}$)/6144 + (2116475*x^5* $\sqrt{3-x+2x^2}$)/10752 + (57375*x^6* $\sqrt{3-x+2x^2}$)/448 + (625*x^7* $\sqrt{3-x+2x^2}$)/16 + (2899366573* $\text{ArcSinh}[(1-4x)/\sqrt{23}]$)/(8388608* $\sqrt{2}$)

Rubi [A] time = 0.312208, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3x^7} + \frac{57375}{448}\sqrt{2x^2-x+3x^6} + \frac{2116475\sqrt{2x^2-x+3x^5}}{10752} + \frac{686531\sqrt{2x^2-x+3x^4}}{6144} - \frac{19750457\sqrt{2x^2-x}}{229376}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2], x]

[Out] (16493087661* $\sqrt{3-x+2x^2}$)/29360128 + (1572007407*x* $\sqrt{3-x+2x^2}$)/7340032 - (15428243*x^2* $\sqrt{3-x+2x^2}$)/131072 - (19750457*x^3* $\sqrt{3-x+2x^2}$)/229376 + (686531*x^4* $\sqrt{3-x+2x^2}$)/6144 + (2116475*x^5* $\sqrt{3-x+2x^2}$)/10752 + (57375*x^6* $\sqrt{3-x+2x^2}$)/448 + (625*x^7* $\sqrt{3-x+2x^2}$)/16 + (2899366573* $\text{ArcSinh}[(1-4x)/\sqrt{23}]$)/(8388608* $\sqrt{2}$)

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q-1)*(a+b*x+c*x^2)^(p+1))/(c*(q+2*p+1)), x] + Dist[1/(c*(q+2*p+1)), Int[(a+b*x+c*x^2)^p*ExpandToSum[c*(q+2*p+1)*Pq - a*e*(q-1)*x^(q-2) - b*e*(q+p)*x^(q-1) - c*e*(q+2*p+1)*x^q, x], x]] /; FreeQ[{a, b, c,

p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx &= \frac{625}{16} x^7 \sqrt{3-x+2x^2} + \frac{1}{16} \int \frac{256+1536x+6016x^2+14976x^3+28176x^4+37440x^5+24475x^6}{\sqrt{3-x+2x^2}} \\
&= \frac{57375}{448} x^6 \sqrt{3-x+2x^2} + \frac{625}{16} x^7 \sqrt{3-x+2x^2} + \frac{1}{224} \int \frac{3584+21504x+84224x^2+209664x^3-}{\sqrt{3-x+2x^2}} \\
&= \frac{2116475x^5 \sqrt{3-x+2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3-x+2x^2} + \frac{625}{16} x^7 \sqrt{3-x+2x^2} + \frac{\int \frac{43008+258048x+10106}{\sqrt{3-x+2x^2}}}{10752} \\
&= \frac{686531x^4 \sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5 \sqrt{3-x+2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3-x+2x^2} + \frac{625}{16} x^7 \sqrt{3-x+2x^2} \\
&= -\frac{19750457x^3 \sqrt{3-x+2x^2}}{229376} + \frac{686531x^4 \sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5 \sqrt{3-x+2x^2}}{10752} + \frac{57375}{448} x^6 \sqrt{3-x+2x^2} \\
&= -\frac{15428243x^2 \sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3 \sqrt{3-x+2x^2}}{229376} + \frac{686531x^4 \sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5 \sqrt{3-x+2x^2}}{10752} \\
&= \frac{1572007407x \sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3 \sqrt{3-x+2x^2}}{229376} + \frac{686531x^4 \sqrt{3-x+2x^2}}{6144} \\
&= \frac{16493087661 \sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x \sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3 \sqrt{3-x+2x^2}}{229376} \\
&= \frac{16493087661 \sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x \sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3 \sqrt{3-x+2x^2}}{229376} \\
&= \frac{16493087661 \sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x \sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2 \sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3 \sqrt{3-x+2x^2}}{229376}
\end{aligned}$$

Mathematica [A] time = 0.246325, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2)}{352321536}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2], x]

[Out] $(4\sqrt{3-x+2x^2})(49479262983 + 18864088884x - 10367779296x^2 - 7584175488x^3 + 9842108416x^4 + 17338163200x^5 + 11280384000x^6 + 3440640000x^7) + 60886698033\sqrt{2}\operatorname{ArcSinh}[(1-4x)/\sqrt{23}]/352321536$

Maple [A] time = 0.065, size = 147, normalized size = 0.8

$$\frac{625x^7}{16}\sqrt{2x^2-x+3} + \frac{2116475x^5}{10752}\sqrt{2x^2-x+3} + \frac{686531x^4}{6144}\sqrt{2x^2-x+3} - \frac{2899366573\sqrt{2}}{16777216}\operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((5x^2+3x+2)^4/(2x^2-x+3)^{(1/2)}, x)$

[Out] $625/16x^7(2x^2-x+3)^{(1/2)} + 2116475/10752x^5(2x^2-x+3)^{(1/2)} + 686531/6144x^4(2x^2-x+3)^{(1/2)} - 2899366573/16777216x^2(2x^2-x+3)^{(1/2)} + 19750457/229376x^3(2x^2-x+3)^{(1/2)} - 15428243/131072x^2(2x^2-x+3)^{(1/2)} + 57375/448x^6(2x^2-x+3)^{(1/2)} + 1572007407/7340032x(2x^2-x+3)^{(1/2)} + 16493087661/29360128(2x^2-x+3)^{(1/2)}$

Maxima [A] time = 1.53155, size = 200, normalized size = 1.08

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7 + \frac{57375}{448}\sqrt{2x^2-x+3}x^6 + \frac{2116475}{10752}\sqrt{2x^2-x+3}x^5 + \frac{686531}{6144}\sqrt{2x^2-x+3}x^4 - \frac{19750457}{229376}\sqrt{2x^2-x+3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((5x^2+3x+2)^4/(2x^2-x+3)^{(1/2)}, x, \operatorname{algorithm}="maxima")$

[Out] $625/16\sqrt{2x^2-x+3}x^7 + 57375/448\sqrt{2x^2-x+3}x^6 + 2116475/10752\sqrt{2x^2-x+3}x^5 + 686531/6144\sqrt{2x^2-x+3}x^4 - 19750457/229376\sqrt{2x^2-x+3}x^3 - 15428243/131072\sqrt{2x^2-x+3}x^2 + 1572007407/7340032\sqrt{2x^2-x+3}x - 2899366573/16777216\sqrt{2}\operatorname{arcsinh}(1/23\sqrt{23}(4x-1)) + 16493087661/29360128\sqrt{2x^2-x+3}$

Fricas [A] time = 1.36022, size = 355, normalized size = 1.92

$$\frac{1}{88080384}(3440640000x^7 + 11280384000x^6 + 17338163200x^5 + 9842108416x^4 - 7584175488x^3 - 10367779296x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 9842108416*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sqrt(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^4}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)

Giac [A] time = 1.17971, size = 112, normalized size = 0.61

$\frac{1}{88080384} (4 (8 (4 (16 (100 (120 (140x + 459)x + 84659)x + 4805717)x - 59251371)x - 323993103)x + 4716022221)x +$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x - 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x + 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.80 \quad \int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=143

$$\frac{125}{12}\sqrt{2x^2-x+3x^5} + \frac{1355}{48}\sqrt{2x^2-x+3x^4} + \frac{8185}{256}\sqrt{2x^2-x+3x^3} - \frac{3387\sqrt{2x^2-x+3x^2}}{1024} - \frac{372783\sqrt{2x^2-x+3x}}{8192} - \dots$$

[Out] $(-203373*\text{Sqrt}[3-x+2*x^2])/32768 - (372783*x*\text{Sqrt}[3-x+2*x^2])/8192 - (3387*x^2*\text{Sqrt}[3-x+2*x^2])/1024 + (8185*x^3*\text{Sqrt}[3-x+2*x^2])/256 + (1355*x^4*\text{Sqrt}[3-x+2*x^2])/48 + (125*x^5*\text{Sqrt}[3-x+2*x^2])/12 - (9267707*\text{ArcSinh}[(1-4*x)/\text{Sqrt}[23]])/(65536*\text{Sqrt}[2])$

Rubi [A] time = 0.168162, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3x^5} + \frac{1355}{48}\sqrt{2x^2-x+3x^4} + \frac{8185}{256}\sqrt{2x^2-x+3x^3} - \frac{3387\sqrt{2x^2-x+3x^2}}{1024} - \frac{372783\sqrt{2x^2-x+3x}}{8192} - \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2+3*x+5*x^2)^3/\text{Sqrt}[3-x+2*x^2], x]$

[Out] $(-203373*\text{Sqrt}[3-x+2*x^2])/32768 - (372783*x*\text{Sqrt}[3-x+2*x^2])/8192 - (3387*x^2*\text{Sqrt}[3-x+2*x^2])/1024 + (8185*x^3*\text{Sqrt}[3-x+2*x^2])/256 + (1355*x^4*\text{Sqrt}[3-x+2*x^2])/48 + (125*x^5*\text{Sqrt}[3-x+2*x^2])/12 - (9267707*\text{ArcSinh}[(1-4*x)/\text{Sqrt}[23]])/(65536*\text{Sqrt}[2])$

Rule 1661

$\text{Int}[(\text{Pq}_.)*((\text{a}_.) + (\text{b}_.)*(x_.) + (\text{c}_.)*(x_.)^2)^{(\text{p}_.)}, x_Symbol] :> \text{With}[\{q = \text{Expon}[\text{Pq}, x], e = \text{Coeff}[\text{Pq}, x, \text{Expon}[\text{Pq}, x]]\}, \text{Simp}[(e*x^{(q-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a + b*x + c*x^2)^p*\text{ExpandToSum}[c*(q+2*p+1)*\text{Pq} - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

Rule 640

```
Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^3}{\sqrt{3 - x + 2x^2}} dx &= \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{12} \int \frac{96 + 432x + 1368x^2 + 2484x^3 + 1545x^4 + \frac{6775x^5}{2}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{120} \int \frac{960 + 4320x + 13680x^2 - 15810x^3 + \frac{12277}{4}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{125}{12} x^5 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1355}{48} x^4 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{203373 \sqrt{3 - x + 2x^2}}{32768} - \frac{372783x \sqrt{3 - x + 2x^2}}{8192} - \frac{3387x^2 \sqrt{3 - x + 2x^2}}{1024} + \frac{8185}{256} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{960} \int \frac{7680 + 34560x - 12277}{\sqrt{3 - x + 2x^2}} dx
\end{aligned}$$

Mathematica [A] time = 0.13242, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2 - x + 3} \left(1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119 \right) - 27803121\sqrt{2} \sinh^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{393216}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2], x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 2775040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/393216

Maple [A] time = 0.054, size = 113, normalized size = 0.8

$$\frac{125x^5}{12}\sqrt{2x^2-x+3} + \frac{1355x^4}{48}\sqrt{2x^2-x+3} + \frac{9267707\sqrt{2}}{131072}\operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right) + \frac{8185x^3}{256}\sqrt{2x^2-x+3} - \frac{3387}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2), x)

[Out] 125/12*x^5*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+8185/256*x^3*(2*x^2-x+3)^(1/2)-3387/1024*x^2*(2*x^2-x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-203373/32768*(2*x^2-x+3)^(1/2)

Maxima [A] time = 1.44931, size = 154, normalized size = 1.08

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2x^2-x+3}x^4 + \frac{8185}{256}\sqrt{2x^2-x+3}x^3 - \frac{3387}{1024}\sqrt{2x^2-x+3}x^2 - \frac{372783}{8192}\sqrt{2x^2-x+3}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2), x, algorithm="maxima")

[Out] 125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/256*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/8192*

$\sqrt{2x^2 - x + 3} * x + 9267707/131072 * \sqrt{2} * \operatorname{arcsinh}(1/23 * \sqrt{23} * (4x - 1)) - 203373/32768 * \sqrt{2x^2 - x + 3}$

Fricas [A] time = 1.34021, size = 266, normalized size = 1.86

$\frac{1}{98304} (1024000x^5 + 2775040x^4 + 3143040x^3 - 325152x^2 - 4473396x - 610119) \sqrt{2x^2 - x + 3} + \frac{9267707}{262144} \sqrt{2} \log(-4\sqrt{2}(\sqrt{2x^2 - x + 3} * (4x - 1) - 32x^2 + 16x - 25))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x - 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^3}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)

Giac [A] time = 1.19468, size = 99, normalized size = 0.69

$\frac{1}{98304} (4(8(20(16(100x + 271)x + 4911)x - 10161)x - 1118349)x - 610119) \sqrt{2x^2 - x + 3} - \frac{9267707}{131072} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2x^2 - x + 3} * (4x - 1) - 32x^2 + 16x - 25))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x - 610119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25))

$$*x - \text{sqrt}(2*x^2 - x + 3)) + 1)$$

$$3.81 \quad \int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=101

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3 + \frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{11373\sqrt{2x^2-x+3}}{1024} + \frac{30725 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

[Out] (-11373*sqrt[3 - x + 2*x^2])/1024 + (3443*x*sqrt[3 - x + 2*x^2])/768 + (655*x^2*sqrt[3 - x + 2*x^2])/96 + (25*x^3*sqrt[3 - x + 2*x^2])/8 + (30725*ArcSinh[(1 - 4*x)/sqrt[23]])/(2048*sqrt[2])

Rubi [A] time = 0.0883339, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1661, 640, 619, 215}

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3 + \frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{11373\sqrt{2x^2-x+3}}{1024} + \frac{30725 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2], x]

[Out] (-11373*sqrt[3 - x + 2*x^2])/1024 + (3443*x*sqrt[3 - x + 2*x^2])/768 + (655*x^2*sqrt[3 - x + 2*x^2])/96 + (25*x^3*sqrt[3 - x + 2*x^2])/8 + (30725*ArcSinh[(1 - 4*x)/sqrt[23]])/(2048*sqrt[2])

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b

*e)/(2*c), Int[(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^(p), x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 3x + 5x^2)^2}{\sqrt{3 - x + 2x^2}} dx &= \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{8} \int \frac{32 + 96x + 7x^2 + \frac{655x^3}{2}}{\sqrt{3 - x + 2x^2}} dx \\
 &= \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{48} \int \frac{192 - 1389x + \frac{3443x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\
 &= \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{1}{192} \int \frac{\frac{7257}{4} - \frac{34119x}{8}}{\sqrt{3 - x + 2x^2}} dx \\
 &= -\frac{11373\sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} - \frac{307}{192} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
 &= -\frac{11373\sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} - \frac{307}{192} \sqrt{3 - x + 2x^2} \\
 &= -\frac{11373\sqrt{3 - x + 2x^2}}{1024} + \frac{3443}{768} x \sqrt{3 - x + 2x^2} + \frac{655}{96} x^2 \sqrt{3 - x + 2x^2} + \frac{25}{8} x^3 \sqrt{3 - x + 2x^2} + \frac{307}{192} \sqrt{3 - x + 2x^2}
 \end{aligned}$$

Mathematica [A] time = 0.0755216, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2 - x + 3}(9600x^3 + 20960x^2 + 13772x - 34119) + 92175\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2], x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288

Maple [A] time = 0.051, size = 79, normalized size = 0.8

$$\frac{25x^3}{8}\sqrt{2x^2-x+3} + \frac{655x^2}{96}\sqrt{2x^2-x+3} + \frac{3443x}{768}\sqrt{2x^2-x+3} - \frac{11373}{1024}\sqrt{2x^2-x+3} - \frac{30725\sqrt{2}}{4096}\operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2), x)

[Out] 25/8*x^3*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+3443/768*x*(2*x^2-x+3)^(1/2)-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))

Maxima [A] time = 1.48942, size = 108, normalized size = 1.07

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3 + \frac{655}{96}\sqrt{2x^2-x+3}x^2 + \frac{3443}{768}\sqrt{2x^2-x+3}x - \frac{30725}{4096}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11373}{1024}\sqrt{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2), x, algorithm="maxima")

[Out] 25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*sqrt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 11373/1024*sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.37724, size = 211, normalized size = 2.09

$$\frac{1}{3072}(9600x^3 + 20960x^2 + 13772x - 34119)\sqrt{2x^2-x+3} + \frac{30725}{8192}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 30725/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**2/sqrt(2*x**2 - x + 3), x)

Giac [A] time = 1.16484, size = 85, normalized size = 0.84

$$\frac{1}{3072} (4 (40 (60x + 131)x + 3443)x - 34119) \sqrt{2x^2 - x + 3} + \frac{30725}{4096} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2x^2 - x + 3} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 30725/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.82 \quad \int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx$$

Optimal. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3x} + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])

Rubi [A] time = 0.032932, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3x} + \frac{39}{16}\sqrt{2x^2-x+3} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]

[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx &= \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{1}{4} \int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}} dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17}{32} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}} dx, x, -1+4x}\right)}{32\sqrt{46}} \\
&= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{17 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0382121, size = 45, normalized size = 0.76

$$\frac{1}{64} \left(4\sqrt{2x^2 - x + 3}(20x + 39) + 17\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2], x]
```

```
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/64
```

Maple [A] time = 0.051, size = 45, normalized size = 0.8

$$\frac{5x}{4}\sqrt{2x^2-x+3} + \frac{39}{16}\sqrt{2x^2-x+3} - \frac{17\sqrt{2}}{64}\operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)`

[Out] `5/4*x*(2*x^2-x+3)^(1/2)+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arsinh(4/23*23^(1/2)*(x-1/4))`

Maxima [A] time = 1.53221, size = 62, normalized size = 1.05

$$\frac{5}{4}\sqrt{2x^2-x+3}x - \frac{17}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39}{16}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out] `5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arsinh(1/23*sqrt(23)*(4*x - 1)) + 39/16*sqrt(2*x^2 - x + 3)`

Fricas [A] time = 1.32083, size = 163, normalized size = 2.76

$$\frac{1}{16}\sqrt{2x^2-x+3}(20x+39) + \frac{17}{128}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out] `1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 3x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2), x)

[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)

Giac [A] time = 1.15619, size = 72, normalized size = 1.22

$$\frac{1}{16} \sqrt{2x^2 - x + 3}(20x + 39) + \frac{17}{64} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)

$$3.83 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=148

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left(\frac{\sqrt{\frac{11}{31(10\sqrt{2}-13)}}((13-10\sqrt{2})x+3\sqrt{2}-7)}{\sqrt{2x^2-x+3}} \right)$$

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))])*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))])*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rubi [A] time = 0.314359, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}(13+10\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}}((13+10\sqrt{2})x+3\sqrt{2}+7)}{\sqrt{2x^2-x+3}} \right) - \sqrt{\frac{1}{682}(10\sqrt{2}-13)} \tanh^{-1} \left(\frac{\sqrt{\frac{11}{31(10\sqrt{2}-13)}}((13-10\sqrt{2})x+3\sqrt{2}-7)}{\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))])*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))])*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
```

b*f, 0] && NegQ[b^2 - 4*a*c]

Rule 1029

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx &= -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{22\sqrt{2}} \\ &= -\left(\frac{1}{2} \left(11(20-13\sqrt{2})\right)\right) \text{Subst}\left(\int \frac{1}{-3751(13-10\sqrt{2})-11x^2} dx, x, \frac{11(7-3\sqrt{2})}{\sqrt{3-x+2x^2}}\right) \\ &= \sqrt{\frac{1}{682} (13+10\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(13+10\sqrt{2})}} (7+3\sqrt{2}+(13+10\sqrt{2})x)}{\sqrt{3-x+2x^2}}\right) - \sqrt{\frac{1}{682}} \left(\dots\right) \end{aligned}$$

Mathematica [C] time = 0.34685, size = 176, normalized size = 1.19

$$\frac{\sqrt{13+i\sqrt{31}}(\sqrt{31}+13i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}\sqrt{2x^2-x+3}}}\right) + \sqrt{13-i\sqrt{31}}(\sqrt{31}-13i) \tanh^{-1}\left(\frac{(-22+4i\sqrt{31})x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}\sqrt{2x^2-x+3}}}\right)}{20\sqrt{682}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)), x]

[Out]
$$\frac{-(\sqrt{13 + I\sqrt{31}})(13I + \sqrt{31})\operatorname{ArcTanh}\left[\frac{63 + I\sqrt{31} + (-22 - 4I)\sqrt{31}}{2\sqrt{286 + (22I)\sqrt{31}}}\sqrt{3 - x + 2x^2}\right] + \sqrt{13 - I\sqrt{31}}(-13I + \sqrt{31})\operatorname{ArcTanh}\left[\frac{63 - I\sqrt{31} + (-22 + 4I)\sqrt{31}}{2\sqrt{286 - (22I)\sqrt{31}}}\sqrt{3 - x + 2x^2}\right]}{2\sqrt{682}}$$

Maple [B] time = 0.108, size = 684, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2), x)

[Out]
$$\frac{1}{21142} \frac{8(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 2^{2(1/2)+8-3 \cdot 2^{1/2}})^{1/2} \cdot 2^{1/2} \cdot (369(-775687+549362 \cdot 2^{1/2}))^{1/2} \cdot 2^{1/2} \cdot (-8866+6820 \cdot 2^{1/2})^{1/2} \cdot \arctan\left(\frac{1}{11692487}(-775687+549362 \cdot 2^{1/2})\right)^{1/2} \cdot (-23(8+3 \cdot 2^{1/2})) \cdot (-23(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+24 \cdot 2^{1/2}-41)^{1/2} \cdot (6485(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 \cdot 2^{1/2}+10368(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+22379 \cdot 2^{1/2}+32016)}{(23(2^{1/2}-1+x)^4(2^{1/2}+1-x)^4+82(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+23) \cdot (8+3 \cdot 2^{1/2}) \cdot (2^{1/2}-1+x)/(2^{1/2}+1-x)+520(-775687+549362 \cdot 2^{1/2})^{1/2} \cdot (-8866+6820 \cdot 2^{1/2})^{1/2} \cdot \arctan\left(\frac{1}{11692487}(-775687+549362 \cdot 2^{1/2})\right)^{1/2} \cdot (-23(8+3 \cdot 2^{1/2})) \cdot (-23(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+24 \cdot 2^{1/2}-41)^{1/2} \cdot (6485(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 \cdot 2^{1/2}+10368(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+22379 \cdot 2^{1/2}+32016)}{(23(2^{1/2}-1+x)^4(2^{1/2}+1-x)^4+82(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+23) \cdot (8+3 \cdot 2^{1/2}) \cdot (2^{1/2}-1+x)/(2^{1/2}+1-x)+465124 \cdot \operatorname{arctanh}\left(\frac{31}{2} \cdot (8(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 \cdot 2^{1/2}+8-3 \cdot 2^{1/2})^{1/2}\right)}{(-8866+6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} - 866822 \cdot \operatorname{arctanh}\left(\frac{31}{2} \cdot (8(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 \cdot 2^{1/2}+8-3 \cdot 2^{1/2})^{1/2}\right)}{(-8866+6820 \cdot 2^{1/2})^{1/2}} \cdot \frac{(8(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2(2^{1/2}+1-x)^2 \cdot 2^{1/2}+8-3 \cdot 2^{1/2})^{1/2}}{(1+(2^{1/2}-1+x)/(2^{1/2}+1-x))^2} \cdot \frac{1}{(1+(2^{1/2}-1+x)/(2^{1/2}+1-x))} \cdot \frac{1}{(8+3 \cdot 2^{1/2})} \cdot \frac{1}{(-8866+6820 \cdot 2^{1/2})^{1/2}}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)

Fricas [B] time = 4.6852, size = 7101, normalized size = 47.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/845680*\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{13*\sqrt{2} + 20}*(13*\sqrt{2} - 20)*\log(1240*(\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{2*x^2 - x + 3})*(\sqrt{2}*(4*x - 1) - 3*x - 5)*\sqrt{13*\sqrt{2} + 20} + 7595*x^2 + 6820*\sqrt{2}*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{13*\sqrt{2} + 20}*(13*\sqrt{2} - 20)*\log(-1240*(\sqrt{341}*200^{(1/4)}*\sqrt{31}*\sqrt{5}*\sqrt{2*x^2 - x + 3})*(\sqrt{2}*(4*x - 1) - 3*x - 5)*\sqrt{13*\sqrt{2} + 20} - 7595*x^2 - 6820*\sqrt{2}*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) - 1/6820*\sqrt{341}*200^{(1/4)}*\sqrt{5}*\sqrt{2}*\sqrt{13*\sqrt{2} + 20}*\arctan(1/2762875*(14260*\sqrt{341}*\sqrt{5}*\sqrt{2*x^2 - x + 3})*(11*200^{(3/4)}*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - \sqrt{2}*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^{(1/4)}*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - \sqrt{2}*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*\sqrt{13*\sqrt{2} + 20} + 7843000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{310}*(\sqrt{341}*\sqrt{5}*\sqrt{2*x^2 - x + 3})*(11*200^{(3/4)}*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - \sqrt{2}*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^{(1/4)}*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - \end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \cdot (38627x^7 - 500012x^6 + 1934180x^5 - 2560320x^4 + 3506400x^3 \\
& + 1202688x^2 - 1202688x) - 4478976x) \cdot \sqrt{13\sqrt{2} + 20} + 550\sqrt{3} \\
& \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 \\
& + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 \\
& - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276 \\
& 288x) + 25\sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 \\
& + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488\sqrt{2} \cdot (4x^8 - 7 \\
& 6x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144 \\
& 820224x) \cdot \sqrt{-(\sqrt{341}) \cdot 200^{1/4} \cdot \sqrt{31} \cdot \sqrt{5} \cdot \sqrt{2x^2 - x + 3} \cdot \\
& (\sqrt{2} \cdot (4x - 1) - 3x - 5) \cdot \sqrt{13\sqrt{2} + 20} - 7595x^2 - 6820\sqrt{2} \cdot \\
& (2x^2 - x + 3) + 23405x - 31000)/x^2} + 89125\sqrt{31} \cdot (2828123x^8 - \\
& 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 \\
& + 37981440x^2 - 7744\sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + \\
& 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) \\
&) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 4 \\
& 4249088x^3 - 34615296x^2 - 24772608x + 18579456) - 1/6820\sqrt{341} \cdot 200 \\
& ^{1/4} \cdot \sqrt{5} \cdot \sqrt{2} \cdot \sqrt{13\sqrt{2} + 20} \cdot \arctan(1/2762875 \cdot (14260\sqrt{3} \\
& 41) \cdot \sqrt{5} \cdot \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4} \cdot (8056x^7 - 28976x^6 + 61838 \\
& x^5 - 93342x^4 + 45376x^3 - 18288x^2 - \sqrt{2} \cdot (4702x^7 - 19541x^6 + \\
& 40352x^5 - 68777x^4 + 35480x^3 - 19080x^2 - 34560x + 27648) - 55296x \\
& + 34560) + 5 \cdot 200^{1/4} \cdot (18463x^7 - 280047x^6 + 1453472x^5 - 3238500x^4 \\
& + 4140576x^3 - 2378592x^2 - \sqrt{2} \cdot (11418x^7 - 177633x^6 + 957180x^5 \\
& - 2237548x^4 + 2920320x^3 - 2005920x^2 - 1990656x + 1534464) - 3068928x \\
& + 1990656) \cdot \sqrt{13\sqrt{2} + 20} - 7843000\sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - \\
& 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x \\
& ^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 \\
& - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2\sqrt{3} \\
& \sqrt{310} \cdot (\sqrt{341}) \cdot \sqrt{5} \cdot \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4} \cdot (30876x^7 - \\
& 44014x^6 + 139674x^5 - 42464x^4 + 38736x^3 + 89856x^2 - \sqrt{2} \cdot (15454 \\
& x^7 - 22399x^6 + 73509x^5 - 37360x^4 + 52200x^3 + 13824x^2 - 13824x) \\
& - 89856x) + 5 \cdot 200^{1/4} \cdot (69479x^7 - 898236x^6 + 3454740x^5 - 4394304x \\
& ^4 + 5347296x^3 + 4478976x^2 - \sqrt{2} \cdot (38627x^7 - 500012x^6 + 1934180x^5 \\
& - 2560320x^4 + 3506400x^3 + 1202688x^2 - 1202688x) - 4478976x) \cdot \sqrt{13\sqrt{2} + 20} - \\
& 550\sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 - 914152x^7 + 15788 \\
& 88x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (155 \\
& 50x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + \\
& 1209600x^2 - 1036800x) + 3276288x) - 25\sqrt{31} \cdot (254591x^8 - 4815126x^7 \\
& + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 16895692 \\
& 8x^2 - 15488\sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 361 \\
& 8x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{(\sqrt{341}) \cdot 200^{1/4} \cdot \sqrt{3} \\
& 1) \cdot \sqrt{5} \cdot \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (4x - 1) - 3x - 5) \cdot \sqrt{13\sqrt{2} \\
& + 20} + 7595x^2 + 6820\sqrt{2} \cdot (2x^2 - x + 3) - 23405x + 31000)/x^2} - \\
& 89125\sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + \\
& 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744\sqrt{2} \cdot (1348x^8 - 26 \\
& 92x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x -
\end{aligned}$$

5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)

[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.84 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(2343727+1678700\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}}((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}}{\sqrt{2x^2-x+3}}\right)}{1364} - \frac{\sqrt{\frac{1}{682}}}{1364}$$

[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 + 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))])*(2119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/1364 - (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1678700*Sqrt[2]))])*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/1364

Rubi [A] time = 0.428841, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682(5x^2+3x+2)} + \frac{\sqrt{\frac{1}{682}(2343727+1678700\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}}((5751+3935\sqrt{2})x+1816\sqrt{2}+2119)}}{\sqrt{2x^2-x+3}}\right)}{1364} - \frac{\sqrt{\frac{1}{682}}}{1364}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2), x]

[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 + 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))])*(2119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/1364 - (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1678700*Sqrt[2]))])*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/1364

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*

```

a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c
e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]

```

Rule 1029

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 204

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[

```

$-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx &= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{7502} \\ &= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{\int \frac{\frac{121}{2}(537-332\sqrt{2})-\frac{121}{2}(127-205\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{165044\sqrt{2}} + \frac{\int \frac{\frac{121}{2}(537+332\sqrt{2})-\frac{121}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{165044} \\ &= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} - \frac{1}{496} \left(11 \left(3357400 - 2343727\sqrt{2} \right) \right) \text{Subst} \left(\int \frac{-\frac{453871}{4} (2+3x+5x^2)}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx \right) \\ &= \frac{(4+65x)\sqrt{3-x+2x^2}}{682(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682} (2343727 + 1678700\sqrt{2})} \tan^{-1} \left(\sqrt{\frac{11}{31(2343727+1678700\sqrt{2})}} \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} \right)}{1364} \end{aligned}$$

Mathematica [C] time = 1.0074, size = 287, normalized size = 1.53

$$25 \frac{\left(i\sqrt{286+22i\sqrt{31}}(224\sqrt{31}+1023i) \tanh^{-1} \left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}} \right) \right)}{(\sqrt{31}-13i)^2} + \frac{10i \left(1364(\sqrt{31}+13i)(65x+4)\sqrt{2x^2-x+3} - 5\sqrt{286-22i\sqrt{31}}(787\sqrt{31}-1271i)(5x^2+3x+2) \right)}{(\sqrt{31}+13i)^2(10ix+\sqrt{31}+3i)(5(\sqrt{31}-13i)x+8\sqrt{31}-4i)}$$

116281

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2), x]

[Out] (25*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1023*I + 224*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]))/(-13*I + Sqrt[31])^2 + ((10*I)*(1364*(13*I + Sqrt[31])*(4 + 65*x)*Sqrt[3 - x + 2*x^2] - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-1271*I + 787*Sqrt[31]))*(2 + 3*x + 5*x^2)*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x]/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]))/((13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I)*x)*(-4*I + 8*Sqrt[31] + 5*(-13*I + Sqrt[31])*x)))/116281

Maple [B] time = 0.153, size = 5225, normalized size = 27.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)`

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^2 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)`

Fricas [B] time = 5.14088, size = 8625, normalized size = 45.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out] `1/263043507934399808*(8422204*563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)*
(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2) + 3357400)*arctan(1/710190022151725
4683789*(47876524*sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(2950932*x^7 -
11691762*x^6 + 24397746*x^5 - 40053004*x^4 + 20309552*x^3 - 10145376*x^2 -
sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17801494*x^5 - 27869789*x^4 + 13808040
*x^3 - 6172200*x^2 - 15724800*x + 10596096) - 21192192*x + 15724800) + 5203
97*563606738^(1/4)*sqrt(341)*(226651*x^7 - 3496629*x^6 + 18614024*x^5 - 428
60780*x^4 + 55586592*x^3 - 36274464*x^2 - sqrt(2)*(168871*x^7 - 2579646*x^6`

$$\begin{aligned}
& + 13533020x^5 - 30582616x^4 + 39345120x^3 - 23947200x^2 - 28449792x + \\
& 19450368) - 38900736x + 28449792) * \sqrt{2x^2 - x + 3} * \sqrt{2343727\sqrt{2} + 3357400} + 20160232886887690715272 * \sqrt{31} * \sqrt{2} * (28180x^8 - 25466 \\
& 6x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} * (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752 \\
& 088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2 * \sqrt{33 \\
& 574/2191} * (\sqrt{33574}) * (22 * 563606738^{3/4} * \sqrt{341}) * (10257392x^7 - 147733 \\
& 68x^6 + 47877288x^5 - 20710528x^4 + 26321472x^3 + 17079552x^2 - \sqrt{2} * (8292238x^7 - 11867543x^6 + 37968813x^5 - 13449840x^4 + 14570280x^3 \\
& + 20176128x^2 - 20176128x) - 17079552x) + 520397 * 563606738^{1/4} * \sqrt{34 \\
& 1} * (795513x^7 - 10292932x^6 + 39734380x^5 - 51864768x^4 + 68281632x^3 \\
& + 34255872x^2 - 8 * \sqrt{2} * (77213x^7 - 998548x^6 + 3846220x^5 - 4943520x \\
& x^4 + 6215760x^3 + 4318272x^2 - 4318272x) - 34255872x) * \sqrt{2x^2 - x \\
& + 3} * \sqrt{2343727\sqrt{2} + 3357400} + 421088065768678 * \sqrt{31} * \sqrt{2} * (12 \\
& 3408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 \\
& - 3822336x^2 - \sqrt{2} * (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 \\
& + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 19140 \\
& 366625849 * \sqrt{31} * (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 \\
& + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 * \sqrt{2} * (4x^8 - 76x \\
& x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 14482 \\
& 0224x) * \sqrt{-(563606738^{1/4} * \sqrt{33574}) * \sqrt{341}) * \sqrt{31} * \sqrt{2x^2 - \\
& x + 3} * (\sqrt{2} * (1123x + 898) - 2021x - 225) * \sqrt{2343727\sqrt{2} + 3357 \\
& 400} - 1731948347213x^2 - 1555218924028 * \sqrt{2} * (2x^2 - x + 3) + 53372285 \\
& 80187x - 7069176927400) / x^2) + 229093555532814667219 * \sqrt{31} * (2828123x^8 \\
& - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x \\
& ^3 + 37981440x^2 - 7744 * \sqrt{2} * (1348x^8 - 2692x^7 + 9789x^6 - 10070x^ \\
& 5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 948879 \\
& 36) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 \\
& + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 8422204 * 563606738 \\
& ^{1/4} * \sqrt{33574} * \sqrt{341} * \sqrt{2} * (5x^2 + 3x + 2) * \sqrt{2343727\sqrt{2} + 3357400} * \arctan(1/7101900221517254683789 * (47876524 * \sqrt{33574}) * (22 * 56360 \\
& 6738^{3/4} * \sqrt{341}) * (2950932x^7 - 11691762x^6 + 24397746x^5 - 40053004x \\
& x^4 + 20309552x^3 - 10145376x^2 - \sqrt{2} * (2248634x^7 - 8421787x^6 + 17 \\
& 801494x^5 - 27869789x^4 + 13808040x^3 - 6172200x^2 - 15724800x + 10596 \\
& 096) - 21192192x + 15724800) + 520397 * 563606738^{1/4} * \sqrt{341} * (226651x^ \\
& 7 - 3496629x^6 + 18614024x^5 - 42860780x^4 + 55586592x^3 - 36274464x^2 \\
& - \sqrt{2} * (168871x^7 - 2579646x^6 + 13533020x^5 - 30582616x^4 + 393451 \\
& 20x^3 - 23947200x^2 - 28449792x + 19450368) - 38900736x + 28449792) * \sqrt{2x^2 - x + 3} * \sqrt{2343727\sqrt{2} + 3357400} - 20160232886887690715272 \\
& * \sqrt{31} * \sqrt{2} * (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549 \\
& 144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} * (8746x^8 - 102335x^7 + 396104x \\
& x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 53913 \\
& 6) + 1154304x - 456192) - 2 * \sqrt{33574/2191} * (\sqrt{33574}) * (22 * 563606738^{3 \\
& /4} * \sqrt{341}) * (10257392x^7 - 14773368x^6 + 47877288x^5 - 20710528x^4 + \\
& 26321472x^3 + 17079552x^2 - \sqrt{2} * (8292238x^7 - 11867543x^6 + 3796881
\end{aligned}$$


```

3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
- 225)*sqrt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028
*sqrt(2)*(2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 22909355
5532814667219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*(16787000*
x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x + 2) + 10072200*x + 6714800)*sqrt(234372
7*sqrt(2) + 3357400)*log(335740000/2191*(563606738^(1/4)*sqrt(33574)*sqrt(3
41)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sq
rt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028*sqrt(2)*(
2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 563606738^(1/4)*sq
rt(33574)*sqrt(341)*sqrt(31)*(16787000*x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x +
2) + 10072200*x + 6714800)*sqrt(2343727*sqrt(2) + 3357400)*log(-335740000/
2191*(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(s
qrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357400) - 173
1948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 5337228580187*x -
7069176927400)/x^2) + 385694293158944*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^
2 + 3*x + 2)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.85 \quad \int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364(5x^2+3x+2)^2} + \frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)} + \frac{25\sqrt{\frac{1}{682}(6414867847+4536374600\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(6414867847+4536374600\sqrt{2})}}{\sqrt{31(6414867847+4536374600\sqrt{2})}}\right)}{3720992}$$

[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 86265*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(6414867847 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*Sqrt[2]))])*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))])*(123161 - 85754*Sqrt[2] + (294669 - 208915*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/3720992

Rubi [A] time = 0.469453, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364(5x^2+3x+2)^2} + \frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)} + \frac{25\sqrt{\frac{1}{682}(6414867847+4536374600\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{31(6414867847+4536374600\sqrt{2})}}{\sqrt{31(6414867847+4536374600\sqrt{2})}}\right)}{3720992}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3), x]

[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 86265*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(6414867847 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*Sqrt[2]))])*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))])*(123161 - 85754*Sqrt[2] + (294669 - 208915*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])]/3720992

Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a

```

```
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3} dx &= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5775 + \frac{6479x}{2} - 2860x^2}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx}{15004} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{-\frac{28220225}{2} + \frac{22521125x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{112560008} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{\int \frac{\frac{33275}{4}(26103-18658\sqrt{2})}{\sqrt{3-x+2x^2}} dx}{247632} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} - \frac{(6875(9072749200 - \dots))}{\dots} \\
&= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496(2+3x+5x^2)} + \frac{25\sqrt{\frac{1}{682}}(6414867847 \dots)}{\dots}
\end{aligned}$$

Mathematica [C] time = 6.23773, size = 1279, normalized size = 5.74

result too large to display

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3), x]

[Out] (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I + Sqrt[31])*(3 - I*Sqrt[31] + 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 - I*Sqrt[31])*(3 - I*Sqrt[31] + 10*x)) - (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I - Sqrt[31])*(3 + I*Sqrt[31] + 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 + I*Sqrt[31])*(3 + I*Sqrt[31] + 10*x)) - (375*Sqrt[(2*(13 - I*Sqrt[31]))/11]*(11 - (2*I)*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 2*(11 - (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 - I*Sqrt[31]])*Sqrt[3 - x + 2*x^2]])/(10571*(13*I + Sqrt[31])^2) - (750*Sqrt[(2*(13 - I*Sqrt[31]))/341]*ArcTanh[(63 - I*Sqrt[31] - 2*(11 - (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 - I*Sqrt[31]])*Sqrt[3 - x + 2*x^2]])/(961*(13*I + Sqrt[31])) + (750*Sqrt[(2*(13 + I*Sqrt[31]))/341]*ArcTanh[(63 + I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 + I*Sqrt[31]])*Sqrt[3 - x + 2*x^2]])/(961*(13*I - Sqrt[31])) - (375*Sqrt[(2*(13 + I*Sqrt[31]))

```

/11]*(11 + (2*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31]
)]*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]Sqrt[3 - x + 2*x^2]])/(10571*(13*I - S
qrt[31])^2) + (((500*I)/31)*(-((20*(-3 + I*Sqrt[31]) - 10*(27 - (4*I)*Sqrt
[31]))*Sqrt[3 - x + 2*x^2])/((300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[3
1])^2)*(-3 + I*Sqrt[31] - 10*x))) + (2*Sqrt[22*(13 - I*Sqrt[31])]*(20*(-3 +
I*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(-3 + I*Sqrt[31])*(27
- (4*I)*Sqrt[31])))*ArcTanh[(-63 + I*Sqrt[31] - (-10 + 4*(-3 + I*Sqrt[31]))
*x)/(2*Sqrt[22*(13 - I*Sqrt[31])]Sqrt[3 - x + 2*x^2]])/((300 - 10*(-3 + I
*Sqrt[31]) + 2*(-3 + I*Sqrt[31])^2)*(1200 - 40*(-3 + I*Sqrt[31]) + 8*(-3 +
I*Sqrt[31])^2))))/(Sqrt[31]*(300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31
])^2)) + (((500*I)/31)*(-(((-20*(3 + I*Sqrt[31]) + 10*(-27 - (4*I)*Sqrt[31]
))*Sqrt[3 - x + 2*x^2])/((300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2)
*(3 + I*Sqrt[31] + 10*x))) + (2*Sqrt[22*(13 + I*Sqrt[31])]*(-20*(3 + I*Sqrt
[31]) - 10*(-27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(3 + I*Sqrt[31])*(-27 - (4*I
)*Sqrt[31])))*ArcTanh[(63 + I*Sqrt[31] - (10 + 4*(3 + I*Sqrt[31]))*x)/(2*Sq
rt[22*(13 + I*Sqrt[31])]Sqrt[3 - x + 2*x^2]])/((300 + 10*(3 + I*Sqrt[31])
+ 2*(3 + I*Sqrt[31])^2)*(1200 + 40*(3 + I*Sqrt[31]) + 8*(3 + I*Sqrt[31])^2
))))/(Sqrt[31]*(300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2))

```

Maple [B] time = 0.191, size = 13040, normalized size = 58.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^3 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)

Fricas [B] time = 5.17811, size = 9927, normalized size = 44.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out]
$$-1/212344000027477426346822144*(46113488900*4115738902305032^{(1/4)}*\sqrt{22681873}*\sqrt{341}*\sqrt{2}*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*\sqrt{6414867847*\sqrt{2} + 9072749200}*\arctan(1/3836668309294009530058322373948769*(64688701796*\sqrt{22681873}*(11*4115738902305032^{(3/4)}*\sqrt{341}*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 2070733376*x^4 + 1037098288*x^3 - 489164544*x^2 - \sqrt{2}*(112700446*x^7 - 434839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*4115738902305032^{(1/4)}*\sqrt{341}*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 - 2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - \sqrt{2}*(8564099*x^7 - 131508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400*x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*\sqrt{2*x^2 - x + 3}*\sqrt{6414867847*\sqrt{2} + 9072749200} + 10891187458641059311133302222822312*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{45363746/479849}*(\sqrt{22681873}*(11*4115738902305032^{(3/4)}*\sqrt{341}*(576322648*x^7 - 827050092*x^6 + 2660713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - \sqrt{2}*(403157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*4115738902305032^{(1/4)}*\sqrt{341}*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*\sqrt{2}*(15328963*x^7 - 198290348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 - 755350272*x) - 2197767168*x))*\sqrt{2*x^2 - x + 3}*\sqrt{6414867847*\sqrt{2} + 9072749200} + 168363055004367262339322*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 7652866136562148288151*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(4115738902305032^{(1/4)}*\sqrt{22681873}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^2 - x + 3}*(\sqrt{2}*(67187*x + 26012) - 93199*x - 41175)*\sqrt{6414867847*\sqrt{2} + 9072749200} - 512510746420187753*x^2 - 460213731479352268*\sqrt{2}*(2*x^2 - x + 3) + 1579369851213231647*$$

$$\begin{aligned}
& x - 2091880597633419400)/x^2) + 123763493848193855808332979804799*\sqrt{31}* \\
& (2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - \\
& 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^ \\
& 6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406 \\
& 4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1 \\
& 3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 46113 \\
& 488900*4115738902305032^{(1/4)}*\sqrt{22681873}*\sqrt{341}*\sqrt{2}*(25*x^4 + 30 \\
& *x^3 + 29*x^2 + 12*x + 4)*\sqrt{6414867847*\sqrt{2} + 9072749200}*\arctan(1/38 \\
& 36668309294009530058322373948769*(64688701796*\sqrt{22681873})*(11*4115738902 \\
& 305032^{(3/4)}*\sqrt{341}*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 20 \\
& 70733376*x^4 + 1037098288*x^3 - 489164544*x^2 - \sqrt{2}*(112700446*x^7 - 43 \\
& 4839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^ \\
& 2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*411573 \\
& 8902305032^{(1/4)}*\sqrt{341}*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 - \\
& 2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - \sqrt{2}*(8564099*x^7 - 1 \\
& 31508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400 \\
& *x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*\sqrt{2*x^2 \\
& - x + 3}*\sqrt{6414867847*\sqrt{2} + 9072749200) - 10891187458641059311133302 \\
& 222822312*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x \\
& ^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 \\
& + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x \\
& - 539136) + 1154304*x - 456192) - 2*\sqrt{45363746/479849}*(\sqrt{22681873} \\
& *(11*4115738902305032^{(3/4)}*\sqrt{341}*(576322648*x^7 - 827050092*x^6 + 2660 \\
& 713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - \sqrt{2}*(40 \\
& 3157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^ \\
& 3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*41157389023050 \\
& 32^{(1/4)}*\sqrt{341}*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241 \\
& 792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*\sqrt{2}*(15328963*x^7 - 19829 \\
& 0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 - \\
& 755350272*x) - 2197767168*x))*\sqrt{2*x^2 - x + 3}*\sqrt{6414867847*\sqrt{2} \\
& + 9072749200) - 168363055004367262339322*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914 \\
& 152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 \\
& - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 \\
& - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 7652866136562148288 \\
& 151*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087 \\
& 81920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + \\
& 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x \\
&))*\sqrt{((4115738902305032^{(1/4)}*\sqrt{22681873}*\sqrt{341}*\sqrt{31}*\sqrt{2*x^ \\
& 2 - x + 3}*(\sqrt{2}*(67187*x + 26012) - 93199*x - 41175)*\sqrt{6414867847*\sqrt{2} \\
& + 9072749200) + 512510746420187753*x^2 + 460213731479352268*\sqrt{2}*(\\
& 2*x^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) - 123763 \\
& 493848193855808332979804799*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560* \\
& x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2} \\
& *(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + \\
& 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 466120
\end{aligned}$$

```

0*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*
x^2 - 24772608*x + 18579456)) - 25*4115738902305032^(1/4)*sqrt(22681873)*sq
rt(341)*sqrt(31)*(226818730000*x^4 + 272182476000*x^3 + 263109726800*x^2 -
6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 108872990400*x +
36290996800)*sqrt(6414867847*sqrt(2) + 9072749200)*log(1134093650000000/47
9849*(4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sqrt(
2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(2*x
^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) + 25*411573
8902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*(226818730000*x^4 + 2721
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.86 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=166

$$\frac{625}{24}\sqrt{2x^2-x+3x^5} + \frac{10075}{96}\sqrt{2x^2-x+3x^4} + \frac{79425}{512}\sqrt{2x^2-x+3x^3} - \frac{111315\sqrt{2x^2-x+3x^2}}{2048} - \frac{8992487\sqrt{2x^2-x+3x^2}}{16384}$$

[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 - x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])

Rubi [A] time = 0.203946, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3x^5} + \frac{10075}{96}\sqrt{2x^2-x+3x^4} + \frac{79425}{512}\sqrt{2x^2-x+3x^3} - \frac{111315\sqrt{2x^2-x+3x^2}}{2048} - \frac{8992487\sqrt{2x^2-x+3x^2}}{16384}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2), x]

[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 - x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(

$2*c*f - b*g$), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :=> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + \frac{83375x^5}{8} + \frac{14375}{4}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \frac{1}{138} \int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x^2}{16} + \frac{430905x^3}{8} + \frac{42328395}{32} - \frac{9917715x}{16} - \frac{7324005}{8}}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} + \frac{1}{138} \int \frac{42328395 - 9917715x - 7324005}{\sqrt{3 - x + 2x^2}} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} + \frac{625}{24}x^5\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} + \frac{10075}{96}x^4\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048} \\
&= -\frac{14641(101 + 79x)}{1472\sqrt{3 - x + 2x^2}} - \frac{31009685\sqrt{3 - x + 2x^2}}{65536} - \frac{8992487x\sqrt{3 - x + 2x^2}}{16384} - \frac{111315x^2\sqrt{3 - x + 2x^2}}{2048}
\end{aligned}$$

Mathematica [A] time = 0.376634, size = 95, normalized size = 0.57

$$\sqrt{2x^2 - x + 3} \left(\frac{625x^5}{24} + \frac{10075x^4}{96} + \frac{79425x^3}{512} - \frac{111315x^2}{2048} - \frac{14641(79x + 101)}{1472(2x^2 - x + 3)} - \frac{8992487x}{16384} - \frac{31009685}{65536} \right) + \frac{31044558}{138}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2), x]

[Out] $\text{Sqrt}[3 - x + 2x^2] * (-31009685/65536 - (8992487x)/16384 - (111315x^2)/2048 + (79425x^3)/512 + (10075x^4)/96 + (625x^5)/24 - (14641(101 + 79x))/(1472(3 - x + 2x^2))) + (310445587 * \text{ArcSinh}[(-1 + 4x)/\text{Sqrt}[23]]) / (131072 * \text{Sqrt}[2])$

Maple [A] time = 0.066, size = 166, normalized size = 1.

$$\frac{8825x^6}{48} \frac{1}{\sqrt{2x^2-x+3}} + \frac{217675x^5}{768} \frac{1}{\sqrt{2x^2-x+3}} + \frac{52235x^4}{1024} \frac{1}{\sqrt{2x^2-x+3}} + \frac{310445587\sqrt{2}}{262144} \text{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((5x^2+3x+2)^4/(2x^2-x+3)^{(3/2)}, x)$

[Out] $8825/48x^6/(2x^2-x+3)^{(1/2)} + 217675/768x^5/(2x^2-x+3)^{(1/2)} + 52235/1024x^4/(2x^2-x+3)^{(1/2)} + 310445587/262144x^3/(2x^2-x+3)^{(1/2)} * \text{arcsinh}(4/23 * 23^{(1/2)} * (x-1/4)) + 1234044515/12058624 * (-1+4x)/(2x^2-x+3)^{(1/2)} + 625/12x^7/(2x^2-x+3)^{(1/2)} - 310445587/131072x/(2x^2-x+3)^{(1/2)} - 4734827/8192x^3/(2x^2-x+3)^{(1/2)} - 18367831/32768x^2/(2x^2-x+3)^{(1/2)} - 1217267299/524288/(2x^2-x+3)^{(1/2)}$

Maxima [A] time = 1.49527, size = 200, normalized size = 1.2

$$\frac{625x^7}{12\sqrt{2x^2-x+3}} + \frac{8825x^6}{48\sqrt{2x^2-x+3}} + \frac{217675x^5}{768\sqrt{2x^2-x+3}} + \frac{52235x^4}{1024\sqrt{2x^2-x+3}} - \frac{4734827x^3}{8192\sqrt{2x^2-x+3}} - \frac{18367831x^2}{32768\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((5x^2+3x+2)^4/(2x^2-x+3)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $625/12x^7/\text{sqrt}(2x^2-x+3) + 8825/48x^6/\text{sqrt}(2x^2-x+3) + 217675/768x^5/\text{sqrt}(2x^2-x+3) + 52235/1024x^4/\text{sqrt}(2x^2-x+3) - 4734827/8192x^3/\text{sqrt}(2x^2-x+3) - 18367831/32768x^2/\text{sqrt}(2x^2-x+3) + 310445587/262144 * \text{sqrt}(2) * \text{arcsinh}(1/23 * \text{sqrt}(23) * (4x-1)) - 2953101993/1507328 * x/\text{sqrt}(2x^2-x+3) - 3653899049/1507328/\text{sqrt}(2x^2-x+3)$

Fricas [A] time = 1.41592, size = 385, normalized size = 2.32

$$21420745503 \sqrt{2} (2x^2 - x + 3) \log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25) + 8(235520000x^7 + 831385600x^6 + 36175872(2x^5 + 10x^4 + 10x^3 + 5x^2 + 5x + 2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^6 + 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 8859305979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)

Giac [A] time = 1.1978, size = 111, normalized size = 0.67

$$-\frac{310445587}{262144} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(4(40(20(16(100x + 353)x + 8707)x + 31341)x - 14204481)x - 55103493)x - 8859305979)x - 10961697147)}{4521984 \sqrt{2} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x - 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x + 3)

$$3.87 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=124

$$\frac{125}{16}\sqrt{2x^2-x+3}x^3 + \frac{1825}{64}\sqrt{2x^2-x+3}x^2 + \frac{15565}{512}\sqrt{2x^2-x+3}x - \frac{181561\sqrt{2x^2-x+3}}{2048} - \frac{1331(17-45x)}{368\sqrt{2x^2-x+3}} + \frac{1168881}{4096\sqrt{2}}$$

[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2])/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])

Rubi [A] time = 0.127095, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16}\sqrt{2x^2-x+3}x^3 + \frac{1825}{64}\sqrt{2x^2-x+3}x^2 + \frac{15565}{512}\sqrt{2x^2-x+3}x - \frac{181561\sqrt{2x^2-x+3}}{2048} - \frac{1331(17-45x)}{368\sqrt{2x^2-x+3}} + \frac{1168881}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2), x]

[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2])/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}} dx &= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{-\frac{110285}{64} - \frac{19067x}{32} + \frac{22195x^2}{16} + \frac{13225x^3}{8} + \frac{2875x^4}{4}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{92} \int \frac{-\frac{110285}{8} - \frac{19067x}{4} + \frac{18515x^2}{4} + \frac{125925x^3}{8}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{1825}{64} x^2 \sqrt{3-x+2x^2} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{492177x}{4}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{15565}{512} x \sqrt{3-x+2x^2} + \frac{1825}{64} x^2 \sqrt{3-x+2x^2} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{492177x}{4}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512} x \sqrt{3-x+2x^2} + \frac{1825}{64} x^2 \sqrt{3-x+2x^2} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{492177x}{4}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512} x \sqrt{3-x+2x^2} + \frac{1825}{64} x^2 \sqrt{3-x+2x^2} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{492177x}{4}}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512} x \sqrt{3-x+2x^2} + \frac{1825}{64} x^2 \sqrt{3-x+2x^2} + \frac{125}{16} x^3 \sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{-\frac{330855}{4} - \frac{492177x}{4}}{\sqrt{3-x+2x^2}} dx
\end{aligned}$$

Mathematica [A] time = 0.228601, size = 65, normalized size = 0.52

$$\frac{4(736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965)}{\sqrt{2x^2-x+3}} - 26884263\sqrt{2} \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)$$

188416

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2), x]

[Out] ((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736000*x^5))/Sqrt[3 - x + 2*x^2] - 26884263*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/188416

Maple [A] time = 0.059, size = 132, normalized size = 1.1

$$\frac{125x^5}{8} \frac{1}{\sqrt{2x^2-x+3}} + \frac{1575x^4}{32} \frac{1}{\sqrt{2x^2-x+3}} - \frac{1168881\sqrt{2}}{8192} \operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right) + \frac{-5392543 + 21570172x}{376832} \frac{1}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x)`

[Out] $125/8*x^5/(2*x^2-x+3)^{(1/2)}+1575/32*x^4/(2*x^2-x+3)^{(1/2)}-1168881/8192*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+5392543/376832*(-1+4*x)/(2*x^2-x+3)^{(1/2)}+1168881/4096*x/(2*x^2-x+3)^{(1/2)}+14265/256*x^3/(2*x^2-x+3)^{(1/2)}-125091/1024*x^2/(2*x^2-x+3)^{(1/2)}-5130399/16384/(2*x^2-x+3)^{(1/2)}$

Maxima [A] time = 1.52628, size = 154, normalized size = 1.24

$$\frac{125x^5}{8\sqrt{2x^2-x+3}} + \frac{1575x^4}{32\sqrt{2x^2-x+3}} + \frac{14265x^3}{256\sqrt{2x^2-x+3}} - \frac{125091x^2}{1024\sqrt{2x^2-x+3}} - \frac{1168881}{8192}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")`

[Out] $125/8*x^5/\operatorname{sqrt}(2*x^2-x+3)+1575/32*x^4/\operatorname{sqrt}(2*x^2-x+3)+14265/256*x^3/\operatorname{sqrt}(2*x^2-x+3)-125091/1024*x^2/\operatorname{sqrt}(2*x^2-x+3)-1168881/8192*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1))+16138403/47104*x/\operatorname{sqrt}(2*x^2-x+3)-15423965/47104/\operatorname{sqrt}(2*x^2-x+3)$

Fricas [A] time = 1.32687, size = 311, normalized size = 2.51

$$\frac{26884263\sqrt{2}(2x^2-x+3)\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+8(736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965)*\operatorname{sqrt}(2*x^2-x+3)}{376832(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")`

[Out] $1/376832*(26884263*\operatorname{sqrt}(2)*(2*x^2-x+3)*\log(4*\operatorname{sqrt}(2)*\operatorname{sqrt}(2*x^2-x+3)*(4*x-1)-32*x^2+16*x-25)+8*(736000*x^5+2318400*x^4+2624760*x^3-5754186*x^2+16138403*x-15423965)*\operatorname{sqrt}(2*x^2-x+3))/(2*x^2-x+3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)

Giac [A] time = 1.20565, size = 97, normalized size = 0.78

$$\frac{1168881}{8192} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(20(40(20x + 63)x + 2853)x - 125091)x + 16138403)x - 15423965}{47104 \sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] 1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x - 15423965)/sqrt(2*x^2 - x + 3)

$$3.88 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 + (25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])

Rubi [A] time = 0.0708539, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}} + \frac{25}{8}x\sqrt{2x^2-x+3} + \frac{415}{32}\sqrt{2x^2-x+3} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2), x]

[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 + (25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{(2 + 3x + 5x^2)^2}{(3 - x + 2x^2)^{3/2}} dx &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{\frac{1173}{16} + \frac{1955x}{8} + \frac{575x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{1}{46} \int \frac{-138 + \frac{9545x}{8}}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{223}{64} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\ &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} + \frac{223 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{23}} dx, x, -1 + 4x}\right)}{64\sqrt{46}} \\ &= \frac{121(19 - 7x)}{92\sqrt{3 - x + 2x^2}} + \frac{415}{32}\sqrt{3 - x + 2x^2} + \frac{25}{8}x\sqrt{3 - x + 2x^2} - \frac{223 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.140896, size = 55, normalized size = 0.67

$$\frac{4600x^3 + 16790x^2 - 9421x + 47027}{736\sqrt{2x^2 - x + 3}} + \frac{223 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2), x]

[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (223*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])

Maple [A] time = 0.052, size = 98, normalized size = 1.2

$$\frac{25x^3}{4\sqrt{2x^2-x+3}} + \frac{365x^2}{16\sqrt{2x^2-x+3}} - \frac{223x}{64\sqrt{2x^2-x+3}} + \frac{15761}{256\sqrt{2x^2-x+3}} - \frac{-13713+54852x}{5888\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2), x)

[Out] 25/4*x^3/(2*x^2-x+3)^(1/2)+365/16*x^2/(2*x^2-x+3)^(1/2)-223/64*x/(2*x^2-x+3)^(1/2)+15761/256/(2*x^2-x+3)^(1/2)-13713/5888*(-1+4*x)/(2*x^2-x+3)^(1/2)+23/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))

Maxima [A] time = 1.48402, size = 108, normalized size = 1.32

$$\frac{25x^3}{4\sqrt{2x^2-x+3}} + \frac{365x^2}{16\sqrt{2x^2-x+3}} + \frac{223}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{9421x}{736\sqrt{2x^2-x+3}} + \frac{47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2), x, algorithm="maxima")

[Out] 25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) + 47027/736/sqrt(2*x^2 - x + 3)

Fricas [A] time = 1.3002, size = 251, normalized size = 3.06

$$\frac{5129\sqrt{2}(2x^2 - x + 3)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(4600x^3 + 16790x^2 - 9421x + 47027)}{5888(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)

Giac [A] time = 1.18148, size = 84, normalized size = 1.02

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(230(20x + 73)x - 9421)x + 47027}{736\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)

$$3.89 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=45

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

[Out] $(-11*(5 + 3*x))/(23*\text{Sqrt}[3 - x + 2*x^2]) - (5*\text{ArcSinh}[(1 - 4*x)/\text{Sqrt}[23]])/(2*\text{Sqrt}[2])$

Rubi [A] time = 0.0291616, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1660, 12, 619, 215}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^{(3/2)}, x]$

[Out] $(-11*(5 + 3*x))/(23*\text{Sqrt}[3 - x + 2*x^2]) - (5*\text{ArcSinh}[(1 - 4*x)/\text{Sqrt}[23]])/(2*\text{Sqrt}[2])$

Rule 1660

$\text{Int}[(\text{Pq}_*)*((\text{a}_*) + (\text{b}_*)*(\text{x}_*) + (\text{c}_*)*(\text{x}_*)^2)^{(p_*)}, \text{x_Symbol}] \rightarrow \text{With}[\{\text{Q} = \text{PolynomialQuotient}[\text{Pq}, \text{a} + \text{b}*x + \text{c}*x^2, x], \text{f} = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, \text{a} + \text{b}*x + \text{c}*x^2, x], x, 0], \text{g} = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, \text{a} + \text{b}*x + \text{c}*x^2, x], x, 1]\}, \text{Simp}[\text{((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; \text{FreeQ}\{\{a, b, c\}, x\} \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

Rule 12

$\text{Int}[(\text{a}_*)*(\text{u}_*), \text{x_Symbol}] \rightarrow \text{Dist}[\text{a}, \text{Int}[\text{u}, x], x] /; \text{FreeQ}[\text{a}, x] \&\& \text{!Match}[\text{Q}[\text{u}, (\text{b}_*)*(\text{v}_*)] /; \text{FreeQ}[\text{b}, x]]$

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}} dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{115}{4\sqrt{3-x+2x^2}} dx \\
 &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{5}{2} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
 &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} + \frac{5 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x \right)}{2\sqrt{46}} \\
 &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5 \sinh^{-1} \left(\frac{1-4x}{\sqrt{23}} \right)}{2\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.0770898, size = 45, normalized size = 1.

$$\frac{5 \sinh^{-1} \left(\frac{4x-1}{\sqrt{23}} \right)}{2\sqrt{2}} - \frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2), x]

[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(2*Sqrt[2])

Maple [A] time = 0.049, size = 64, normalized size = 1.4

$$-\frac{5x}{2} \frac{1}{\sqrt{2x^2-x+3}} - \frac{17}{8} \frac{1}{\sqrt{2x^2-x+3}} + \frac{-49+196x}{184} \frac{1}{\sqrt{2x^2-x+3}} + \frac{5\sqrt{2}}{4} \operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x)

[Out] -5/2*x/(2*x^2-x+3)^(1/2)-17/8/(2*x^2-x+3)^(1/2)+49/184*(-1+4*x)/(2*x^2-x+3)^(1/2)+5/4*2^(1/2)*arsinh(4/23*23^(1/2)*(x-1/4))

Maxima [A] time = 1.50947, size = 62, normalized size = 1.38

$$\frac{5}{4} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x-1)\right) - \frac{33x}{23\sqrt{2x^2-x+3}} - \frac{55}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 5/4*sqrt(2)*arsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3) - 55/23/sqrt(2*x^2 - x + 3)

Fricas [B] time = 1.30075, size = 209, normalized size = 4.64

$$\frac{115\sqrt{2}(2x^2-x+3)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)-88\sqrt{2x^2-x+3}(3x+5)}{184(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x + 3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)

[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)

Giac [A] time = 1.22882, size = 72, normalized size = 1.6

$$-\frac{5}{4} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{11(3x + 5)}{23 \sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] -5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23*(3*x + 5)/sqrt(2*x^2 - x + 3)

$$3.90 \quad \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22}\sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right) - \frac{1}{22}\sqrt{\frac{1}{682}(500\sqrt{2})}$$

[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(247 + 500*Sqrt[2]))])*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-247 + 500*Sqrt[2]))])*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/22

Rubi [A] time = 0.407876, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}} + \frac{1}{22}\sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}((69+65\sqrt{2})x+4\sqrt{2}+61)}{\sqrt{2x^2-x+3}} \right) - \frac{1}{22}\sqrt{\frac{1}{682}(500\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]

[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(247 + 500*Sqrt[2]))])*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-247 + 500*Sqrt[2]))])*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/22

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*x*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e

```

- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]

```

Rule 1029

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 204

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

```

a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx &= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{2783} \\
 &= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}(3+8\sqrt{2})-\frac{2783}{2}(13-5\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}(3-8\sqrt{2})-\frac{2783}{2}(13+5\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{61226\sqrt{2}} \\
 &= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8} \left(253(1000-247\sqrt{2}) \right) \text{Subst} \left(\int \frac{1}{-\frac{240097759}{4}(247-500\sqrt{2})x} dx \right) \\
 &= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22} \sqrt{\frac{1}{682}(247+500\sqrt{2})} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(247+500\sqrt{2})}}(61+4\sqrt{2})}{\sqrt{3-x+2x^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 1.36352, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}} - 23\sqrt{682(13+i\sqrt{31})}(13\sqrt{31}+69i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 23\sqrt{682(13-i\sqrt{31})}(13\sqrt{31}-69i)}{6901840}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^(3/2)*(2+3*x+5*x^2)),x]

[Out] ((-27280*(-13+6*x))/Sqrt[3-x+2*x^2]-23*Sqrt[682*(13+I*Sqrt[31])]*(69*I+13*Sqrt[31])*ArcTanh[(63+I*Sqrt[31]+(-22-(4*I)*Sqrt[31])*x)/(2*Sqrt[286+(22*I)*Sqrt[31]]*Sqrt[3-x+2*x^2])]-23*Sqrt[682*(13-I*Sqrt[31])]*(-69*I+13*Sqrt[31])*ArcTanh[(63-I*Sqrt[31]+(-22+(4*I)*Sqrt[31])*x)/(2*Sqrt[286-(22*I)*Sqrt[31]]*Sqrt[3-x+2*x^2])])/6901840

Maple [B] time = 0.124, size = 718, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)`

[Out]
$$\frac{1}{465124} \cdot \frac{(8 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 3 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 8 - 3 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} \cdot (2197 \cdot (-775687 + 549362 \cdot 2^{1/2}))^{1/2} \cdot 2^{1/2} \cdot (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{1/2}))^{1/2} \cdot (-23 \cdot (8 + 3 \cdot 2^{1/2})) \cdot (-23 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 24 \cdot 2^{1/2} - 41))^{1/2} \cdot (6485 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 10368 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (2^{1/2} - 1 + x)^4 / (2^{1/2} + 1 - x)^4 + 82 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{1/2}) \cdot (2^{1/2} - 1 + x) / (2^{1/2} + 1 - x) + 3070 \cdot (-775687 + 549362 \cdot 2^{1/2})^{1/2} \cdot (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot \arctan(1/11692487 \cdot (-775687 + 549362 \cdot 2^{1/2}))^{1/2} \cdot (-23 \cdot (8 + 3 \cdot 2^{1/2})) \cdot (-23 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 24 \cdot 2^{1/2} - 41))^{1/2} \cdot (6485 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 10368 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 22379 \cdot 2^{1/2} + 32016) / (23 \cdot (2^{1/2} - 1 + x)^4 / (2^{1/2} + 1 - x)^4 + 82 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 23) \cdot (8 + 3 \cdot 2^{1/2}) \cdot (2^{1/2} - 1 + x) / (2^{1/2} + 1 - x) + 1712502 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 3 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 8 - 3 \cdot 2^{1/2}))^{1/2} / (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} - 6617446 \cdot \operatorname{arctanh}(31/2 \cdot (8 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 3 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 8 - 3 \cdot 2^{1/2}))^{1/2} / (-8866 + 6820 \cdot 2^{1/2})^{1/2} \cdot 2^{1/2} / ((8 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 + 3 \cdot (2^{1/2} - 1 + x)^2 / (2^{1/2} + 1 - x)^2 \cdot 2^{1/2} + 8 - 3 \cdot 2^{1/2}))^{1/2} / (1 + (2^{1/2} - 1 + x) / (2^{1/2} + 1 - x))^2)^{1/2} / (1 + (2^{1/2} - 1 + x) / (2^{1/2} + 1 - x)) / (8 + 3 \cdot 2^{1/2}) / (-8866 + 6820 \cdot 2^{1/2})^{1/2} + 1/22 / (2 \cdot x^2 - x + 3)^{1/2} - 3/506 \cdot (-1 + 4 \cdot x) / (2 \cdot x^2 - x + 3)^{1/2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)`

Fricas [B] time = 4.79084, size = 7618, normalized size = 43.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")

[Out]
$$-1/50921775520*(339388*\sqrt{341}*50^{(1/4)}*\sqrt{10}*\sqrt{2}*(2*x^2 - x + 3)*\sqrt{247*\sqrt{2} + 1000}*\arctan(1/328782125*(14260*\sqrt{341})*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22*50^{(3/4)}*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 46944*x^2 - \sqrt{2}*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^{(1/4)}*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - \sqrt{2}*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888))*\sqrt{247*\sqrt{2} + 1000} + 933317000*\sqrt{31}*\sqrt{2}*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - \sqrt{2}*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*\sqrt{310/119}*(\sqrt{341}*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22*50^{(3/4)}*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - \sqrt{2}*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^{(1/4)}*(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*\sqrt{2}*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x) - 46199808*x))*\sqrt{247*\sqrt{2} + 1000} + 65450*\sqrt{31}*\sqrt{2}*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - \sqrt{2}*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 2975*\sqrt{31}*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*\sqrt{2}*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\sqrt{-(\sqrt{341})*50^{(1/4)}*\sqrt{31}*\sqrt{10}*\sqrt{2*x^2 - x + 3})*(\sqrt{2}*(37*x - 38) + x - 75)*\sqrt{247*\sqrt{2} + 1000} - 903805*x^2 - 811580*\sqrt{2}*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*\sqrt{31}*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\sqrt{2}*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339388*\sqrt{341}*50^{(1/4)}*\sqrt{10}*\sqrt{2}*(2*x^2 - x + 3)*\sqrt{247*\sqrt{2} + 1000}*\arctan(1/328782125*(14260*\sqrt{341})*\sqrt{10}*\sqrt{2*x^2 - x + 3}*(22*50^{(3/4)}*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 46944*x^2 - \sqrt{2}*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^{(1/4)}*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - \sqrt{2}*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 +$$

```

17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888
))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
- 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
- x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/
(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2 - x + 3) - 1000*x + 3000)*s
qrt(247*sqrt(2) + 1000)*log(3100000/119*(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(1
0)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 10
00) + 903805*x^2 + 811580*sqrt(2)*(2*x^2 - x + 3) - 2785195*x + 3689000)/x^
2) + 23*sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2
- x + 3) - 1000*x + 3000)*sqrt(247*sqrt(2) + 1000)*log(-3100000/119*(sqrt(
341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) +
x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x +
3) + 2785195*x - 3689000)/x^2) + 201271840*sqrt(2*x^2 - x + 3)*(6*x - 13))
/(2*x^2 - x + 3)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.91 \quad \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=211

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682}(129694447 + 103775000\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(129694447 + 103775000\sqrt{2})}}}{\sqrt{\frac{11}{31(129694447 + 103775000\sqrt{2})}}}\right)}{30008}$$

[Out] $-(6315 - 2306*x)/(345092*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(682*\text{Sqrt}[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (\text{Sqrt}[(129694447 + 103775000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(129694447 + 103775000*\text{Sqrt}[2]))])*(12611 + 16454*\text{Sqrt}[2] + (45519 + 29065*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])]/30008 - (\text{Sqrt}[(-12969447 + 103775000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(-129694447 + 103775000*\text{Sqrt}[2]))])*(12611 - 16454*\text{Sqrt}[2] + (45519 - 29065*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])]/30008$

Rubi [A] time = 0.473226, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{6315 - 2306x}{345092\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)} + \frac{\sqrt{\frac{1}{682}(129694447 + 103775000\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{\frac{11}{31(129694447 + 103775000\sqrt{2})}}}{\sqrt{\frac{11}{31(129694447 + 103775000\sqrt{2})}}}\right)}{30008}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2), x]

[Out] $-(6315 - 2306*x)/(345092*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(682*\text{Sqrt}[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (\text{Sqrt}[(129694447 + 103775000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(129694447 + 103775000*\text{Sqrt}[2]))])*(12611 + 16454*\text{Sqrt}[2] + (45519 + 29065*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])]/30008 - (\text{Sqrt}[(-12969447 + 103775000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(-129694447 + 103775000*\text{Sqrt}[2]))])*(12611 - 16454*\text{Sqrt}[2] + (45519 - 29065*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])]/30008$

Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1060

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a

```

```
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx &= \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{(3-x+2x^2)^{3/2} (2+3x+5x^2)} dx}{7502} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{\int \frac{-\frac{3002857}{2}+\frac{694358x}{4}}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{20878066} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} + \frac{\int \frac{\frac{30613}{4}(4653+2158x)}{\sqrt{3-x+2x^2} (2+3x+5x^2)} dx}{459} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} - \frac{(253(207550000 + \int \frac{1}{\sqrt{3-x+2x^2}} dx))}{\sqrt{\frac{1}{682}} (12969444)} \\
&= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2} (2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}} (12969444)}{\sqrt{\frac{1}{682}} (12969444)}
\end{aligned}$$

Mathematica [C] time = 1.48804, size = 740, normalized size = 3.51

$$100 \left(\frac{682((22-4i\sqrt{31})x+i\sqrt{31}+52)}{(\sqrt{31}+13i)(10ix+\sqrt{31}+3i)\sqrt{2x^2-x+3}} + \frac{682((22+4i\sqrt{31})x-i\sqrt{31}+52)}{(\sqrt{31}-13i)(-10ix+\sqrt{31}-3i)\sqrt{2x^2-x+3}} + \frac{22(2(11\sqrt{31}-62i)x+52\sqrt{31}+31i)}{(\sqrt{31}+13i)\sqrt{2x^2-x+3}} + \frac{22(2(11\sqrt{31}+62i)x+52\sqrt{31}-31i)}{(\sqrt{31}-13i)\sqrt{2x^2-x+3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^(3/2)*(2+3*x+5*x^2)^2),x]

[Out] (100*((682*(52 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x))/((13*I + Sqrt[31])* (3*I + Sqrt[31] + (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (682*(52 - I*Sqrt[31] + (22 + (4*I)*Sqrt[31])*x))/((-13*I + Sqrt[31])*(-3*I + Sqrt[31] - (10*I)*x)* Sqrt[3 - x + 2*x^2]) + (22*(31*I + 52*Sqrt[31] + 2*(-62*I + 11*Sqrt[31])*x))/((13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + (22*(-31*I + 52*Sqrt[31] + 2*(6 2*I + 11*Sqrt[31])*x))/((-13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + ((575*I)* Sqrt[682*(13 + I*Sqrt[31])]*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x]/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]))/(-13*I + Sqrt[3 1])^2 + (155*(44*(16353 + (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[

$$286 + (22*I)*\text{Sqrt}[31]]*(-29 + (17*I)*\text{Sqrt}[31] + 10*(11 + (2*I)*\text{Sqrt}[31])*x) \\ * \text{ArcTanh}[(63 + I*\text{Sqrt}[31] + (-22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 + (22*I)* \\ \text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2]])]/(22*(-13*I + \text{Sqrt}[31])^3*(-3*I + \text{Sqrt}[31] \\ - (10*I)*x)) + (155*(44*(16353 - (581*I)*\text{Sqrt}[31))*\text{Sqrt}[3 - x + 2*x^2] + 3 \\ 45*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*(29 + (17*I)*\text{Sqrt}[31] + (-110 + (20*I)*\text{Sqrt}[\\ 31])*x)*\text{ArcTanh}[(-63 + I*\text{Sqrt}[31] + (22 - (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - \\ (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - x + 2*x^2]])]/(22*(13*I + \text{Sqrt}[31])^3*(3*I + \text{Sqr} \\ t[31] + (10*I)*x)) - ((575*I)*\text{Sqrt}[682*(13 - I*\text{Sqrt}[31])])*\text{ArcTanh}[(63 - I*S \\ \text{qrt}[31] + (-22 + (4*I)*\text{Sqrt}[31])*x)/(2*\text{Sqrt}[286 - (22*I)*\text{Sqrt}[31]]*\text{Sqrt}[3 - \\ x + 2*x^2]])/(13*I + \text{Sqrt}[31])^2))/2674463$$

Maple [B] time = 0.179, size = 5942, normalized size = 28.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)

Fricas [B] time = 5.23242, size = 9287, normalized size = 44.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 5 - 311740976x^4 + 161753488x^3 - 89046144x^2 - \sqrt{2}*(18515146x^7 - \\
& 65709803x^6 + 140687186x^5 - 209710441x^4 + 101256360x^3 - 39198600x^2 \\
& - 126316800x + 76909824) - 153819648x + 126316800) + 643405*3446160200^{(} \\
& 1/4)*\sqrt{341}*(1637219x^7 - 25548801x^6 + 138274456x^5 - 324967420x^4 \\
& + 425065248x^3 - 297030816x^2 - \sqrt{2}*(1361849x^7 - 20608224x^6 + 106 \\
& 575580x^5 - 236322704x^4 + 301502880x^3 - 169632000x^2 - 225358848x + \\
& 143534592) - 287069184x + 225358848))*\sqrt{2x^2 - x + 3}*\sqrt{129694447*s} \\
& \text{qrt}(2) + 207550000) - 6920408156831705561333000*\sqrt{31}*\sqrt{2}*(28180x^8 \\
& - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496 \\
& *x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x \\
& ^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2 \\
& *\sqrt{41510/397951}*(\sqrt{20755}*(11*3446160200^{(3/4)*\sqrt{341}*(66710248*x} \\
& ^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028 \\
& 8*x^2 - \sqrt{2}*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^ \\
& 4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446 \\
& 160200^{(1/4)*\sqrt{341}*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358 \\
& 592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*\sqrt{2}*(2586013*x^7 - 33428948 \\
& *x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737 \\
& 05472*x) - 144903168*x))*\sqrt{2x^2 - x + 3}*\sqrt{129694447*\sqrt{2} + 20755} \\
& 0000) - 116912097033204550*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x^7 + 1578 \\
& 888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15 \\
& 550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 \\
& + 1209600x^2 - 1036800x) + 3276288x) - 5314186228782025*\sqrt{31}*(254591 \\
& *x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328 \\
& *x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + \\
& 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224*x))*\sqrt{(3446160200^{(} \\
& 1/4)*\sqrt{20755}*\sqrt{341}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(6137*x + \\
& 12812) - 18949*x + 6675)*\sqrt{129694447*\sqrt{2} + 207550000) + 38893032433} \\
& 2445*x^2 + 349243556543420*\sqrt{2}*(2x^2 - x + 3) - 1198540387228555*x + 1 \\
& 587470711561000)/x^2) - 78641001782178472287875*\sqrt{31}*(2828123x^8 - 969 \\
& 6916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 3 \\
& 7981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15 \\
& 569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064*x - 94887936))/(\\
& 2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 4424 \\
& 9088x^3 - 34615296x^2 - 24772608*x + 18579456)) + 23*3446160200^{(1/4)*sqr} \\
& \text{t}(20755)*\sqrt{341}*\sqrt{31}*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^ \\
& 2 - 129694447*\sqrt{2}*(10x^4 + x^3 + 16x^2 + 7x + 6) + 1452850000*x + 12 \\
& 45300000)*\sqrt{129694447*\sqrt{2} + 207550000)*\log(1037750000000/397951*(344} \\
& 6160200^{(1/4)*\sqrt{20755}*\sqrt{341}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(\\
& 6137*x + 12812) - 18949*x + 6675)*\sqrt{129694447*\sqrt{2} + 207550000) + 388} \\
& 930324332445*x^2 + 349243556543420*\sqrt{2}*(2x^2 - x + 3) - 11985403872285} \\
& 55*x + 1587470711561000)/x^2) - 23*3446160200^{(1/4)*\sqrt{20755}*\sqrt{341}*s} \\
& \text{qrt}(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*\sqrt{2} \\
&)*(10x^4 + x^3 + 16x^2 + 7x + 6) + 1452850000*x + 1245300000)*\sqrt{12969} \\
& 4447*\sqrt{2} + 207550000)*\log(-1037750000000/397951*(3446160200^{(1/4)*\sqrt{2}*(}
\end{aligned}$$

```
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.92 \quad \int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3} dx$$

Optimal. Leaf size=246

$$-\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}}(13874275807943 + 9819738650000\sqrt{2})}{682}$$

[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/81861824

Rubi [A] time = 0.525246, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{4353943 - 6508666x}{941410976\sqrt{2x^2 - x + 3}} + \frac{5(17315x + 7318)}{1860496\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)} + \frac{65x + 4}{1364\sqrt{2x^2 - x + 3}(5x^2 + 3x + 2)^2} + \frac{3\sqrt{\frac{1}{682}}(13874275807943 + 9819738650000\sqrt{2})}{682}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3), x]

[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2])])]*(5538393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/81861824

393 - 4123702*sqrt[2] + (13785797 - 9662095*sqrt[2])*x)/sqrt[3 - x + 2*x^2
]]/81861824

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(!IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1060

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(!IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^3} dx &= \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2} (2+3x+5x^2)} - \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5}{1860496\sqrt{3-x+2x^2}} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5}{1860496\sqrt{3-x+2x^2}} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5}{1860496\sqrt{3-x+2x^2}} \\
&= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2} (2+3x+5x^2)^2} + \frac{5}{1860496\sqrt{3-x+2x^2}}
\end{aligned}$$

Mathematica [C] time = 2.24957, size = 231, normalized size = 0.94

$$\frac{27280(162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044)}{\sqrt{2x^2-x+3}(5x^2+3x+2)^2} + 69\sqrt{286-22i\sqrt{31}}(13785797\sqrt{31}+14026539i)$$

2568169142

Antiderivative was successfully verified.

[In] Integrate[1/((3-x+2*x^2)^(3/2)*(2+3*x+5*x^2)^3),x]

[Out] ((27280*(22374044+161806828*x+175833195*x^2+277167774*x^3+86411405*x^4+162716650*x^5))/(Sqrt[3-x+2*x^2]*(2+3*x+5*x^2)^2)+69*Sqrt[286-(22*I)*Sqrt[31]]*(14026539*I+13785797*Sqrt[31])*ArcTan[(63*I+Sqrt[31]-2*(11*I+2*Sqrt[31])*x)/(2*Sqrt[286-(22*I)*Sqrt[31]]*Sqrt[3-x+2*x^2])]- (69*I)*Sqrt[286+(22*I)*Sqrt[31]]*(-14026539*I+13785797*Sqrt[31]))

31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]])]/25681691425280

Maple [B] time = 0.266, size = 18981, normalized size = 77.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)

Fricas [B] time = 5.4732, size = 11105, normalized size = 45.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/33652296632397026886019646994897920*(920746859815884*1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arctan(1/2252270155289097943751876925347228391692375*(2800589462980*sqrt(1963947

$$\begin{aligned}
& 730) * (22 * 1928545343086076450^{(3/4)} * \text{sqrt}(341) * (7361410004 * x^7 - 28555361914 * \\
& x^6 + 59872788262 * x^5 - 96593638888 * x^4 + 48573560944 * x^3 - 23355012672 * x^2 \\
& - \text{sqrt}(2) * (5311119598 * x^7 - 20292577289 * x^6 + 42695479118 * x^5 - 6800681868 \\
& 3 * x^4 + 33985514680 * x^3 - 15860251800 * x^2 - 37489478400 * x + 26167456512) - \\
& 52334913024 * x + 37489478400) + 30441189815 * 1928545343086076450^{(1/4)} * \text{sqrt}(3 \\
& 41) * (560592897 * x^7 - 8616399363 * x^6 + 45618625128 * x^5 - 104316505460 * x^4 + \\
& 134890825824 * x^3 - 85859939808 * x^2 - \text{sqrt}(2) * (402019087 * x^7 - 6162703212 * x^ \\
& 6 + 32499503540 * x^5 - 73942829952 * x^4 + 95407993440 * x^3 - 59600016000 * x^2 - \\
& 68177562624 * x + 47773380096) - 95546760192 * x + 68177562624)) * \text{sqrt}(2 * x^2 - \\
& x + 3) * \text{sqrt}(13874275807943 * \text{sqrt}(2) + 19639477300000) + 63935410859819554532 \\
& 31134497759874144159000 * \text{sqrt}(31) * \text{sqrt}(2) * (28180 * x^8 - 254666 * x^7 + 704270 * x \\
& ^6 - 1385256 * x^5 + 1549144 * x^4 - 642048 * x^3 - 98496 * x^2 - \text{sqrt}(2) * (8746 * x^8 \\
& - 102335 * x^7 + 396104 * x^6 - 783113 * x^5 + 1320710 * x^4 - 752088 * x^3 + 396144 \\
& * x^2 + 546048 * x - 539136) + 1154304 * x - 456192) - 2 * \text{sqrt}(1963947730/3471424 \\
& 919) * (\text{sqrt}(1963947730) * (22 * 1928545343086076450^{(3/4)} * \text{sqrt}(341) * (26184810824 \\
& * x^7 - 37618468196 * x^6 + 121297463436 * x^5 - 48741866816 * x^4 + 58784153184 * x \\
& ^3 + 51583129344 * x^2 - \text{sqrt}(2) * (19194187986 * x^7 - 27528525721 * x^6 + 8845761 \\
& 3411 * x^5 - 33685377680 * x^4 + 38926767960 * x^3 + 41764674816 * x^2 - 4176467481 \\
& 6 * x) - 51583129344 * x) + 30441189815 * 1928545343086076450^{(1/4)} * \text{sqrt}(341) * (19 \\
& 98926311 * x^7 - 25858659004 * x^6 + 99738083860 * x^5 - 129415692096 * x^4 + 16744 \\
& 6420704 * x^3 + 96037622784 * x^2 - 22 * \text{sqrt}(2) * (65886479 * x^7 - 852213084 * x^6 + \\
& 3285070260 * x^5 - 4244909760 * x^4 + 5424792480 * x^3 + 3393259776 * x^2 - 3393259 \\
& 776 * x) - 96037622784 * x)) * \text{sqrt}(2 * x^2 - x + 3) * \text{sqrt}(13874275807943 * \text{sqrt}(2) + \\
& 19639477300000) + 2282926923240949861309948624550 * \text{sqrt}(31) * \text{sqrt}(2) * (123408 * \\
& x^8 - 914152 * x^7 + 1578888 * x^6 - 3293072 * x^5 + 396480 * x^4 + 798336 * x^3 - 38 \\
& 22336 * x^2 - \text{sqrt}(2) * (15550 * x^8 - 118051 * x^7 + 244047 * x^6 - 707374 * x^5 + 105 \\
& 3960 * x^4 - 1667952 * x^3 + 1209600 * x^2 - 1036800 * x) + 3276288 * x) + 1037694056 \\
& 01861357332270392025 * \text{sqrt}(31) * (254591 * x^8 - 4815126 * x^7 + 32303580 * x^6 - 90 \\
& 866808 * x^5 + 108781920 * x^4 - 74219328 * x^3 - 168956928 * x^2 - 15488 * \text{sqrt}(2) * (\\
& 4 * x^8 - 76 * x^7 + 517 * x^6 - 1536 * x^5 + 2385 * x^4 - 3618 * x^3 + 2268 * x^2 - 1944 \\
& * x) + 144820224 * x)) * \text{sqrt}(-(1928545343086076450^{(1/4)} * \text{sqrt}(1963947730) * \text{sqrt}(\\
& 341) * \text{sqrt}(31) * \text{sqrt}(2 * x^2 - x + 3) * (\text{sqrt}(2) * (2995431 * x + 1523456) - 4518887 * \\
& x - 1471975) * \text{sqrt}(13874275807943 * \text{sqrt}(2) + 19639477300000) - 16051926912456 \\
& 8199977215 * x^2 - 144139751866959199979540 * \text{sqrt}(2) * (2 * x^2 - x + 3) + 4946614 \\
& 21179791799929785 * x - 655180690304359999907000) / x^2) + 72653875977067675604 \\
& 899255656362206183625 * \text{sqrt}(31) * (2828123 * x^8 - 9696916 * x^7 + 53385560 * x^6 - \\
& 142835344 * x^5 + 254146592 * x^4 - 249300096 * x^3 + 37981440 * x^2 - 7744 * \text{sqrt}(2) \\
& * (1348 * x^8 - 2692 * x^7 + 9789 * x^6 - 10070 * x^5 + 15569 * x^4 - 5568 * x^3 + 1080 * \\
& x^2 + 4320 * x - 5184) + 223064064 * x - 94887936) / (2585191 * x^8 - 4661200 * x^7 \\
& + 14191920 * x^6 + 490880 * x^5 - 13562944 * x^4 + 44249088 * x^3 - 34615296 * x^2 - \\
& 24772608 * x + 18579456) + 920746859815884 * 1928545343086076450^{(1/4)} * \text{sqrt}(19 \\
& 63947730) * \text{sqrt}(341) * \text{sqrt}(2) * (50 * x^6 + 35 * x^5 + 103 * x^4 + 85 * x^3 + 83 * x^2 + \\
& 32 * x + 12) * \text{sqrt}(13874275807943 * \text{sqrt}(2) + 19639477300000) * \arctan(1/225227015 \\
& 5289097943751876925347228391692375 * (2800589462980 * \text{sqrt}(1963947730) * (22 * 1928 \\
& 545343086076450^{(3/4)} * \text{sqrt}(341) * (7361410004 * x^7 - 28555361914 * x^6 + 5987278
\end{aligned}$$

$$\begin{aligned}
& 8262x^5 - 96593638888x^4 + 48573560944x^3 - 23355012672x^2 - \sqrt{2}*(5 \\
& 311119598x^7 - 20292577289x^6 + 42695479118x^5 - 68006818683x^4 + 33985 \\
& 514680x^3 - 15860251800x^2 - 37489478400x + 26167456512) - 52334913024x \\
& + 37489478400) + 30441189815*1928545343086076450^{(1/4)}*\sqrt{341}*(56059289 \\
& 7x^7 - 8616399363x^6 + 45618625128x^5 - 104316505460x^4 + 134890825824x \\
& x^3 - 85859939808x^2 - \sqrt{2}*(402019087x^7 - 6162703212x^6 + 324995035 \\
& 40x^5 - 73942829952x^4 + 95407993440x^3 - 59600016000x^2 - 68177562624x \\
& x + 47773380096) - 95546760192x + 68177562624))*\sqrt{2x^2 - x + 3}*\sqrt{(1 \\
& 3874275807943*\sqrt{2} + 19639477300000) - 639354108598195545323113449775987 \\
& 4144159000*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x \\
& x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 \\
& + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048 \\
& *x - 539136) + 1154304x - 456192) - 2*\sqrt{1963947730/3471424919}*(\sqrt{(19 \\
& 63947730)*(22*1928545343086076450^{(3/4)}*\sqrt{341}*(26184810824x^7 - 376184 \\
& 68196x^6 + 121297463436x^5 - 48741866816x^4 + 58784153184x^3 + 51583129 \\
& 344x^2 - \sqrt{2}*(19194187986x^7 - 27528525721x^6 + 88457613411x^5 - 33 \\
& 685377680x^4 + 38926767960x^3 + 41764674816x^2 - 41764674816x) - 515831 \\
& 29344x) + 30441189815*1928545343086076450^{(1/4)}*\sqrt{341}*(1998926311x^7 \\
& - 25858659004x^6 + 99738083860x^5 - 129415692096x^4 + 167446420704x^3 + \\
& 96037622784x^2 - 22*\sqrt{2}*(65886479x^7 - 852213084x^6 + 3285070260x^5 \\
& 5 - 4244909760x^4 + 5424792480x^3 + 3393259776x^2 - 3393259776x) - 9603 \\
& 7622784x))*\sqrt{2x^2 - x + 3}*\sqrt{(13874275807943*\sqrt{2} + 1963947730000 \\
& 0) - 2282926923240949861309948624550*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x \\
& x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - s \\
& \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 16 \\
& 67952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 10376940560186135733227 \\
& 0392025*\sqrt{31}*(254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + \\
& 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 \\
& 7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 1448202 \\
& 24x))*\sqrt{((1928545343086076450^{(1/4)}*\sqrt{1963947730}*\sqrt{341}*\sqrt{31})* \\
& \sqrt{2x^2 - x + 3}*(\sqrt{2}*(2995431x + 1523456) - 4518887x - 1471975)*s \\
& \sqrt{13874275807943*\sqrt{2} + 19639477300000) + 160519269124568199977215x^2 \\
& + 144139751866959199979540*\sqrt{2}*(2x^2 - x + 3) - 494661421179791799929 \\
& 785x + 655180690304359999907000)/x^2) - 7265387597706767560489925565636220 \\
& 6183625*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 \\
& + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2 \\
& 692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - \\
& 5184) + 223064064x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 \\
& + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 1 \\
& 8579456)) + 69*1928545343086076450^{(1/4)}*\sqrt{1963947730}*\sqrt{341}*\sqrt{31} \\
&)*(981973865000000x^6 + 687381705500000x^5 + 2022866161900000x^4 + 16693 \\
& 55570500000x^3 + 1630076615900000x^2 - 13874275807943*\sqrt{2}*(50x^6 + 3 \\
& 5x^5 + 103x^4 + 85x^3 + 83x^2 + 32x + 12) + 628463273600000x + 235673 \\
& 727600000)*\sqrt{(13874275807943*\sqrt{2} + 19639477300000)*\log(17675529570000 \\
& 00000/3471424919*(1928545343086076450^{(1/4)}*\sqrt{1963947730}*\sqrt{341}*\sqrt{
\end{aligned}$$

```
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 14719
75)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 16051926912456819997721
5*x^2 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 4946614211797917
99929785*x + 655180690304359999907000)/x^2) - 69*1928545343086076450^(1/4)*
sqrt(1963947730)*sqrt(341)*sqrt(31)*(981973865000000*x^6 + 687381705500000*
x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 -
13874275807943*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12) + 628463273600000*x + 235673727600000)*sqrt(13874275807943*sqrt(2) +
19639477300000)*log(-1767552957000000000/3471424919*(1928545343086076450^(1
/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(29954
31*x + 1523456) - 4518887*x - 1471975)*sqrt(13874275807943*sqrt(2) + 196394
77300000) - 160519269124568199977215*x^2 - 144139751866959199979540*sqrt(2)
*(2*x^2 - x + 3) + 494661421179791799929785*x - 655180690304359999907000)/x
^2) + 35746658463005881594925920*(162716650*x^5 + 86411405*x^4 + 277167774*
x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x^2 - x + 3))/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.93 \quad \int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=147

$$\frac{625}{32}\sqrt{2x^2-x+3}x^3 + \frac{38375}{384}\sqrt{2x^2-x+3}x^2 + \frac{526075\sqrt{2x^2-x+3}x}{3072} - \frac{1308645\sqrt{2x^2-x+3}}{4096} + \frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}$$

[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (625*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2])

Rubi [A] time = 0.167403, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32}\sqrt{2x^2-x+3}x^3 + \frac{38375}{384}\sqrt{2x^2-x+3}x^2 + \frac{526075\sqrt{2x^2-x+3}x}{3072} - \frac{1308645\sqrt{2x^2-x+3}}{4096} + \frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2), x]

[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (625*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[

$(a + b*x + c*x^2)^{(p + 1)}$ *ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 3x + 5x^2)^4}{(3 - x + 2x^2)^{5/2}} dx &= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} + \frac{250125x^5}{8} + \frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16} + \frac{991875}{16}}{(3 - x + 2x^2)^{3/2}} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16} + \frac{991875}{16}}{\sqrt{3-x+2x^2}}}{1587} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{625}{32}x^3\sqrt{3 - x + 2x^2} + \frac{\int \frac{-\frac{141812733}{32} - \frac{1880595x}{2} + \frac{2}{2}}{\sqrt{3-x+2x^2}}}{317} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{38375}{384}x^2\sqrt{3 - x + 2x^2} + \frac{625}{32}x^3\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} + \frac{38375}{384}x^2\sqrt{3 - x + 2x^2} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072} \\
&= -\frac{14641(101 + 79x)}{4416(3 - x + 2x^2)^{3/2}} + \frac{1331(7409 + 116368x)}{101568\sqrt{3 - x + 2x^2}} - \frac{1308645\sqrt{3 - x + 2x^2}}{4096} + \frac{526075x\sqrt{3 - x + 2x^2}}{3072}
\end{aligned}$$

Mathematica [A] time = 0.517884, size = 75, normalized size = 0.51

$$\frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x}{6500352(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2), x]

[Out] $(-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x + 2*x^2)^{(3/2)}) - (16955197*\text{ArcSinh}[(-1 + 4*x)/\text{Sqrt}[23]])/(8192*\text{Sqrt}[2])$

Maple [A] time = 0.071, size = 214, normalized size = 1.5

$$\frac{138025 x^5}{256} (2x^2 - x + 3)^{-\frac{3}{2}} + \frac{16955197 x^3}{12288} (2x^2 - x + 3)^{-\frac{3}{2}} - \frac{67488035 x^2}{16384} (2x^2 - x + 3)^{-\frac{3}{2}} - \frac{16955197 \sqrt{2}}{16384} \text{Arcsinh} \left(\frac{4x - 1}{\sqrt{23}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)`

[Out] $138025/256*x^5/(2*x^2-x+3)^{(3/2)}+16955197/12288*x^3/(2*x^2-x+3)^{(3/2)}-67488035/16384*x^2/(2*x^2-x+3)^{(3/2)}-16955197/16384*2^{(1/2)}*\text{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+992926033/13000704*(-1+4*x)/(2*x^2-x+3)^{(1/2)}+5141612725/36175872*(-1+4*x)/(2*x^2-x+3)^{(3/2)}+55167267/131072*x/(2*x^2-x+3)^{(3/2)}-799745/1024*x^4/(2*x^2-x+3)^{(3/2)}+30875/96*x^6/(2*x^2-x+3)^{(3/2)}+16955197/8192*x/(2*x^2-x+3)^{(1/2)}+16955197/32768/(2*x^2-x+3)^{(1/2)}-2149616639/524288/(2*x^2-x+3)^{(3/2)}+625/8*x^7/(2*x^2-x+3)^{(3/2)}$

Maxima [B] time = 1.91995, size = 342, normalized size = 2.33

$$\frac{625 x^7}{8 (2x^2 - x + 3)^{\frac{3}{2}}} + \frac{30875 x^6}{96 (2x^2 - x + 3)^{\frac{3}{2}}} + \frac{138025 x^5}{256 (2x^2 - x + 3)^{\frac{3}{2}}} - \frac{799745 x^4}{1024 (2x^2 - x + 3)^{\frac{3}{2}}} - \frac{16955197}{13000704} x \left(\frac{284 x}{\sqrt{2x^2 - x + 3}} - \frac{3174 x^2}{(2x^2 - x + 3)^{(3/2)}} - \frac{71}{\sqrt{2x^2 - x + 3}} + \frac{805 x}{(2x^2 - x + 3)^{(3/2)}} - \frac{3243}{(2x^2 - x + 3)^{(3/2)}} \right) - \frac{16955197}{16384} \sqrt{2} \text{arcsinh} \left(\frac{1}{23} \sqrt{23} (4x - 1) \right) + \frac{1203818987}{6500352} \sqrt{2x^2 - x + 3} + \frac{3536205583}{3250176} x / \sqrt{2x^2 - x + 3} - \frac{2638851}{512} x^2 / (2x^2 - x + 3)^{(3/2)} + \frac{257773037}{1083392} / \sqrt{2x^2 - x + 3} + \frac{29484067}{23552} x / (2x^2 - x + 3)^{(3/2)} - \frac{374445479}{70656} / (2x^2 - x + 3)^{(3/2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")`

[Out] $625/8*x^7/(2*x^2 - x + 3)^{(3/2)} + 30875/96*x^6/(2*x^2 - x + 3)^{(3/2)} + 138025/256*x^5/(2*x^2 - x + 3)^{(3/2)} - 799745/1024*x^4/(2*x^2 - x + 3)^{(3/2)} - 16955197/13000704*x*(284*x/\text{sqrt}(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^{(3/2)} - 71/\text{sqrt}(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^{(3/2)} - 3243/(2*x^2 - x + 3)^{(3/2)}) - 16955197/16384*\text{sqrt}(2)*\text{arcsinh}(1/23*\text{sqrt}(23)*(4*x - 1)) + 1203818987/6500352*\text{sqrt}(2*x^2 - x + 3) + 3536205583/3250176*x/\text{sqrt}(2*x^2 - x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^{(3/2)} + 257773037/1083392/\text{sqrt}(2*x^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^{(3/2)} - 374445479/70656/(2*x^2 - x + 3)^{(3/2)}$

$$(2 - x + 3)^{(3/2)}$$

Fricas [A] time = 1.37283, size = 441, normalized size = 3.

$$\frac{26907897639 \sqrt{2} (4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x - 18974698519)\sqrt{2x^2 - x + 3}}{52002816}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*x^7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^4}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)

Giac [A] time = 1.15042, size = 109, normalized size = 0.74

$$\frac{16955197}{16384} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{((4(2645(20(40(60x + 247)x + 16563)x - 479847)x + 9962225$$

6500352(2x^2 - x + 3))

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x + 9962225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x + 3)^(3/2)
```

$$3.94 \quad \int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=105

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 - x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])

Rubi [A] time = 0.105366, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679 - 6744x)}{8464\sqrt{2x^2 - x + 3}} + \frac{125}{16}x\sqrt{2x^2 - x + 3} + \frac{3175}{64}\sqrt{2x^2 - x + 3} - \frac{1331(17 - 45x)}{1104(2x^2 - x + 3)^{3/2}} - \frac{7495 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2), x]

[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 - x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} dx &= -\frac{1331(17-45x)}{1104(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{91275}{64} - \frac{57201x}{32} + \frac{66585x^2}{16} + \frac{39675x^3}{8} + \frac{8625x^4}{4}}{(3-x+2x^2)^{3/2}} dx \\
&= -\frac{1331(17-45x)}{1104(3-x+2x^2)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{4 \int \frac{\frac{1452105}{64} + \frac{277725x}{8} + \frac{198375x^2}{16}}{\sqrt{3-x+2x^2}} dx}{1587} \\
&= -\frac{1331(17-45x)}{1104(3-x+2x^2)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{125}{16} x \sqrt{3-x+2x^2} + \frac{\int \frac{\frac{214245}{4} + \frac{5038725x}{32}}{\sqrt{3-x+2x^2}} dx}{1587} \\
&= -\frac{1331(17-45x)}{1104(3-x+2x^2)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64} \sqrt{3-x+2x^2} + \frac{125}{16} x \sqrt{3-x+2x^2} + \frac{749}{128} \sqrt{3-x+2x^2} \\
&= -\frac{1331(17-45x)}{1104(3-x+2x^2)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64} \sqrt{3-x+2x^2} + \frac{125}{16} x \sqrt{3-x+2x^2} + \frac{749}{128} \sqrt{3-x+2x^2}
\end{aligned}$$

Mathematica [A] time = 0.335113, size = 65, normalized size = 0.62

$$\frac{3174000x^5 + 16980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565}{101568(2x^2 - x + 3)^{3/2}} + \frac{7495 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2), x]

[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) + (7495*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(128*Sqrt[2])

Maple [B] time = 0.057, size = 180, normalized size = 1.7

$$\frac{125x^5}{4} (2x^2 - x + 3)^{-\frac{3}{2}} - \frac{7495x^3}{192} (2x^2 - x + 3)^{-\frac{3}{2}} + \frac{222809x^2}{256} (2x^2 - x + 3)^{-\frac{3}{2}} + \frac{7495\sqrt{2}}{256} \operatorname{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x - \frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x)`

[Out] $125/4*x^5/(2*x^2-x+3)^{(3/2)}-7495/192*x^3/(2*x^2-x+3)^{(3/2)}+222809/256*x^2/(2*x^2-x+3)^{(3/2)}+7495/256*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))-3391139/203136*(-1+4*x)/(2*x^2-x+3)^{(1/2)}-14081711/565248*(-1+4*x)/(2*x^2-x+3)^{(3/2)}-281177/2048*x/(2*x^2-x+3)^{(3/2)}+2675/16*x^4/(2*x^2-x+3)^{(3/2)}-7495/128*x/(2*x^2-x+3)^{(1/2)}-7495/512/(2*x^2-x+3)^{(1/2)}+20961031/24576/(2*x^2-x+3)^{(3/2)}$

Maxima [B] time = 1.50078, size = 296, normalized size = 2.82

$$\frac{125x^5}{4(2x^2-x+3)^{\frac{3}{2}}} + \frac{2675x^4}{16(2x^2-x+3)^{\frac{3}{2}}} + \frac{7495}{203136}x \left(\frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{\frac{3}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")`

[Out] $125/4*x^5/(2*x^2-x+3)^{(3/2)}+2675/16*x^4/(2*x^2-x+3)^{(3/2)}+7495/203136*x*(284*x/\operatorname{sqrt}(2*x^2-x+3)-3174*x^2/(2*x^2-x+3)^{(3/2)}-71/\operatorname{sqrt}(2*x^2-x+3)+805*x/(2*x^2-x+3)^{(3/2)}-3243/(2*x^2-x+3)^{(3/2)})+7495/256*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1))-532145/101568*\operatorname{sqrt}(2*x^2-x+3)-4515389/50784*x/\operatorname{sqrt}(2*x^2-x+3)+7197/8*x^2/(2*x^2-x+3)^{(3/2)}+396211/50784/\operatorname{sqrt}(2*x^2-x+3)-269783/1104*x/(2*x^2-x+3)^{(3/2)}+1002137/1104/(2*x^2-x+3)^{(3/2)}$

Fricas [A] time = 1.37072, size = 370, normalized size = 3.52

$$\frac{11894565\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+8(3174000x^5+16812544(4x^4-4x^3+13x^2-6x+9))}{812544(4x^4-4x^3+13x^2-6x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")`

[Out] $1/812544*(11894565*\operatorname{sqrt}(2)*(4*x^4-4*x^3+13*x^2-6*x+9)*\log(-4*\operatorname{sqrt}(2)*\operatorname{sqrt}(2*x^2-x+3)*(4*x-1)-32*x^2+16*x-25)+8*(3174000*x^5+16812544*(4*x^4-4*x^3+13*x^2-6*x+9)))$

$$\frac{980900x^4 - 29423976x^3 + 101546529x^2 - 62463282x + 89784565}{(4x^4 - 4x^3 + 13x^2 - 6x + 9)} \sqrt{2x^2 - x + 3}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^3}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2), x)

[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)

Giac [A] time = 1.22387, size = 97, normalized size = 0.92

$$-\frac{7495}{256} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{3((4(13225(20x + 107)x - 2451998)x + 33848843)x - 20821094)x}{101568(2x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2), x, algorithm="giac")

[Out] -7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 20821094)*x + 89784565)/(2*x^2 - x + 3)^(3/2)

$$3.95 \quad \int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*Sqrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])

Rubi [A] time = 0.0612398, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}} - \frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2), x]

[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*Sqrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}} dx &= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{131}{16} + \frac{5865x}{8} + \frac{1725x^2}{4}}{(3-x+2x^2)^{3/2}} dx \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{4}{1587} \int \frac{39675}{16\sqrt{3-x+2x^2}} dx \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25}{4} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+4x\right)}{4\sqrt{46}} \\
&= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} - \frac{25 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.240769, size = 55, normalized size = 0.81

$$\frac{25 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}} - \frac{11(2336x^3 + 6183x^2 + 714x + 8623)}{3174(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2), x]

[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) + (25*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])

Maple [B] time = 0.052, size = 146, normalized size = 2.2

$$-\frac{25x^3}{6}(2x^2-x+3)^{-\frac{3}{2}} - \frac{145x^2}{8}(2x^2-x+3)^{-\frac{3}{2}} - \frac{319x}{64}(2x^2-x+3)^{-\frac{3}{2}} - \frac{15775}{768}(2x^2-x+3)^{-\frac{3}{2}} + \frac{-8493+33972}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2), x)

[Out] -25/6*x^3/(2*x^2-x+3)^(3/2)-145/8*x^2/(2*x^2-x+3)^(3/2)-319/64*x/(2*x^2-x+3)^(3/2)-15775/768/(2*x^2-x+3)^(3/2)+8493/5888*(-1+4*x)/(2*x^2-x+3)^(3/2)+2267/2116*(-1+4*x)/(2*x^2-x+3)^(1/2)-25/4*x/(2*x^2-x+3)^(1/2)-25/16/(2*x^2-x+3)^(1/2)+25/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))

Maxima [B] time = 1.77602, size = 250, normalized size = 3.68

$$\frac{25}{6348}x \left(\frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2x^2-x+3)^{\frac{3}{2}}} \right) + \frac{25}{8}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2), x, algorithm="maxima")

[Out] 25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*x^2 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/2) - 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/138/(2*x^2 - x + 3)^(3/2)

Fricas [B] time = 1.37113, size = 302, normalized size = 4.44

$$\frac{39675 \sqrt{2} (4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(-4 \sqrt{2} \sqrt{2x^2 - x + 3} (4x - 1) - 32x^2 + 16x - 25) - 88 (2336x^3 + 6183x^2 + 714x + 8623) \sqrt{2x^2 - x + 3}}{25392 (4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x^2 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(5x^2 + 3x + 2)^2}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)

[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)

Giac [A] time = 1.26898, size = 82, normalized size = 1.21

$$-\frac{25}{8} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2x} - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{11 \left(\left(2336x + 6183\right)x + 714\right)x + 8623}{3174 \left(2x^2 - x + 3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/3174*((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)

$$3.96 \quad \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x + 2*x^2])

Rubi [A] time = 0.0223468, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}} - \frac{11(3x+5)}{69(2x^2-x+3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2), x]

[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x + 2*x^2])

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}} dx &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{213}{4(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} + \frac{71}{46} \int \frac{1}{(3-x+2x^2)^{3/2}} dx \\ &= -\frac{11(5+3x)}{69(3-x+2x^2)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}} \end{aligned}$$

Mathematica [A] time = 0.104221, size = 33, normalized size = 0.7

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)}{1587(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2), x]
```

```
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

Maple [A] time = 0.045, size = 30, normalized size = 0.6

$$\frac{1704x^3 - 1278x^2 + 2010x - 1904}{1587} (2x^2 - x + 3)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2), x)
```

```
[Out] 2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

Maxima [A] time = 1.07712, size = 80, normalized size = 1.7

$$\frac{284x}{529\sqrt{2x^2-x+3}} - \frac{71}{529\sqrt{2x^2-x+3}} - \frac{11x}{23(2x^2-x+3)^{\frac{3}{2}}} - \frac{55}{69(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^2 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)

Fricas [A] time = 1.34092, size = 132, normalized size = 2.81

$$\frac{2(852x^3 - 639x^2 + 1005x - 952)\sqrt{2x^2 - x + 3}}{1587(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)

[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)

Giac [A] time = 1.15732, size = 39, normalized size = 0.83

$$\frac{2(3(71(4x-3)x+335)x-952)}{1587(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] 2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)

$$3.97 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)} dx$$

Optimal. Leaf size=199

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(25000\sqrt{2}-15457)}} ((247 + 345\sqrt{2})\sqrt{2x^2 - x + 3}}{\sqrt{2x^2 - x + 3}} \right)$$

[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x + 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457 + 25000*Sqrt[2]))])*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(15457 + 25000*Sqrt[2]))])*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/484

Rubi [A] time = 0.455598, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603 - 658x}{128018\sqrt{2x^2 - x + 3}} + \frac{13 - 6x}{759(2x^2 - x + 3)^{3/2}} + \frac{1}{484} \sqrt{\frac{1}{682} (25000\sqrt{2} - 15457)} \tan^{-1} \left(\frac{\sqrt{\frac{11}{31(25000\sqrt{2}-15457)}} ((247 + 345\sqrt{2})\sqrt{2x^2 - x + 3}}{\sqrt{2x^2 - x + 3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]

[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x + 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457 + 25000*Sqrt[2]))])*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(15457 + 25000*Sqrt[2]))])*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x)]/Sqrt[3 - x + 2*x^2])/484

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*

```

a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
0]

```

Rule 1060

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rule 1035

```

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) +
(e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d

```


$$- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x, x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0] \&\& NeQ[b*d - a*e, 0] \&\& NegQ[b^2 - 4*a*c]$$

Rule 1029

$$\text{Int}[\frac{(g_{.}) + (h_{.})*(x_{.})}{((a_{.}) + (b_{.})*(x_{.}) + (c_{.})*(x_{.})^2)*Sqrt[(d_{.}) + (e_{.})*(x_{.}) + (f_{.})*(x_{.})^2]}], x_Symbol] \rightarrow \text{Dist}[-2*g*(g*b - 2*a*h), \text{Subst}[\text{Int}[1/\text{Simp}[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, \text{Simp}[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[e^2 - 4*d*f, 0] \&\& NeQ[b*d - a*e, 0] \&\& EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]$$

Rule 206

$$\text{Int}[\frac{(a_{.}) + (b_{.})*(x_{.})^2}{(x_{.})^2}^{-1}, x_Symbol] \rightarrow \text{Simp}[\frac{1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]]}{\text{Rt}[a, 2]*\text{Rt}[-b, 2]}], x] /; FreeQ[{a, b}, x] \&\& NegQ[a/b] \&\& (GtQ[a, 0] || LtQ[b, 0])$$

Rule 204

$$\text{Int}[\frac{(a_{.}) + (b_{.})*(x_{.})^2}{(x_{.})^2}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]}{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]}], x] /; FreeQ[{a, b}, x] \&\& PosQ[a/b] \&\& (LtQ[a, 0] || LtQ[b, 0])$$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx &= \frac{13-6x}{759(3-x+2x^2)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)} dx}{8349} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{-\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{23235267} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}(11-54\sqrt{2})-\frac{2112297}{4}(119-65\sqrt{2})}{\sqrt{3-x+2x^2}(2+3x+5x^2)} dx}{511175874\sqrt{2}} \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32} \left(17457(50000-15457\sqrt{2}) \right) S \\
&= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484} \sqrt{\frac{1}{682}(-15457+25000\sqrt{2})}
\end{aligned}$$

Mathematica [C] time = 0.909804, size = 218, normalized size = 1.1

$$\frac{-3948x^3 + 23592x^2 - 19767x + 39005}{384054(2x^2 - x + 3)^{3/2}} - \frac{\sqrt{\frac{1}{682}(13 + i\sqrt{31})}(119\sqrt{31} + 247i) \tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680} + \frac{\sqrt{\frac{1}{682}(13 - i\sqrt{31})}(-247i + 119\sqrt{31}) \tanh^{-1}\left(\frac{(-63+i\sqrt{31})+(22-4i\sqrt{31})x}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]

[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) - (Sqrt[(13 + I*Sqrt[31])/682]*(247*I + 119*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/9680 + (Sqrt[(13 - I*Sqrt[31])/682]*(-247*I + 119*Sqrt[31])*ArcTanh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/9680

Maple [B] time = 0.121, size = 751, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)`

[Out] $\frac{1}{66}(2x^2-x+3)^{3/2} - \frac{1}{506}(-1+4x)(2x^2-x+3)^{3/2} - \frac{329}{256036}(-1+4x) / (2x^2-x+3)^{1/2} + \frac{1}{10232728}(8(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2^{1/2}+8-3*2^{1/2}))^{1/2} * 2^{1/2} * (10111*(-775687+549362*2^{1/2}))^{1/2} * 2^{1/2} * (-8866+6820*2^{1/2})^{1/2} * \arctan(1/11692487 * (-775687+549362*2^{1/2}))^{1/2} * (-23*(8+3*2^{1/2})) * (-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2} * (6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})) * (2^{1/2}-1+x)/(2^{1/2}+1-x) + 13910 * (-775687+549362*2^{1/2})^{1/2} * (-8866+6820*2^{1/2})^{1/2} * \arctan(1/11692487 * (-775687+549362*2^{1/2}))^{1/2} * (-23*(8+3*2^{1/2})) * (-23*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+24*2^{1/2}-41))^{1/2} * (6485*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2^{1/2}+10368*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+22379*2^{1/2}+32016)/(23*(2^{1/2}-1+x)^4/(2^{1/2}+1-x)^4+82*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+23)*(8+3*2^{1/2})) * (2^{1/2}-1+x)/(2^{1/2}+1-x) - 993674 * \operatorname{arctanh}(31/2*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2*2^{1/2}+8-3*2^{1/2}))^{1/2} / (-8866+6820*2^{1/2})^{1/2} * 2^{1/2} - 42685698 * \operatorname{arctanh}(31/2*(8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2*2^{1/2}+8-3*2^{1/2}))^{1/2} / (-8866+6820*2^{1/2})^{1/2} * 2^{1/2} / ((8*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+3*(2^{1/2}-1+x)^2/(2^{1/2}+1-x)^2+2*2^{1/2}+8-3*2^{1/2})) / (1+(2^{1/2}-1+x)/(2^{1/2}+1-x))^2)^{1/2} / (1+(2^{1/2}-1+x)/(2^{1/2}+1-x)) / (8+3*2^{1/2}) / (-8866+6820*2^{1/2})^{1/2} + 13/484 / (2*x^2-x+3)^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)`

Fricas [B] time = 4.94658, size = 8280, normalized size = 41.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")`

[Out]
$$\frac{1}{370971467791584000} \cdot (1123856268 \sqrt{341}) \cdot 200^{1/4} \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) \sqrt{-772850000 \sqrt{2} + 2500000000} \arctan\left(-\frac{1}{7889389562500} \cdot (71300 \sqrt{341}) \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4}) \cdot (347404x^7 - 907814x^6 + 2112962x^5 - 2166688x^4 + 787344x^3 + 304128x^2 - \sqrt{2} \cdot (35898x^7 - 441939x^6 + 782418x^5 - 2117233x^4 + 1272680x^3 - 1081800x^2 - 518400x + 1043712) - 2087424x + 518400) + 5 \cdot 200^{1/4} \cdot (712757x^7 - 10233303x^6 + 48529768x^5 - 94500260x^4 + 113086944x^3 - 22282848x^2 - \sqrt{2} \cdot (158647x^7 - 2935272x^6 + 19428740x^5 - 55765712x^4 + 78380640x^3 - 84096000x^2 - 37407744x + 53208576) - 106417152x + 37407744)\right) \sqrt{-772850000 \sqrt{2} + 2500000000} + 22395686500000 \sqrt{31} \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - \sqrt{310/5711} \cdot (\sqrt{341}) \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4}) \cdot (1665224x^7 - 2325796x^6 + 7065036x^5 - 196416x^4 - 2176416x^3 + 8895744x^2 + \sqrt{2} \cdot (167914x^7 - 195429x^6 + 331239x^5 + 1685680x^4 - 3693960x^3 + 4195584x^2 - 4195584x) - 8895744x) + 5 \cdot 200^{1/4} \cdot (3246491x^7 - 41888524x^6 + 159670660x^5 - 190080576x^4 + 180496224x^3 + 376648704x^2 - 2 \sqrt{2} \cdot (40239x^7 - 558044x^6 + 2804660x^5 - 9524160x^4 + 34843680x^3 - 74006784x^2 + 74006784x) - 376648704x) \sqrt{-772850000 \sqrt{2} + 2500000000} + 314105000 \sqrt{31} \sqrt{2} \cdot (123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 14277500 \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488 \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \sqrt{-(\sqrt{341}) \cdot 200^{1/4} \sqrt{31} \sqrt{2x^2 - x + 3} \cdot (\sqrt{2} \cdot (281x - 444) + 163x - 725) \sqrt{-772850000 \sqrt{2} + 2500000000} - 4337504500x^2 - 3894902000 \sqrt{2} \cdot (2x^2 - x + 3) + 13366595500x - 17704100000)/x^2} + 254496437500 \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \sqrt{2} \cdot (1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456) + 1123856268 \sqrt{341} \cdot 200^{1/4} \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) \sqrt{-772850000 \sqrt{2} + 2500000000} \arctan\left(-\frac{1}{7889389562500} \cdot (71300 \sqrt{341}) \sqrt{2x^2 - x + 3} \cdot (11 \cdot 200^{3/4}) \cdot (347404x^7 - 907814x^6 + 2112962x^5 - 2166688x^4 + 787344x^3 + 304128x^2 - \sqrt{2} \cdot (35898x^7 - 441939x^6 + 782418x^5 - 2117233x^4 + 1272680x^3 - 1081800x^2 - 518400x + 1043712) - 2087424x + 518400\right)$$

$$\begin{aligned}
& 0) + 5 \cdot 200^{(1/4)} \cdot (712757x^7 - 10233303x^6 + 48529768x^5 - 94500260x^4 + \\
& 113086944x^3 - 22282848x^2 - \sqrt{2} \cdot (158647x^7 - 2935272x^6 + 1942874 \\
& 0x^5 - 55765712x^4 + 78380640x^3 - 84096000x^2 - 37407744x + 53208576) \\
& - 106417152x + 37407744) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 2239568 \\
& 6500000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 \\
& + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2} \cdot (8746x^8 - 102335x^7 + \\
& 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x \\
& - 539136) + 1154304x - 456192) - \sqrt{310/5711} \cdot (\sqrt{341} \cdot \sqrt{2x^2 - x \\
& + 3}) \cdot (11 \cdot 200^{(3/4)} \cdot (1665224x^7 - 2325796x^6 + 7065036x^5 - 196416x^4 - \\
& 2176416x^3 + 8895744x^2 + \sqrt{2} \cdot (167914x^7 - 195429x^6 + 331239x^5 + \\
& 1685680x^4 - 3693960x^3 + 4195584x^2 - 4195584x) - 8895744x) + 5 \cdot 200^{(1/4)} \\
& \cdot (3246491x^7 - 41888524x^6 + 159670660x^5 - 190080576x^4 + 1804962 \\
& 24x^3 + 376648704x^2 - 2 \cdot \sqrt{2} \cdot (40239x^7 - 558044x^6 + 2804660x^5 - \\
& 9524160x^4 + 34843680x^3 - 74006784x^2 + 74006784x) - 376648704x) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 314105000 \cdot \sqrt{31} \cdot \sqrt{2} \cdot (123408x^8 \\
& - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 38223 \\
& 36x^2 - \sqrt{2} \cdot (15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 105396 \\
& 0x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) - 14277500 \cdot \sqrt{31} \cdot (254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 \\
& - 74219328x^3 - 168956928x^2 - 15488 \cdot \sqrt{2} \cdot (4x^8 - 76x^7 + 517x^6 \\
& - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x) \cdot \sqrt{(\sqrt{341} \cdot 200^{(1/4)} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3}) \cdot (\sqrt{2} \cdot (281x - 444) + \\
& 163x - 725) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} + 4337504500x^2 + 38949 \\
& 02000 \cdot \sqrt{2} \cdot (2x^2 - x + 3) - 13366595500x + 17704100000)/x^2} - 2544964 \\
& 37500 \cdot \sqrt{31} \cdot (2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + \\
& 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744 \cdot \sqrt{2} \cdot (1348x^8 - 269 \\
& 2x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5 \\
& 184) + 223064064x - 94887936) / (2585191x^8 - 4661200x^7 + 14191920x^6 + \\
& 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 185 \\
& 79456) + 1587 \cdot \sqrt{341} \cdot 200^{(1/4)} \cdot \sqrt{31} \cdot (200000x^4 - 200000x^3 + 6500 \\
& 00x^2 + 15457 \cdot \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) - 300000x + 4500 \\
& 00) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} \cdot \log(77500000/5711 \cdot (\sqrt{341} \cdot 200^{(1/4)} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3}) \cdot (\sqrt{2} \cdot (281x - 444) + 163x - 725) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} + 4337504500x^2 + 3894902000 \cdot \sqrt{2} \cdot (2x^2 - x + 3) - 13366595500x + 17704100000)/x^2} - 1587 \cdot \sqrt{341} \cdot 200^{(1/4)} \cdot \sqrt{31} \cdot (200000x^4 - 200000x^3 + 650000x^2 + 15457 \cdot \sqrt{2} \cdot (4x^4 - 4x^3 + 13x^2 - 6x + 9) - 300000x + 450000) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} \cdot \log(-77500000/5711 \cdot (\sqrt{341} \cdot 200^{(1/4)} \cdot \sqrt{31} \cdot \sqrt{2x^2 - x + 3}) \cdot (\sqrt{2} \cdot (281x - 444) + 163x - 725) \cdot \sqrt{-772850000 \cdot \sqrt{2} + 2500000000} - 4337504500x^2 - 3894902000 \cdot \sqrt{2} \cdot (2x^2 - x + 3) + 13366595500x - 17704100000)/x^2} - 965935696000 \cdot (3948x^3 - 23592x^2 + 19767x - 39005) \cdot \sqrt{2x^2 - x + 3}) / (4x^4 - 4x^3 + 13x^2 - 6x + 9)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)

[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.98 \quad \int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2} dx$$

Optimal. Leaf size=234

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463 + 23600\sqrt{2})}}{1035276(2x^2 - x + 3)^{3/2}}$$

[Out] $-(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^{(3/2)}) - (3133427 + 1352542*x)/(523849656*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)) + (625*\text{Sqrt}[(30463 + 23600*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(30463 + 23600*\text{Sqrt}[2]))])*(203 + 242*\text{Sqrt}[2] + (687 + 445*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])/660176 - (625*\text{Sqrt}[(-30463 + 23600*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(-30463 + 23600*\text{Sqrt}[2]))])*(203 - 242*\text{Sqrt}[2] + (687 - 445*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])/660176$

Rubi [A] time = 0.543007, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{15101 - 8654x}{1035276(2x^2 - x + 3)^{3/2}} - \frac{1352542x + 3133427}{523849656\sqrt{2x^2 - x + 3}} + \frac{65x + 4}{682(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} + \frac{625\sqrt{\frac{1}{682}(30463 + 23600\sqrt{2})}}{1035276(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((3 - x + 2*x^2)^{(5/2)}*(2 + 3*x + 5*x^2)^2), x]$

[Out] $-(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^{(3/2)}) - (3133427 + 1352542*x)/(523849656*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)) + (625*\text{Sqrt}[(30463 + 23600*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(30463 + 23600*\text{Sqrt}[2]))])*(203 + 242*\text{Sqrt}[2] + (687 + 445*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])/660176 - (625*\text{Sqrt}[(-30463 + 23600*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(-30463 + 23600*\text{Sqrt}[2]))])*(203 - 242*\text{Sqrt}[2] + (687 - 445*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2])/660176$

Rule 974

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

```

Rule 1035

```

Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2], x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a

```



```
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx &= \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \frac{\int \frac{-1738+\frac{441x}{2}-5720x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx}{7502} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} - \frac{\int \frac{-\frac{9406661}{2}+3}{(3-x+2x^2)^{5/2} (2+3x+5x^2)} dx}{6} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)} \\
&= -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2} (2+3x+5x^2)}
\end{aligned}$$

Mathematica [C] time = 1.16162, size = 296, normalized size = 1.26

$$5456(13525420x^5 + 32686812x^4 + 2879479x^3 + 84671384x^2 - 5712309x + 31010342) + 198375i\sqrt{286 + 22i\sqrt{31}}(687\sqrt{31} - 63) \sqrt{3-x+2x^2} + (-22 - (4i)\sqrt{31})x / (2\sqrt{286 + (22i)\sqrt{31}})\sqrt{3-x+2x^2} + 198375\sqrt{286 - (22i)\sqrt{31}}(31 + (687i)\sqrt{31})\sqrt{3-x+2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2), x]

[Out] -(5456*(31010342 - 5712309*x + 84671384*x^2 + 2879479*x^3 + 32686812*x^4 + 13525420*x^5) + (198375*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(31*I + 687*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 198375*Sqrt[286 - (22*I)*Sqrt[31]]*(31 + (687*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2]

```
rt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh[(-63 + I*Sqrt[3
1] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])]/(2858123723136*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2))
```

Maple [B] time = 0.175, size = 5975, normalized size = 25.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^2 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

Fricas [B] time = 4.83618, size = 8498, normalized size = 36.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/25604335602537914112*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*
```

$$\begin{aligned}
& 321084x^7 - 1338894x^6 + 2762802x^5 - 4721048x^4 + 2438224x^3 - 1317312x^2 - \sqrt{2}*(277258x^7 - 994619x^6 + 2123978x^5 - 3198193x^4 + 1552680x^3 - 621000x^2 - 1900800x + 1181952) - 2363904x + 1900800) + 1829*6962^{(1/4)}*\sqrt{341}*(25187x^7 - 392073x^6 + 2114488x^5 - 4948060x^4 + 6460704x^3 - 4452768x^2 - \sqrt{2}*(20477x^7 - 310452x^6 + 1610140x^5 - 3584192x^4 + 4580640x^3 - 2620800x^2 - 3400704x + 2198016) - 4396032x + 3400704))*\sqrt{2x^2 - x + 3}*\sqrt{30463*\sqrt{2} + 47200} + 31558548641224*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2*\sqrt{118/79}*(\sqrt{118}*(22*6962^{(3/4)}*\sqrt{341}*(1050904x^7 - 1523916x^6 + 5005956x^5 - 2572736x^4 + 3615264x^3 + 877824x^2 - \sqrt{2}*(1065206x^7 - 1518091x^6 + 4815081x^5 - 1448880x^4 + 1303560x^3 + 3131136x^2 - 3131136x) - 877824x) + 1829*6962^{(1/4)}*\sqrt{341}*(84981x^7 - 1100084x^6 + 4256060x^5 - 5639616x^4 + 7745184x^3 + 2571264x^2 - 242*\sqrt{2}*(319x^7 - 4124x^6 + 15860x^5 - 20160x^4 + 24480x^3 + 20736x^2 - 20736x) - 2571264x))*\sqrt{2x^2 - x + 3}*\sqrt{30463*\sqrt{2} + 47200} + 187549318*\sqrt{31}*\sqrt{2}*(123408x^8 - 914152x^7 + 1578888x^6 - 3293072x^5 + 396480x^4 + 798336x^3 - 3822336x^2 - \sqrt{2}*(15550x^8 - 118051x^7 + 244047x^6 - 707374x^5 + 1053960x^4 - 1667952x^3 + 1209600x^2 - 1036800x) + 3276288x) + 8524969*\sqrt{31}*(254591x^8 - 4815126x^7 + 32303580x^6 - 90866808x^5 + 108781920x^4 - 74219328x^3 - 168956928x^2 - 15488*\sqrt{2}*(4x^8 - 76x^7 + 517x^6 - 1536x^5 + 2385x^4 - 3618x^3 + 2268x^2 - 1944x) + 144820224x))*\sqrt{-(6962^{(1/4)}*\sqrt{341}*\sqrt{118}*\sqrt{31}*\sqrt{2x^2 - x + 3}*(\sqrt{2}*(101x + 176) - 277x + 75))*\sqrt{30463*\sqrt{2} + 47200} - 219481829x^2 - 197085724*\sqrt{2}*(2x^2 - x + 3) + 676362371x - 895844200)/x^2) + 358619870923*\sqrt{31}*(2828123x^8 - 9696916x^7 + 53385560x^6 - 142835344x^5 + 254146592x^4 - 249300096x^3 + 37981440x^2 - 7744*\sqrt{2}*(1348x^8 - 2692x^7 + 9789x^6 - 10070x^5 + 15569x^4 - 5568x^3 + 1080x^2 + 4320x - 5184) + 223064064x - 94887936))/(2585191x^8 - 4661200x^7 + 14191920x^6 + 490880x^5 - 13562944x^4 + 44249088x^3 - 34615296x^2 - 24772608x + 18579456)) + 301208632500*6962^{(1/4)}*\sqrt{341}*\sqrt{118}*\sqrt{2}*(20x^6 - 8x^5 + 61x^4 + x^3 + 53x^2 + 15x + 18))*\sqrt{30463*\sqrt{2} + 47200}*\arctan(1/11117215998613*(168268*\sqrt{118}*(22*6962^{(3/4)}*\sqrt{341}*(321084x^7 - 1338894x^6 + 2762802x^5 - 4721048x^4 + 2438224x^3 - 1317312x^2 - \sqrt{2}*(277258x^7 - 994619x^6 + 2123978x^5 - 3198193x^4 + 1552680x^3 - 621000x^2 - 1900800x + 1181952) - 2363904x + 1900800) + 1829*6962^{(1/4)}*\sqrt{341}*(25187x^7 - 392073x^6 + 2114488x^5 - 4948060x^4 + 6460704x^3 - 4452768x^2 - \sqrt{2}*(20477x^7 - 310452x^6 + 1610140x^5 - 3584192x^4 + 4580640x^3 - 2620800x^2 - 3400704x + 2198016) - 4396032x + 3400704))*\sqrt{2x^2 - x + 3}*\sqrt{30463*\sqrt{2} + 47200} - 31558548641224*\sqrt{31}*\sqrt{2}*(28180x^8 - 254666x^7 + 704270x^6 - 1385256x^5 + 1549144x^4 - 642048x^3 - 98496x^2 - \sqrt{2}*(8746x^8 - 102335x^7 + 396104x^6 - 783113x^5 + 1320710x^4 - 752088x^3 + 396144x^2 + 546048x - 539136) + 1154304x - 456192) - 2*\sqrt{11
\end{aligned}$$

```

8/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(341)*(1050904*x^7 - 1523916*x^6 + 5005
956*x^5 - 2572736*x^4 + 3615264*x^3 + 877824*x^2 - sqrt(2)*(1065206*x^7 - 1
518091*x^6 + 4815081*x^5 - 1448880*x^4 + 1303560*x^3 + 3131136*x^2 - 313113
6*x) - 877824*x) + 1829*6962^(1/4)*sqrt(341)*(84981*x^7 - 1100084*x^6 + 425
6060*x^5 - 5639616*x^4 + 7745184*x^3 + 2571264*x^2 - 242*sqrt(2)*(319*x^7 -
4124*x^6 + 15860*x^5 - 20160*x^4 + 24480*x^3 + 20736*x^2 - 20736*x) - 2571
264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) - 187549318*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
+ 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 8524969*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 9086680
8*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8
- 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) +
144820224*x))*sqrt((6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x
+ 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sqrt(30463*sqrt(2) + 47200) + 21
9481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x + 3) - 676362371*x + 895844200)/
x^2) - 358619870923*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14
2835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(
1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^
2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24
772608*x + 18579456)) + 991875*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*(944
000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 - 30463*sqrt(2
)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 708000*x + 849600)
*sqrt(30463*sqrt(2) + 47200)*log(7375000000000/79*(6962^(1/4)*sqrt(341)*sq
rt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sq
rt(30463*sqrt(2) + 47200) + 219481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x +
3) - 676362371*x + 895844200)/x^2) - 991875*6962^(1/4)*sqrt(341)*sqrt(118)*
sqrt(31)*(944000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 -
30463*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 70800
0*x + 849600)*sqrt(30463*sqrt(2) + 47200)*log(-7375000000000/79*(6962^(1/4)
*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) -
277*x + 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)
*(2*x^2 - x + 3) + 676362371*x - 895844200)/x^2) - 48877259552*(13525420*x^
5 + 32686812*x^4 + 2879479*x^3 + 84671384*x^2 - 5712309*x + 31010342)*sqrt(
2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.99 \quad \int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^3} dx$$

Optimal. Leaf size=269

$$-\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1}{1364(2x^2 - x + 3)}$$

[Out] $-(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^{(3/2)}) - (1134826571 - 1504660754*x)/(476353953856*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)) + (35*\text{Sqrt}[(2243059557247 + 2011748500000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(2243059557247 + 2011748500000*\text{Sqrt}[2]))])*(1432939 + 2428746*\text{Sqrt}[2] + (6290431 + 3861685*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2]))/1800960128 - (35*\text{Sqrt}[(-2243059557247 + 2011748500000*\text{Sqrt}[2])/682]*\text{ArcTanh}[(\text{Sqrt}[11/(31*(-2243059557247 + 2011748500000*\text{Sqrt}[2]))])*(1432939 - 2428746*\text{Sqrt}[2] + (6290431 - 3861685*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2]))/1800960128$

Rubi [A] time = 0.589435, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1134826571 - 1504660754x}{476353953856\sqrt{2x^2 - x + 3}} + \frac{86885x + 46386}{1860496(2x^2 - x + 3)^{3/2}(5x^2 + 3x + 2)} - \frac{12280939 - 19536786x}{2824232928(2x^2 - x + 3)^{3/2}} + \frac{1}{1364(2x^2 - x + 3)}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3), x]

[Out] $-(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^{(3/2)}) - (1134826571 - 1504660754*x)/(476353953856*\text{Sqrt}[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x + 2*x^2)^{(3/2)}*(2 + 3*x + 5*x^2)) + (35*\text{Sqrt}[(2243059557247 + 2011748500000*\text{Sqrt}[2])/682]*\text{ArcTan}[(\text{Sqrt}[11/(31*(2243059557247 + 2011748500000*\text{Sqrt}[2]))])*(1432939 + 2428746*\text{Sqrt}[2] + (6290431 + 3861685*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2]))/1800960128 - (35*\text{Sqrt}[(-2243059557247 + 2011748500000*\text{Sqrt}[2])/682]*\text{ArcTanh}[(\text{Sqrt}[11/(31*(-2243059557247 + 2011748500000*\text{Sqrt}[2]))])*(1432939 - 2428746*\text{Sqrt}[2] + (6290431 - 3861685*\text{Sqrt}[2])*x)]/\text{Sqrt}[3 - x + 2*x^2]))/1800960128$

$$\frac{x^2}{1800960128} - \frac{(35\sqrt{-2243059557247 + 2011748500000\sqrt{2}}/682) \cdot \text{ArcTanh}\left(\frac{\sqrt{11/(31(-2243059557247 + 2011748500000\sqrt{2}))}}{(1432939 - 2428746\sqrt{2} + (6290431 - 3861685\sqrt{2})x)}\right)}{\sqrt{3 - x + 2x^2}}}{1800960128}$$

Rule 974

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]

Rule 1060

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !

IGtQ[q, 0]

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int
[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^3} dx &= \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} - \frac{\int \frac{-5687 + \frac{8635x}{2} - 8580x^2}{(3-x+2x^2)^{5/2} (2+3x+5x^2)^2} dx}{15004} \\
&= \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} + \frac{46386+86885x}{1860496(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} + \frac{4+65x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} + \frac{46386+86885x}{1860496(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{46386+86885x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{46386+86885x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{46386+86885x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2} \\
&= -\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{46386+86885x}{1364(3-x+2x^2)^{3/2} (2+3x+5x^2)^2}
\end{aligned}$$

Mathematica [C] time = 1.97068, size = 242, normalized size = 0.9

$$\frac{5456(225699113100x^7 - 12234606480x^6 + 592923725931x^5 + 174241614961x^4 + 519223213785x^3 + 178650961091x^2 + 218659985088x + 9739335532)}{(2x^2-x+3)^{3/2}(5x^2+3x+2)^2} + 11109\sqrt{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3), x]

[Out] ((5456*(9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 + 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7))

$$\frac{1}{((3 - x + 2x^2)^{3/2} * (2 + 3x + 5x^2)^2) + 11109 * \text{Sqrt}[286 + (22 * I) * \text{Sqrt}[31]] * (4541903 - (6290431 * I) * \text{Sqrt}[31]) * \text{ArcTanh}[(63 + I * \text{Sqrt}[31] + (-22 - (4 * I) * \text{Sqrt}[31]) * x) / (2 * \text{Sqrt}[286 + (22 * I) * \text{Sqrt}[31]]) * \text{Sqrt}[3 - x + 2 * x^2]]] - (11109 * I) * \text{Sqrt}[286 - (22 * I) * \text{Sqrt}[31]] * (-4541903 * I + 6290431 * \text{Sqrt}[31]) * \text{ArcTanh}[(-63 + I * \text{Sqrt}[31] + (22 - (4 * I) * \text{Sqrt}[31]) * x) / (2 * \text{Sqrt}[286 - (22 * I) * \text{Sqrt}[31]]) * \text{Sqrt}[3 - x + 2 * x^2]]]}{7796961516715008}$$

Maple [B] time = 0.277, size = 19014, normalized size = 70.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 3x + 2)^3 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)

Fricas [B] time = 5.31818, size = 11142, normalized size = 41.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

```

[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/452534011574628261925237033857859439*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 12846126780182995822393825477
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 5088576640840418594376929*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 1921101946251381781783*x^2 - 1725071135409404048948*sqrt(2)*(2*x^2 -
x + 3) + 5920130487427727531617*x - 7841232433679109313400)/x^2) + 1459787
1341117040707265710769608369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560
*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*
sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3
+ 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612
00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296
*x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)*
sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390

```

$$\begin{aligned}
& *x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*\text{sqrt}(2243059557247*\text{sqrt}(2) + 40234970 \\
& 00000)*\arctan(1/452534011574628261925237033857859439*(11475013444*\text{sqrt}(4023 \\
& 497)*(11*129508224872072^{(3/4)}*\text{sqrt}(341)*(2673027292*x^7 - 11768684222*x^6 \\
& + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - \text{s} \\
& \text{qrt}(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4 \\
& + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206 \\
& 82752*x + 17594323200) + 124728407*129508224872072^{(1/4)}*\text{sqrt}(341)*(2145837 \\
& 31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x \\
& ^3 - 41786349984*x^2 - \text{sqrt}(2)*(190078101*x^7 - 2862100476*x^6 + 1468800342 \\
& 0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x \\
& + 19060075008) - 38120150016*x + 31156503552))*\text{sqrt}(2*x^2 - x + 3)*\text{sqrt}(22 \\
& 43059557247*\text{sqrt}(2) + 4023497000000) - 128461267801829958223938254772553647 \\
& 2*\text{sqrt}(31)*\text{sqrt}(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154 \\
& 9144*x^4 - 642048*x^3 - 98496*x^2 - \text{sqrt}(2)*(8746*x^8 - 102335*x^7 + 396104 \\
& *x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391 \\
& 36) + 1154304*x - 456192) - 2*\text{sqrt}(8046994/10139750351)*(\text{sqrt}(4023497)*(11* \\
& 129508224872072^{(3/4)}*\text{sqrt}(341)*(8140972152*x^7 - 11907581308*x^6 + 3977730 \\
& 3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - \text{sqrt}(2)*(10 \\
& 387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242 \\
& 91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12 \\
& 9508224872072^{(1/4)}*\text{sqrt}(341)*(692762453*x^7 - 8972954292*x^6 + 34803726780 \\
& *x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*\text{sqrt}(2)*(367 \\
& 903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014 \\
& 40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*\text{sqrt}(2*x^2 - x \\
& + 3)*\text{sqrt}(2243059557247*\text{sqrt}(2) + 4023497000000) - 111948686098489209076292 \\
& 438*\text{sqrt}(31)*\text{sqrt}(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + \\
& 396480*x^4 + 798336*x^3 - 3822336*x^2 - \text{sqrt}(2)*(15550*x^8 - 118051*x^7 + \\
& 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800 \\
& *x) + 3276288*x) - 5088576640840418594376929*\text{sqrt}(31)*(254591*x^8 - 4815126 \\
& *x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569 \\
& 28*x^2 - 15488*\text{sqrt}(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36 \\
& 18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*\text{sqrt}((129508224872072^{(1/4)}*\text{sqr} \\
& \text{t}(4023497)*\text{sqrt}(341)*\text{sqrt}(31)*\text{sqrt}(2*x^2 - x + 3)*(\text{sqrt}(2)*(643213*x + 2195 \\
& 288) - 2838501*x + 1552075))*\text{sqrt}(2243059557247*\text{sqrt}(2) + 4023497000000) + 1 \\
& 921101946251381781783*x^2 + 1725071135409404048948*\text{sqrt}(2)*(2*x^2 - x + 3) \\
& - 5920130487427727531617*x + 7841232433679109313400)/x^2) - 145978713411170 \\
& 40707265710769608369*\text{sqrt}(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1 \\
& 42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*\text{sqrt}(2)* \\
& (1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x \\
& ^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + \\
& 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2 \\
& 4772608*x + 18579456) + 55545*129508224872072^{(1/4)}*\text{sqrt}(4023497)*\text{sqrt}(341 \\
&)*\text{sqrt}(31)*(4023497000000000*x^8 + 80469940000000*x^7 + 1291542537000000*x^6 \\
& + 692041484000000*x^5 + 156916383000000*x^4 + 949545292000000*x^3 + 96966 \\
& 2777000000*x^2 - 2243059557247*\text{sqrt}(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
\end{aligned}$$

```

5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589
2000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(246439191250000000
0000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sq
rt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*sqrt
(2243059557247*sqrt(2) + 4023497000000) + 1921101946251381781783*x^2 + 1725
071135409404048948*sqrt(2)*(2*x^2 - x + 3) - 5920130487427727531617*x + 784
1232433679109313400)/x^2) - 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(
341)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
00000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 1921101946251381781783*x^2 -
1725071135409404048948*sqrt(2)*(2*x^2 - x + 3) + 5920130487427727531617*x -
7841232433679109313400)/x^2) + 427817641581532204139104*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{5}{2}} (5x^2 + 3x + 2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)

[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.100 $\int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$

Optimal. Leaf size=436

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (8c^2 (2a^2 f^2 + 12abef + 5b^2 (2df + e^2)) - 56b^2 cf(af + be) - 32c^3 (a(2df + e^2) + 4bde) + 21b^4 f^2)}{512c^5}$$

[Out] $((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b + 2*c*x)*\text{Sqrt}[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x + c*x^2)^{(3/2)})/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^{(3/2)})/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*(a + b*x + c*x^2)^{(3/2)})/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^{(3/2)})/(6*c) - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(1024*c^{(11/2)})$

Rubi [A] time = 0.789373, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (8c^2 (2a^2 f^2 + 12abef + 5b^2 (2df + e^2)) - 56b^2 cf(af + be) - 32c^3 (a(2df + e^2) + 4bde) + 21b^4 f^2)}{512c^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + b*x + c*x^2]*(d + e*x + f*x^2)^2, x]$

[Out] $((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b + 2*c*x)*\text{Sqrt}[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x + c*x^2)^{(3/2)})/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^{(3/2)})/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*(a + b*x + c*x^2)^{(3/2)})/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^{(3/2)})/(6*c) - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(1024*c$

$^{(11/2)}$

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+bx+cx^2} (d+ex+fx^2)^2 dx &= \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (6cd^2+12cdex-3(af^2-2c(e^2+2df))) dx}{6c} \\
&= \frac{f(8ce-3bf)x^2 (a+bx+cx^2)^{3/2}}{20c^2} + \frac{f^2 x^3 (a+bx+cx^2)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2} (30cd^2+6cdex-3(af^2-2c(e^2+2df))) dx}{6c} \\
&= \frac{(21b^2 f^2 - 4cf(14be+5af) + 40c^2(e^2+2df)) x (a+bx+cx^2)^{3/2}}{160c^3} + \frac{f(8ce-3bf)}{6c} \int \sqrt{a+bx+cx^2} dx \\
&= \frac{(640c^3 de - 105b^3 f^2 + 28bcf(10be+7af) - 8c^2(32aef+25b(e^2+2df))) (a+bx+cx^2)^{3/2}}{960c^4} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df))) + 8c^2(12abef)}{512c^5} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df))) + 8c^2(12abef)}{512c^5} \\
&= \frac{(128c^4 d^2 + 21b^4 f^2 - 56b^2 cf(be+af) - 32c^3(4bde+a(e^2+2df))) + 8c^2(12abef)}{512c^5}
\end{aligned}$$

Mathematica [A] time = 0.964738, size = 657, normalized size = 1.51

$$-f^2 \left(-15(16a^2c^2 - 56ab^2c + 21b^4) \left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)} - (b^2 - 4ac) \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right) + 16c^{3/2}(-196abc) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]

[Out] (3840*c^(9/2)*d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + 10240*c^(9/2)*d*e*(a + x*(b + c*x))^(3/2) + 3840*c^(9/2)*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(3/2) + 6144*c^(9/2)*e*f*x^2*(a + x*(b + c*x))^(3/2) + 2560*c^(9/2)*f^2*x^3*(a + x*(b + c*x))^(3/2) - 1920*c^4*(b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 1920*b*c^3*d*e*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) + 8*c*e*f*(-16*c^(3/2)*(-35*b^2 + 32*a*c + 42*b*c*x)*(a + x*(b + c*x))^(3/2) - 15*b*(7*b^2 - 12*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - 40*c^2*(e^2 + 2*d*f)*(80*b*c^(3/2)*(a + x*(b + c*x))^(3/2) - 3*(5*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - f^2*(2304*b*c^(7/2)*(a + x*(b + c*x))^(3/2) - 3*(5*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))

$$2)x^2(a + x(b + cx))^{3/2} + 16c^{3/2}(105b^3 - 196abc - 126b^2cx + 120a^2c^2x)(a + x(b + cx))^{3/2} - 15(21b^4 - 56ab^2c + 16a^2c^2)(2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)} - (b^2 - 4ac)\text{ArcTan}h[(b + 2cx)/(2\sqrt{c}\sqrt{a + x(b + cx)})]))/(15360c^{11/2})$$

Maple [B] time = 0.063, size = 1429, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((cx^2+bx+a)^{1/2}(fx^2+ex+d)^2, x)$

[Out] $\frac{3}{8}efb/c^2axx(cx^2+bx+a)^{1/2} - \frac{7}{32}efb^3/c^3xx(cx^2+bx+a)^{1/2} - \frac{5}{16}efb^3/c^{7/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{a+3}{16}efb^2/c^3a(cx^2+bx+a)^{1/2} + \frac{3}{8}efb/c^{5/2}a^2\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{2}{3}d^2e(cx^2+bx+a)^{3/2}/c - \frac{7}{64}f^2b^3/c^4(cx^2+bx+a)^{3/2} + \frac{21}{512}f^2b^5/c^5(cx^2+bx+a)^{1/2} - \frac{21}{1024}f^2b^6/c^{11/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{1}{16}f^2a^3/c^{5/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) - \frac{5}{128}b^4/c^{7/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{e^2-1}{8}a^2/c^{3/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{e^2+1}{4}d^2/c(cx^2+bx+a)^{1/2} + \frac{b+1}{2}d^2/c^{1/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{a-1}{8}d^2/c^{3/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{b^2-7}{32}f^2b^2/c^3axx(cx^2+bx+a)^{1/2} - \frac{7}{20}efb/c^2xx(cx^2+bx+a)^{3/2} - \frac{1}{8}a/c^2(cx^2+bx+a)^{1/2} + \frac{bdf+5}{16}b^2/c^2xx(cx^2+bx+a)^{1/2} + \frac{d^2f-1}{4}a/cxx(cx^2+bx+a)^{1/2} + \frac{d^2f-1}{2}d^2eb/c^{3/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{a+3}{8}b^2/c^{5/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{ad^2f-1}{2}d^2eb/cxx(cx^2+bx+a)^{1/2} + \frac{7}{24}efb^2/c^3(cx^2+bx+a)^{3/2} - \frac{7}{64}efb^4/c^4(cx^2+bx+a)^{1/2} - \frac{1}{4}a^2/c^{3/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{d^2f+1}{2}d^2xx(cx^2+bx+a)^{1/2} - \frac{5}{12}b/c^2(cx^2+bx+a)^{3/2} + \frac{d^2f-1}{16}a/c^2(cx^2+bx+a)^{1/2} + \frac{b^2e^2+1}{8}d^2eb^3/c^{5/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{5}{32}b^2/c^2xx(cx^2+bx+a)^{1/2} + \frac{e^2+5}{32}b^3/c^3(cx^2+bx+a)^{1/2} + \frac{d^2f+3}{16}b^2/c^{5/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{ae^2-5}{64}b^4/c^{7/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{d^2f-1}{8}a/cxx(cx^2+bx+a)^{1/2} + \frac{e^2+2}{5}efx^2(cx^2+bx+a)^{3/2}/c + \frac{1}{2}xx(cx^2+bx+a)^{3/2}/cd^2f - \frac{1}{8}f^2a/c^2xx(cx^2+bx+a)^{3/2} - \frac{1}{4}d^2eb^2/c^2(cx^2+bx+a)^{1/2} - \frac{3}{20}f^2b/c^2xx^2(cx^2+bx+a)^{3/2} + \frac{1}{16}f^2a^2/c^2xx(cx^2+bx+a)^{1/2} + \frac{1}{32}f^2a^2/c^3(cx^2+bx+a)^{1/2} + \frac{b+21}{160}f^2b^2/c^3xx(cx^2+bx+a)^{3/2} + \frac{21}{256}f^2b^4/c^4xx(cx^2+bx+a)^{1/2} + \frac{35}{256}f^2b^4/c^{9/2}\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2}) + \frac{a-7}{64}f^2b^3/c^4ax(cx^2+bx+a)^{1/2} - \frac{15}{64}f^2b^2/c^{7/2}a^2\ln((1/2b+cx)/c^{1/2} + (cx^2+bx+a)^{1/2})$

$$b*x+a)^{(1/2)}+49/240*f^2*b/c^3*a*(c*x^2+b*x+a)^{(3/2)}+7/128*e*f*b^5/c^{(9/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/15*e*f*a/c^2*(c*x^2+b*x+a)^{(3/2)}+1/4*x*(c*x^2+b*x+a)^{(3/2)}/c*e^2-5/24*b/c^2*(c*x^2+b*x+a)^{(3/2)}*e^2+5/64*b^3/c^3*(c*x^2+b*x+a)^{(1/2)}*e^2+1/6*f^2*x^3*(c*x^2+b*x+a)^{(3/2)}/c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.83322, size = 2866, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e \\ & + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c + 2 \\ & 40*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2* \\ & c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*\sqrt{c}*\log(-8*c^2*x \\ & ^2 - 8*b*c*x - b^2 - 4*\sqrt{c*x^2 + b*x + a}*(2*c*x + b)*\sqrt{c} - 4*a*c) - \\ & 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*x^4 + \\ & 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e)*f)*x \\ & ^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2 + (31 \\ & 5*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*b*c^5* \\ & e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16*a*c^5 \\ &)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*a*b^2* \\ & c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2*c^4 - \\ & 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*(10*(\\ & b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*\sqrt{c*x^2 + b \\ & *x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a \\ & *b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140 \\ & *a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c \end{aligned}$$

$$\begin{aligned} &^3 + 16a^2c^4)d - (7b^5c - 40ab^3c^2 + 48a^2b^2c^3)e)f)\sqrt{-c} \\ &*\arctan(1/2\sqrt{cx^2 + bx + a}*(2cx + b)\sqrt{-c}/(c^2x^2 + b^2cx + a \\ &*c)) + 2*(1280c^6f^2x^5 + 1920b^2c^5d^2 + 128*(24c^6e^2f + b^2c^5f^2)* \\ &x^4 + 16*(120c^6e^2 - (9b^2c^4 - 20a^2c^5)f^2 + 24*(10c^6d + b^2c^5e \\ &)*f)*x^3 - 640*(3b^2c^4 - 8a^2c^5)d^2e + 40*(15b^3c^3 - 52ab^2c^4)e^2 \\ &+ (315b^5c - 1680ab^3c^2 + 1808a^2b^2c^3)f^2 + 8*(640c^6d^2e + 40* \\ &b^2c^5e^2 + (21b^3c^3 - 68ab^2c^4)f^2 + 8*(10b^2c^5d - (7b^2c^4 - 16 \\ &a^2c^5)e)*f)*x^2 + 8*(10*(15b^3c^3 - 52ab^2c^4)d - (105b^4c^2 - 460* \\ &a^2b^2c^3 + 256a^2c^4)e)*f + 2*(1920c^6d^2 + 640b^2c^5d^2e - 40*(5b^2 \\ &*c^4 - 12a^2c^5)e^2 - (105b^4c^2 - 448ab^2c^3 + 240a^2c^4)f^2 - 8* \\ &(10*(5b^2c^4 - 12a^2c^5)d - (35b^3c^3 - 116ab^2c^4)e)*f)*x)\sqrt{cx^2 \\ &+ bx + a)}/c^6] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)

[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)

Giac [A] time = 1.30159, size = 861, normalized size = 1.97

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2 \left(8 \left(10 f^2 x + \frac{bc^4 f^2 + 24 c^5 f e}{c^5} \right) x + \frac{240 c^5 d f - 9 b^2 c^3 f^2 + 20 a c^4 f^2 + 24 b c^4 f e + 120 c^5 e^2}{c^5} \right) x \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")

[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (b*c^4*f^2 + 24*c^5*f*e)/c^5)*x + (240*c^5*d*f - 9*b^2*c^3*f^2 + 20*a*c^4*f^2 + 24*b*c^4*f*e + 120*c^5*e^2)/c^5)*x + (80*b*c^4*d*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2 + 640*c^5*d*e - 56*b^2*c^3*f*e + 128*a*c^4*f*e + 40*b*c^4*e^2)/c^5)*x + (1920*c^5*d^2 - 400*b^2*c^3*d*f + 960*a*c^4*d*f - 105*b^4*c*f^2 + 448*a*b^2*c^2*f^2 - 240*a^2*c^3*f^2 + 640*b*c^4*d*e + 280*b^3*c^2*f*e - 928*a*b*c^3*f*e - 200*

$$\begin{aligned}
& b^2c^3e^2 + 480ac^4e^2)/c^5)x + (1920b^4c^4d^2 + 1200b^3c^2d^2f - \\
& 4160ab^3c^3d^2f + 315b^5f^2 - 1680ab^3c^3f^2 + 1808a^2b^2c^2f^2 - 19 \\
& 20b^2c^3d^2e + 5120ac^4d^2e - 840b^4c^2f^2e + 3680ab^2c^2f^2e - 2048 \\
& a^2c^3f^2e + 600b^3c^2e^2 - 2080ab^3c^3e^2)/c^5) + 1/1024(128b^2c^4 \\
& ^4d^2 - 512ac^5d^2 + 80b^4c^2d^2f - 384ab^2c^3d^2f + 256a^2c^4d^2 \\
& f + 21b^6f^2 - 140ab^4c^2f^2 + 240a^2b^2c^2f^2 - 64a^3c^3f^2 - \\
& 128b^3c^3d^2e + 512ab^3c^4d^2e - 56b^5c^2f^2e + 320ab^3c^2f^2e - 384 \\
& a^2b^3c^3f^2e + 40b^4c^2e^2 - 192ab^2c^3e^2 + 128a^2c^4e^2)*\log(a \\
& bs(-2*(\sqrt{c})x - \sqrt{cx^2 + bx + a})*\sqrt{c} - b))/c^{(11/2)}
\end{aligned}$$

3.101 $\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$

Optimal. Leaf size=175

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2}(-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3} - \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af + 2be) + 5b^2f + 16cd)}{128c^{7/2}}$$

[Out] $((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*\text{Sqrt}[a + b*x + c*x^2]) / (64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^{(3/2)}) / (24*c^2) + (f*x*(a + b*x + c*x^2)^{(3/2)}) / (4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e + a*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (128*c^{(7/2)})$

Rubi [A] time = 0.163777, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2}(-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3} - \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af + 2be) + 5b^2f + 16cd)}{128c^{7/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + b*x + c*x^2]*(d + e*x + f*x^2), x]$

[Out] $((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*\text{Sqrt}[a + b*x + c*x^2]) / (64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^{(3/2)}) / (24*c^2) + (f*x*(a + b*x + c*x^2)^{(3/2)}) / (4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e + a*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (128*c^{(7/2)})$

Rule 1661

$\text{Int}[(Pq_*)*((a_*) + (b_*)(x_*) + (c_*)(x_*)^2)^{(p_*)}, x_Symbol] := \text{With}[\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^{(q-1)}*(a + b*x + c*x^2)^{(p+1)}) / (c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a + b*x + c*x^2)^p * \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{\int (4cd - af + \frac{1}{2}(8ce - 5bf)x) \sqrt{a + bx + cx^2} dx}{4c} \\
&= \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} + \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{(16c^2d - 8bce + 5b^2f - 4acf)\sqrt{a + bx + cx^2}}{16c^2} \\
&= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)}{24c^2} \\
&= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)}{24c^2} \\
&= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)}{24c^2}
\end{aligned}$$

Mathematica [A] time = 0.292534, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}(4bc(2c(6d+2ex+fx^2)-13af)+8c^2(a(8e+3fx)+2cx(6d+4ex+3fx^2))-2b^2c(12e+5fx)-384c^{7/2}}{384c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]

[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/(384*c^(7/2))

Maple [B] time = 0.052, size = 453, normalized size = 2.6

$$\frac{fx}{4c}(cx^2 + bx + a)^{\frac{3}{2}} - \frac{5bf}{24c^2}(cx^2 + bx + a)^{\frac{3}{2}} + \frac{5b^2fx}{32c^2}\sqrt{cx^2 + bx + a} + \frac{5b^3f}{64c^3}\sqrt{cx^2 + bx + a} + \frac{3b^2fa}{16}\ln\left(\left(\frac{b}{2} + cx\right)\frac{1}{\sqrt{c}} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d), x)

[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*f*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)+5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*f*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*x*(c*x^2+b*x+a)^(1/2)-1/16*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*x*(c*x^2+b*x+a)^(1/2)-1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/16*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*d*x*(c*x^2+b*x+a)^(1/2)+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49495, size = 1061, normalized size = 6.06

$$\frac{3(16(b^2c^2 - 4ac^3)d - 8(b^3c - 4abc^2)e + (5b^4 - 24ab^2c + 16a^2c^2)f)\sqrt{c}\log(-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2 + bx + a}(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
+ 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
+ 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A] time = 1.15989, size = 286, normalized size = 1.63

$$\frac{1}{192} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(6fx + \frac{bc^2f + 8c^3e}{c^3} \right) x + \frac{48c^3d - 5b^2cf + 12ac^2f + 8bc^2e}{c^3} \right) x + \frac{48bc^2d + 15b^3f - 52abcf - 24b^2ce + 64a^2c^2e}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)

$$3.102 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=431

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \dots$$

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

Rubi [A] time = 1.05133, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {989, 621, 206, 1032, 724}

$$\frac{\sqrt{f(2af - b(e - \sqrt{e^2 - 4df})) + c(-e\sqrt{e^2 - 4df} - 2df + e^2)} \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}f\sqrt{e^2 - 4df}} + \dots$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

- 4*d*f])

Rule 989

Int[Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]/((d_) + (e_)*(x_) + (f_)*(x_)^2), x_Symbol] := Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx &= -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{f} + \frac{c \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f} \\
&= \frac{(2c) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df})) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)^2} dx}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{(2(2f(cd-af) - (ce-bf)(e - \sqrt{e^2-4df}))) \text{Subst}\left(\int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df}+2fx)^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f\sqrt{e^2-4df}} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c(e^2-2df-e\sqrt{e^2-4df})+f(2af-b(e-\sqrt{e^2-4df}))} \tanh^{-1}\left(\frac{2af-b(e-\sqrt{e^2-4df})+c(e\sqrt{e^2-4df}-2df+e^2)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af-b(e-\sqrt{e^2-4df}))}+c(e\sqrt{e^2-4df}-2df+e^2)}\right)}{\sqrt{2}f\sqrt{e^2-4df}}
\end{aligned}$$

Mathematica [A] time = 1.25489, size = 417, normalized size = 0.97

$$\frac{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)} \tanh^{-1}\left(\frac{4af-b(\sqrt{e^2-4df}+e)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}}\right)}{\sqrt{2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2), x]

[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])

Maple [B] time = 0.433, size = 6019, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)`

[Out] result too large to display

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)`

[Out] `Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)`

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.103 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx$$

Optimal. Leaf size=488

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \frac{(f(be - 4af) - (\sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) - ((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rubi [A] time = 2.92941, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {971, 1032, 724, 206}

$$\frac{(f(be - 4af) - (e - \sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}} + \frac{(f(be - 4af) - (\sqrt{e^2 - 4df})(ce - bf)) \tanh^{-1}\left(\frac{4af + 2x(bf - c(e - \sqrt{e^2 - 4df})) - b(e - \sqrt{e^2 - 4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}\right)}{\sqrt{2}(e^2 - 4df)^{3/2}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2, x]

[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) - ((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]))/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

```
*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p + 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx &= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx}{-e^2+4df} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} - \frac{(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df}))) \int \frac{1}{(e-\sqrt{e^2-4df})}}{(e^2-4df)^{3/2}} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(2(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df})))) \operatorname{Subst}\left(\frac{1}{2v}\right)}{(e^2-4df)^{3/2}} \\
&= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(ce(e-\sqrt{e^2-4df})+f(4af-b(2e-\sqrt{e^2-4df}))) \tanh^{-1}\left(\frac{e-\sqrt{e^2-4df}}{2v}\right)}{\sqrt{2}(e^2-4df)^{3/2} \sqrt{ce^2-2cdf-bef+2af^2}}
\end{aligned}$$

Mathematica [A] time = 5.4495, size = 555, normalized size = 1.14

$$\frac{4f(e+2fx)\sqrt{a+x(b+cx)}}{(e^2-4df)(\sqrt{e^2-4df}-e-2fx)(\sqrt{e^2-4df}+e+2fx)} + \frac{(ce(\sqrt{e^2-4df}-e)-f(4af+b(\sqrt{e^2-4df}-2e))) \tanh^{-1}\left(\frac{e-\sqrt{e^2-4df}}{2v}\right)}{\sqrt{2}(e^2-4df)^{3/2} \sqrt{f(2af+b(\sqrt{e^2-4df}-2e))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]

[Out] (4*f*(e + 2*f*x)*Sqrt[a + x*(b + c*x)]/((e^2 - 4*d*f)*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + ((c*e*(-e + Sqrt[e^2 - 4*d*f]) - f*(4*a*f + b*(-2*e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(-4*a*f + 2*c*(e - Sqrt[e^2 - 4*d*f])*x + b*(e - Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)]])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - ((c*e*(e + Sqrt[e^2 - 4*d*f]) + f*(4*a*f - b*(2*e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])*Sqrt[a + x*(b + c*x)]])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]))

Maple [B] time = 0.371, size = 22287, normalized size = 45.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)`

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.104 $\int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx$

Optimal. Leaf size=564

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2} (16c^2(3a^2f^2 + 24abef + 14b^2(2df + e^2)) - 72b^2cf(3af + 4be) - 128c^3(a(2df + e^2) + 6bde))}{6144c^5}$$

[Out] $-\left((b^2 - 4ac)(768c^4d^2 + 99b^4f^2 - 72b^2c^2f(4be + 3af) - 128c^3(6bde + a(e^2 + 2df))) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df))\right)(b + 2cx)\sqrt{a + bx + cx^2}/(16384c^6) + \left((768c^4d^2 + 99b^4f^2 - 72b^2c^2f(4be + 3af) - 128c^3(6bde + a(e^2 + 2df))) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df))\right)(b + 2cx)(a + bx + cx^2)^{3/2}/(6144c^5) + \left((5376c^3d^2e - 693b^3f^2 + 36b^2c^2f(56be + 31af) - 32c^2(48aef + 49b(e^2 + 2df)))\right)(a + bx + cx^2)^{5/2}/(13440c^4) + \left((99b^2f^2 - 12c^2f(24be + 7af) + 224c^2(e^2 + 2df))\right)x(a + bx + cx^2)^{5/2}/(1344c^3) + (f(32ce - 11bf))x^2(a + bx + cx^2)^{5/2}/(112c^2) + (f^2x^3(a + bx + cx^2)^{5/2})/(8c) + \left((b^2 - 4ac)^2(768c^4d^2 + 99b^4f^2 - 72b^2c^2f(4be + 3af) - 128c^3(6bde + a(e^2 + 2df))) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df))\right)\text{ArcTanh}\left[\frac{(b + 2cx)}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right]/(32768c^{13/2})$

Rubi [A] time = 0.934847, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2} (16c^2(3a^2f^2 + 24abef + 14b^2(2df + e^2)) - 72b^2cf(3af + 4be) - 128c^3(a(2df + e^2) + 6bde))}{6144c^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]

[Out] $-\left((b^2 - 4ac)(768c^4d^2 + 99b^4f^2 - 72b^2c^2f(4be + 3af) - 128c^3(6bde + a(e^2 + 2df))) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df))\right)(b + 2cx)\sqrt{a + bx + cx^2}/(16384c^6) + \left((768c^4d^2 + 99b^4f^2 - 72b^2c^2f(4be + 3af) - 128c^3(6bde + a(e^2 + 2df))) + 16c^2(24abef + 3a^2f^2 + 14b^2(e^2 + 2df))\right)(b + 2cx)(a + bx + cx^2)^{3/2}/(6144c^5) + \left((5376c^3d^2e - 693b^3f^2 + 36b^2c^2f(56be + 31af) - 32c^2(48aef + 49b(e^2 + 2df)))\right)(a + b$

$$\begin{aligned} & x + c*x^2)^{(5/2)}/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22 \\ & 4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^{(5/2)}/(1344*c^3) + (f*(32*c*e - 1 \\ & 1*b*f)*x^2*(a + b*x + c*x^2)^{(5/2)}/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^ \\ & (5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b \\ & *e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3* \\ & a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b* \\ & x + c*x^2])]/(32768*c^{(13/2)}) \end{aligned}$$

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (d + ex + fx^2)^2 dx &= \frac{f^2 x^3 (a + bx + cx^2)^{5/2}}{8c} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c(e^2 + 2df))) dx}{8c} \\
&= \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} + \frac{f^2 x^3 (a + bx + cx^2)^{5/2}}{8c} + \frac{\int (a + bx + cx^2)^{3/2} (8cd^2 + 16cdex - (3af^2 - 8c(e^2 + 2df))) dx}{8c} \\
&= \frac{(99b^2 f^2 - 12cf(24be + 7af) + 224c^2(e^2 + 2df))x (a + bx + cx^2)^{5/2}}{1344c^3} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} \\
&= \frac{(5376c^3 de - 693b^3 f^2 + 36bcf(56be + 31af) - 32c^2(48aef + 49b(e^2 + 2df)))x (a + bx + cx^2)^{5/2}}{13440c^4} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} \\
&= \frac{(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(48aef + 49b(e^2 + 2df)))x (a + bx + cx^2)^{5/2}}{6144c^5} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(48aef + 49b(e^2 + 2df)))x (a + bx + cx^2)^{5/2}}{16384c^5} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(48aef + 49b(e^2 + 2df)))x (a + bx + cx^2)^{5/2}}{16384c^5} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2} \\
&= -\frac{(b^2 - 4ac)(768c^4 d^2 + 99b^4 f^2 - 72b^2 cf(4be + 3af) - 128c^3(6bde + a(e^2 + 2df)) + 16c^2(48aef + 49b(e^2 + 2df)))x (a + bx + cx^2)^{5/2}}{16384c^5} + \frac{f(32ce - 11bf)x^2 (a + bx + cx^2)^{5/2}}{112c^2}
\end{aligned}$$

Mathematica [A] time = 1.8282, size = 829, normalized size = 1.47

$$\frac{430080f^2(a + x(b + cx))^{5/2}x^3 + 983040ef(a + x(b + cx))^{5/2}x^2 + 573440(e^2 + 2df)(a + x(b + cx))^{5/2}x + 1376256de(a + x(b + cx))^{5/2}}{16384c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]

[Out] (430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b + c*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*x^2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80640*(b^2 - 4*a*c)*d^2*(-2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])]))/c^(3/2) - (2*6880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-256*c^(5/2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*a*c)*sqrt[a + x*(b + c*x)] + 14*b*c*x*(a + x*(b + c*x))^(3/2) + 14*c*(a + x*(b + c*x))^(5/2)))/c^(5/2)

$$a*c)*(16*c^{(3/2)}*(b + 2*c*x)*(a + x*(b + c*x))^{(3/2)} - 3*(b^2 - 4*a*c)*(2*\text{Sqrt}[c]*(b + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)] - (b^2 - 4*a*c)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])])))/c^{(9/2)} - (224*(e^2 + 2*d*f)*(1792*b*c^{(5/2)}*(a + x*(b + c*x))^{(5/2)} - 5*(7*b^2 - 4*a*c)*(16*c^{(3/2)}*(b + 2*c*x)*(a + x*(b + c*x))^{(3/2)} - 3*(b^2 - 4*a*c)*(2*\text{Sqrt}[c]*(b + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)] - (b^2 - 4*a*c)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])]))))/c^{(7/2)} - (3*f^2*(112640*b*c^{(9/2)}*x^2*(a + x*(b + c*x))^{(5/2)} + 256*c^{(5/2)}*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x*(b + c*x))^{(5/2)} - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^{(3/2)}*(b + 2*c*x)*(a + x*(b + c*x))^{(3/2)} - 3*(b^2 - 4*a*c)*(2*\text{Sqrt}[c]*(b + 2*c*x)*\text{Sqrt}[a + x*(b + c*x)] - (b^2 - 4*a*c)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])]))))/c^{(11/2)})/(3440640*c)$$

Maple [B] time = 0.063, size = 2458, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d)^2, x)$

[Out] $-7/512*b^5/c^4*(c*x^2+b*x+a)^{(1/2)}*e^2+7/1024*b^6/c^{(9/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2-1/16*a^3/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2+1/8*d^2/c*(c*x^2+b*x+a)^{(3/2)}*b+3/8*d^2*(c*x^2+b*x+a)^{(1/2)}*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b^3+3/8*d^2/c^{(1/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+3/128*d^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^4+3/128*f^2*a^4/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-99/16384*f^2*b^7/c^6*(c*x^2+b*x+a)^{(1/2)}+33/2048*f^2*b^5/c^5*(c*x^2+b*x+a)^{(3/2)}-33/640*f^2*b^3/c^4*(c*x^2+b*x+a)^{(5/2)}+1/6*x*(c*x^2+b*x+a)^{(5/2)}/c*e^2-7/60*b/c^2*(c*x^2+b*x+a)^{(5/2)}*e^2+7/192*b^3/c^3*(c*x^2+b*x+a)^{(3/2)}*e^2+3/32*e*f*b^2/c^3*a^2*(c*x^2+b*x+a)^{(1/2)}-3/16*d*e*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*a-3/8*d*e*b/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+3/16*d*e*b^3/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-1/4*d*e*b/c*x*(c*x^2+b*x+a)^{(3/2)}+3/32*d*e*b^3/c^2*(c*x^2+b*x+a)^{(1/2)}*x-9/128*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^{(3/2)}-57/512*f^2*b^2/c^3*a^2*(c*x^2+b*x+a)^{(1/2)}*x+153/2048*f^2*b^4/c^4*(c*x^2+b*x+a)^{(1/2)}*x*a-1/8*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x*d*f-1/16*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b*d*f-1/24*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b*d*f-1/12*a/c*x*(c*x^2+b*x+a)^{(3/2)}*d*f+7/48*b^2/c^2*x*(c*x^2+b*x+a)^{(3/2)}*d*f+1/8*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*x*a*e^2-7/128*b^4/c^3*(c*x^2+b*x+a)^{(1/2)}*x*d*f+1/8*b^3/c^3*(c*x^2+b*x+a)^{(1/2)}*a*d*f+9/32*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*d*f-15/128*b^4/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*f+1/4*b^2/c^2*(c*x^2+b*x$

$$\begin{aligned}
& +a)^{(1/2)} * x * a * d * f - 3/8 * d * e * b / c * (c * x^2 + b * x + a)^{(1/2)} * x * a + 3/16 * e * f * b / c^2 * a^2 * (c \\
& * x^2 + b * x + a)^{(1/2)} * x - 3/16 * e * f * b^3 / c^3 * (c * x^2 + b * x + a)^{(1/2)} * x * a + 1/8 * e * f * b / c^2 * \\
& a * x * (c * x^2 + b * x + a)^{(3/2)} + 21/256 * e * f * b^5 / c^9 * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) \\
& * a - 3/32 * e * f * b^3 / c^3 * x * (c * x^2 + b * x + a)^{(3/2)} + 9/256 * e * f * b^5 / c^4 * \\
& (c * x^2 + b * x + a)^{(1/2)} * x - 3/32 * e * f * b^4 / c^4 * (c * x^2 + b * x + a)^{(1/2)} * a + 1/16 * e * f * b^2 / c \\
& ^3 * a * (c * x^2 + b * x + a)^{(3/2)} + 1/4 * d^2 * x * (c * x^2 + b * x + a)^{(3/2)} - 7/30 * b / c^2 * (c * x^2 + b * \\
& x + a)^{(5/2)} * d * f + 7/96 * b^2 / c^2 * x * (c * x^2 + b * x + a)^{(3/2)} * e^2 + 99/32768 * f^2 * b^8 / c^{(1 \\
& 3/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + 2/5 * d * e * (c * x^2 + b * x + a)^{(5/2)} \\
& / c - 15/128 * f^2 * b^2 / c^{(7/2)} * a^3 * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) - \\
& 1/16 * f^2 * a / c^2 * x * (c * x^2 + b * x + a)^{(5/2)} + 1/64 * f^2 * a^2 / c^2 * x * (c * x^2 + b * x + a)^{(3/2)} \\
& + 1/128 * f^2 * a^2 / c^3 * (c * x^2 + b * x + a)^{(3/2)} * b - 9/1024 * e * f * b^7 / c^{(11/2)} * \ln((1/2 * b + \\
& c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + 3/20 * e * f * b^2 / c^3 * (c * x^2 + b * x + a)^{(5/2)} + 2/7 * \\
& e * f * x^2 * (c * x^2 + b * x + a)^{(5/2)} / c + 3/128 * f^2 * a^3 / c^2 * (c * x^2 + b * x + a)^{(1/2)} * x + 3/256 \\
& * f^2 * a^3 / c^3 * (c * x^2 + b * x + a)^{(1/2)} * b + 33/1024 * f^2 * b^4 / c^4 * x * (c * x^2 + b * x + a)^{(3/2)} \\
& - 99/8192 * f^2 * b^6 / c^5 * (c * x^2 + b * x + a)^{(1/2)} * x + 153/4096 * f^2 * b^5 / c^5 * (c * x^2 + b * x \\
& + a)^{(1/2)} * a - 9/256 * f^2 * b^3 / c^4 * a * (c * x^2 + b * x + a)^{(3/2)} - 57/1024 * f^2 * b^3 / c^4 * a^2 \\
& * (c * x^2 + b * x + a)^{(1/2)} + 33/448 * f^2 * b^2 / c^3 * x * (c * x^2 + b * x + a)^{(5/2)} - 11/112 * f^2 * b / \\
& c^2 * x^2 * (c * x^2 + b * x + a)^{(5/2)} + 93/1120 * f^2 * b / c^3 * a * (c * x^2 + b * x + a)^{(5/2)} + 105/102 \\
& 4 * f^2 * b^4 / c^{(9/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a^2 - 63/2048 * f \\
& ^2 * b^6 / c^{(11/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a + 1/3 * x * (c * x^2 + \\
& b * x + a)^{(5/2)} / c * d * f - 3/64 * e * f * b^4 / c^4 * (c * x^2 + b * x + a)^{(3/2)} + 9/512 * e * f * b^6 / c^5 * (\\
& c * x^2 + b * x + a)^{(1/2)} - 4/35 * e * f * a / c^2 * (c * x^2 + b * x + a)^{(5/2)} - 3/16 * d^2 / c^{(3/2)} * \ln((\\
& 1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * b^2 * a - 1/8 * d * e * b^2 / c^2 * (c * x^2 + b * x + a) \\
& ^{(3/2)} + 3/64 * d * e * b^4 / c^3 * (c * x^2 + b * x + a)^{(1/2)} - 3/128 * d * e * b^5 / c^{(7/2)} * \ln((1/2 * b \\
& + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) - 3/32 * d^2 / c * (c * x^2 + b * x + a)^{(1/2)} * x * b^2 + 3/1 \\
& 6 * d^2 / c * (c * x^2 + b * x + a)^{(1/2)} * b * a + 9/64 * b^2 / c^{(5/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * \\
& x^2 + b * x + a)^{(1/2)}) * a^2 * e^2 - 15/256 * b^4 / c^{(7/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + \\
& b * x + a)^{(1/2)}) * a * e^2 + 7/512 * b^6 / c^{(9/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) \\
& * d * f - 1/24 * a / c * x * (c * x^2 + b * x + a)^{(3/2)} * e^2 - 1/48 * a / c^2 * (c * x^2 + b * x + a)^{(3/2)} \\
& * b * e^2 - 1/16 * a^2 / c * (c * x^2 + b * x + a)^{(1/2)} * x * e^2 - 1/32 * a^2 / c^2 * (c * x^2 + b * x + a)^{(1/ \\
& 2)} * b * e^2 - 1/8 * a^3 / c^{(3/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * d * f + 7/ \\
& 96 * b^3 / c^3 * (c * x^2 + b * x + a)^{(3/2)} * d * f - 7/256 * b^4 / c^3 * (c * x^2 + b * x + a)^{(1/2)} * x * e^2 + \\
& 1/16 * b^3 / c^3 * (c * x^2 + b * x + a)^{(1/2)} * a * e^2 - 7/256 * b^5 / c^4 * (c * x^2 + b * x + a)^{(1/2)} * d * \\
& f - 3/14 * e * f * b / c^2 * x * (c * x^2 + b * x + a)^{(5/2)} + 3/16 * e * f * b / c^{(5/2)} * a^3 * \ln((1/2 * b + c * x) \\
& / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) - 15/64 * e * f * b^3 / c^{(7/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} \\
& + (c * x^2 + b * x + a)^{(1/2)}) * a^2 + 1/8 * f^2 * x^3 * (c * x^2 + b * x + a)^{(5/2)} / c
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.81723, size = 5115, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c
```

$$\begin{aligned} &^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*\text{sqrt}(-c)*\text{arctan}(1/2*\text{sqrt}(c*x^2 + \\ &b*x + a)*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2* \\ &x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^ \\ &6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + \\ &2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^ \\ &2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3* \\ &b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 \\ &+ 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 \\ &- 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 1 \\ &28*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - \\ &3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 \\ &+ 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36* \\ &a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(\\ &14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6 \\ &)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(\\ &315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(2 \\ &6880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35 \\ &*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4* \\ &c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^ \\ &2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e \\ &)*f)*x)*\text{sqrt}(c*x^2 + b*x + a))/c^7] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)

[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)

Giac [B] time = 1.33874, size = 1553, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7
*f^2 + 32*c^8*f*e)/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 +
480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 15
6*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7
*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^
4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b
^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x +
(80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2
+ 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7
*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^
3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^
2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6
*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2
- 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*
c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 5
3760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*
b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2
- 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80
640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2
*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e +
23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/
32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c
^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*
b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 +
768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*
e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*
c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 204
8*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(13/2)
```

3.105 $\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$

Optimal. Leaf size=236

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2}(-4acf + 7b^2f - 12bce + 24c^2d)}{192c^3} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2}(-4c(af + 3be) + 7b^2f)}{512c^4}$$

[Out] $-\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4caf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{f(a + bx + cx^2)^{5/2}}{6c} + \frac{(b^2 - 4ac)^2(24c^2d + 7b^2f - 4c(3be + af))\text{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{a + bx + cx^2}}\right]}{1024c^{9/2}}$

Rubi [A] time = 0.230253, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)(a + bx + cx^2)^{3/2}(-4acf + 7b^2f - 12bce + 24c^2d)}{192c^3} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2}(-4c(af + 3be) + 7b^2f)}{512c^4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + bx + cx^2)^{3/2}(d + ex + fx^2), x]$

[Out] $-\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4caf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{f(a + bx + cx^2)^{5/2}}{6c} + \frac{(b^2 - 4ac)^2(24c^2d + 7b^2f - 4c(3be + af))\text{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{a + bx + cx^2}}\right]}{1024c^{9/2}}$

Rule 1661

$\text{Int}[(Pq_*)(a_.) + (b_*)(x_) + (c_*)(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{With}[\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^{(q-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(q + 2*p + 1)), x] + \text{Dist}[1/(c*(q + 2*p + 1)), \text{Int}[(a + b*x + c*x^2)^p * \text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q + 2*p + 1)*x^q, x], x]] /;$ FreeQ[{a, b, c,

p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{\int (6cd - af + \frac{1}{2}(12ce - 7bf)x)(a + bx + cx^2)^{3/2} dx}{6c} \\
&= \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{(2c(6cd - af) - \frac{1}{2}b(12ce - 7bf)(a + bx + cx^2))^{3/2}}{60c^2} \\
&= \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{60c^2} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{60c^2} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{60c^2}
\end{aligned}$$

Mathematica [A] time = 0.703481, size = 392, normalized size = 1.66

$$\frac{360d(b^2 - 4ac)\left(\frac{(b^2 - 4ac)\tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) - 2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)}}{c^{3/2}}\right)}{c^{3/2}} - 60be\left(\frac{3(b^2 - 4ac)\left(\frac{(b^2 - 4ac)\tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) - 2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)}}{c^{5/2}}\right)}{c^{5/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x]

[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(5/2)))/c)/(15360*c)

Maple [B] time = 0.053, size = 862, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d), x)$

[Out] $\frac{1}{8}f*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*x*a-3/16*e*b/c*(c*x^2+b*x+a)^{(1/2)}*x*a+1/4*d*x*(c*x^2+b*x+a)^{(3/2)}+1/5*e*(c*x^2+b*x+a)^{(5/2)}/c-7/256*f*b^4/c^3*(c*x^2+b*x+a)^{(1/2)}*x+1/16*f*b^3/c^3*(c*x^2+b*x+a)^{(1/2)}*a+9/64*f*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2-3/32*d/c*(c*x^2+b*x+a)^{(1/2)}*x*b^2+3/16*d/c*(c*x^2+b*x+a)^{(1/2)}*b*a-1/48*f*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b-1/16*f*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x-1/32*f*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b+7/96*f*b^2/c^2*x*(c*x^2+b*x+a)^{(3/2)}-3/16*e*b/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+3/32*e*b^3/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-1/8*e*b/c*x*(c*x^2+b*x+a)^{(3/2)}-3/16*d/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^{(1/2)}*x-3/32*e*b^2/c^2*(c*x^2+b*x+a)^{(1/2)}*a-15/256*f*b^4/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-1/24*f*a/c*x*(c*x^2+b*x+a)^{(3/2)}-1/16*f*a^3/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-7/60*f*b/c^2*(c*x^2+b*x+a)^{(5/2)}+7/192*f*b^3/c^3*(c*x^2+b*x+a)^{(3/2)}-7/512*f*b^5/c^4*(c*x^2+b*x+a)^{(1/2)}+7/1024*f*b^6/c^{(9/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/8*d/c*(c*x^2+b*x+a)^{(3/2)}*b+3/8*d*(c*x^2+b*x+a)^{(1/2)}*x*a-3/64*d/c^2*(c*x^2+b*x+a)^{(1/2)}*b^3+3/8*d/c^{(1/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+3/128*d/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^4-1/16*e*b^2/c^2*(c*x^2+b*x+a)^{(3/2)}+3/128*e*b^4/c^3*(c*x^2+b*x+a)^{(1/2)}-3/256*e*b^5/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/6*f*x*(c*x^2+b*x+a)^{(5/2)}/c$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 2.63385, size = 1947, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)

[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)

Giac [A] time = 1.29431, size = 563, normalized size = 2.39

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2 \left(8 \left(10 c f x + \frac{13 b c^5 f + 12 c^6 e}{c^5} \right) x + \frac{120 c^6 d + 3 b^2 c^4 f + 140 a c^5 f + 132 b c^5 e}{c^5} \right) x + \frac{360 b c^5 d - 7 b^3 c^3 f + 36 a b c^4 f + 12 b^2 c^4 e + 384 a c^5 e}{c^5} \right) x + \frac{120 b^2 c^4 d + 2400 a c^5 d + 35 b^4 c^2 f - 216 a b^2 c^3 f + 240 a^2 c^4 f - 60 b^3 c^3 e + 336 a b c^4 e}{c^5} \right) x - \frac{360 b^3 c^3 d - 2400 a b c^4 d + 105 b^5 c^3 f - 760 a b^3 c^2 f + 1296 a^2 b c^3 f - 180 b^4 c^2 e + 1200 a b^2 c^3 e - 1536 a^2 c^4 e}{c^5} - \frac{1}{1024} (24 b^4 c^2 d - 192 a b^2 c^3 d + 384 a^2 c^4 d + 7 b^6 f - 60 a b^4 c f + 144 a^2 b^2 c^2 f - 64 a^3 c^3 f - 12 b^5 c^3 e + 96 a b^3 c^2 e - 192 a^2 b c^3 e) \log(\text{abs}(-2(\sqrt{c} x - \sqrt{c x^2 + b x + a}) \sqrt{c} - b)) / c^{9/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b*c^4*d + 105*b^5*c^3*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f - 12*b^5*c^3*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)

$$3.106 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} dx$$

Optimal. Leaf size=679

$$\frac{((e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - df))) \tanh^{-1}\left(\frac{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}{\dots}\right)}{\dots}$$

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 - 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (((c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 11.0327, antiderivative size = 678, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {977, 1076, 621, 206, 1032, 724}

$$\frac{((e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) - c(e^2 - 2df)) - 2f(-f^2(b^2d - a^2f) + 2cdf(be - af) + c^2(-d)(e^2 - df))) \tanh^{-1}\left(\frac{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}{\dots}\right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x]
```

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 - 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (((c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

$$\frac{[a + b*x + c*x^2]}{(8*\text{Sqrt}[c]*f^3) + (((c*e - b*f)*(e - \text{Sqrt}[e^2 - 4*d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f)))*\text{ArcTanh}[(4*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + b*x + c*x^2])]}{(\text{Sqrt}[2]*f^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]) + (((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + \text{Sqrt}[e^2 - 4*d*f]))*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)))*\text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]])*\text{Sqrt}[a + b*x + c*x^2])]}{(\text{Sqrt}[2]*f^3*\text{Sqrt}[e^2 - 4*d*f]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]])}$$

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f))*(2*p + 2*q + 1) + c*(2*d*f - e^2*(2*p + q)))] + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1)))]*x + ((c*e - b*f)^2*(1 - p)*p + c*(p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1032

Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + bx + cx^2)^{3/2}}{d + ex + fx^2} dx &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}(8c^2de - 4acef - bf(5be - 16af) + 4bc(e^2 - d^2))}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{2f^2} \\ &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}f(-4bcde + 5b^2df + 4af(cd - 2af)) - \frac{1}{4}d(-3b^2f^2 + 12cf(be - af) - 8c^2(e^2 - d^2))}{\sqrt{a + bx + cx^2}(d + ex + fx^2)} dx}{2f^2} \\ &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \text{Subst}\left(\int \frac{1}{4c - x^2} dx\right)}{4f^3} \\ &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{b+2x}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^3} \\ &= -\frac{(4ce - 5bf - 2cfx)\sqrt{a + bx + cx^2}}{4f^2} + \frac{(3b^2f^2 - 12cf(be - af) + 8c^2(e^2 - df)) \tanh^{-1}\left(\frac{b+2x}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^3} \end{aligned}$$

Mathematica [A] time = 4.85734, size = 1232, normalized size = 1.81

$$\sqrt{a + x(b + cx)} \left(-4 \left(e + \sqrt{e^2 - 4df} \right)^2 c^2 + 4f \left(e + \sqrt{e^2 - 4df} \right) xc^2 - 16af^2c + 10bf \left(e + \sqrt{e^2 - 4df} \right) c - 4bf^2xc - 2b^2f^2 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x]

[Out]
$$\frac{\left((-2b^2f^2 - 16ac^2f^2 + 10b^2c^2f(e + \sqrt{e^2 - 4df})) - 4c^2(e + \sqrt{e^2 - 4df})^2 - 4b^2c^2f^2x + 4c^2f^2(e + \sqrt{e^2 - 4df})x \right) \sqrt{a + x(b + cx)} + 2\sqrt{a + x(b + cx)}(b^2f^2 - 2c^2(-2e^2 + 4df + 2e\sqrt{e^2 - 4df} + ef^2x - f\sqrt{e^2 - 4df})x) + c^2(8af + b(-5e + 5\sqrt{e^2 - 4df} + 2f^2x)) - ((b^2f^2 + 4c^2(-e^2 + 2df + e\sqrt{e^2 - 4df}) - 4c^2(3af + b(-e + \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)) \right) / (\sqrt{c}f) + ((-bf) + c(e + \sqrt{e^2 - 4df}))(-b^2f^2 + 4c^2(e^2 - 2df + e\sqrt{e^2 - 4df}) - 4c^2(-3af + b(e + \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)) / (\sqrt{c}f) + (8\sqrt{2}c(c^2(e^4 - 4de^2f + 2d^2f^2 + e^3\sqrt{e^2 - 4df} - 2de^2f\sqrt{e^2 - 4df}) + f^2(2a^2f^2 - 2abf(e + \sqrt{e^2 - 4df}) + b^2(e^2 - 2df + e\sqrt{e^2 - 4df})) + 2c^2(af(e^2 - 2df + e\sqrt{e^2 - 4df}) - b(e^3 - 3de^2f + e^2\sqrt{e^2 - 4df} - df\sqrt{e^2 - 4df}))) \operatorname{ArcTanh}\left(\frac{4af - 2c(e + \sqrt{e^2 - 4df})x - b(e + \sqrt{e^2 - 4df} - 2fx)}{2\sqrt{2}\sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df}) + f(2af - b(e + \sqrt{e^2 - 4df}))}\sqrt{a + x(b + cx)}}\right)) / (f\sqrt{c(e^2 - 2df + e\sqrt{e^2 - 4df}) + f(2af - b(e + \sqrt{e^2 - 4df}))}) + (8\sqrt{2}c(c^2(-e^4 + 4de^2f - 2d^2f^2 + e^3\sqrt{e^2 - 4df} - 2de^2f\sqrt{e^2 - 4df}) + f^2(-2a^2f^2 + 2abf(e - \sqrt{e^2 - 4df})) + b^2(-e^2 + 2df + e\sqrt{e^2 - 4df})) + 2c^2(af(-e^2 + 2df + e\sqrt{e^2 - 4df}) + b(e^3 - 3de^2f - e^2\sqrt{e^2 - 4df} + df\sqrt{e^2 - 4df}))) \operatorname{ArcTanh}\left(\frac{4af + 2c(-e + \sqrt{e^2 - 4df})x + b(-e + \sqrt{e^2 - 4df} + 2fx)}{2\sqrt{2}\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))}\sqrt{a + x(b + cx)}}\right)) / (f\sqrt{c(e^2 - 2df - e\sqrt{e^2 - 4df}) + f(2af + b(-e + \sqrt{e^2 - 4df}))}) \right) / (16c^2f^2\sqrt{e^2 - 4df})$$

Maple [B] time = 0.329, size = 22523, normalized size = 33.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)`

[Out] result too large to display

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")`

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}}}{d + ex + fx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.107 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx$$

Optimal. Leaf size=704

$$\frac{\left((e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + 2c^2d(e^2 - 4df)) \right)}{2\sqrt{2}f^2(e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}}$$

[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])

Rubi [A] time = 11.9496, antiderivative size = 704, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$\frac{\left((e - \sqrt{e^2 - 4df})(ce - bf)(f(be - 2af) + 2c(e^2 - 5df)) - 2f(f(-be(3af + cd) + 4af(af + cd) + 2b^2df) + 2c^2d(e^2 - 4df)) \right)}{2\sqrt{2}f^2(e^2 - 4df)^{3/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
qrt[e^2 - 4*d*f]])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2} dx &= -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{\sqrt{a+bx+cx^2} \left(\frac{1}{2}(3be-4af) + (3ce+bf)x + 4cfx^2 \right)}{d+ex+fx^2} dx}{-e^2+4df} \\
&= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} - \frac{\int \frac{cf(2b^2df+4af(cd+af)-be(cd+3d^2))\sqrt{a+bx+cx^2}}{d+ex+fx^2} dx}{-e^2+4df} \\
&= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^2 \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{f^2} - \frac{\int \frac{2c^3d}{\sqrt{a+bx+cx^2}} dx}{f^2} \\
&= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{(2c^2) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \sqrt{a+bx+cx^2}\right)}{f^2} \\
&= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^{3/2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2} \\
&= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)} - \frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)} + \frac{c^{3/2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2}
\end{aligned}$$

Mathematica [B] time = 6.81996, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]

[Out]
$$\begin{aligned}
&(-2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e - \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) - (2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) \\
&- (3*f*(a + x*(b + c*x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c^2*f*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*f^2) - ((2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f])))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/f \\
&+ (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]])*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - \text{Sqrt}[e^2 - 4*d*f]) - 4*a*c*f*(e - \text{Sqrt}[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f])
\end{aligned}$$

$$\begin{aligned} & [e^2 - 4d*f])) + 4*c*(-e + \text{Sqrt}[e^2 - 4d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f \\ & - e*\text{Sqrt}[e^2 - 4d*f]) + 4*c*f*(a*f - b*(e - \text{Sqrt}[e^2 - 4d*f]))) * \text{ArcTan} \\ & h[(-4*a*f - b*(-e + \text{Sqrt}[e^2 - 4d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4d*f] \\ &))) * x] / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - \\ & 4*d*f] + b*f*\text{Sqrt}[e^2 - 4d*f]]*\text{Sqrt}[a + b*x + c*x^2])) / (f*(16*a*f^2 + 8*b \\ & *f*(-e + \text{Sqrt}[e^2 - 4d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4d*f])^2)) / (16*c*f^2)) \\ &) / ((e^2 - 4d*f)*(a + b*x + c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(((- \\ & 4*c*f*(4*a*f - b*(e - \text{Sqrt}[e^2 - 4d*f])) - 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4d*f] \\ &))) * (b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4d*f])) - 4*c*f*(b*f - c*(e - \text{Sqrt}[e^2 - \\ & 4d*f])) * x) * \text{Sqrt}[a + b*x + c*x^2]) / (8*c*f^2) - ((-2*(b*f - c*(e - \text{Sqrt}[e^2 \\ & - 4d*f])) * (b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4d*f]) - 4*c*f*(3* \\ & a*f - b*(e - \text{Sqrt}[e^2 - 4d*f]))) * \text{ArcTanh}[(b + 2*c*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[a + b \\ & *x + c*x^2])]) / (\text{Sqrt}[c]*f) + (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a* \\ & f^2 - c*e*\text{Sqrt}[e^2 - 4d*f] + b*f*\text{Sqrt}[e^2 - 4d*f]]*(-4*(-e + \text{Sqrt}[e^2 - 4 \\ & *d*f])*(b*f - c*(e - \text{Sqrt}[e^2 - 4d*f])) * (b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e* \\ & \text{Sqrt}[e^2 - 4d*f]) - 4*c*f*(3*a*f - b*(e - \text{Sqrt}[e^2 - 4d*f])))) + 4*f*(2*c* \\ & f*(4*a*f - b*(e - \text{Sqrt}[e^2 - 4d*f]))^2 - (e - \text{Sqrt}[e^2 - 4d*f])*(b*f - c* \\ & (e - \text{Sqrt}[e^2 - 4d*f])) * (b^2*f + 4*a*c*f - 2*b*c*(e - \text{Sqrt}[e^2 - 4d*f]))) \\ &) * \text{ArcTanh}[(-4*a*f - b*(-e + \text{Sqrt}[e^2 - 4d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^ \\ & 2 - 4d*f])) * x] / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqr} \\ & t[e^2 - 4d*f] + b*f*\text{Sqrt}[e^2 - 4d*f]]*\text{Sqrt}[a + b*x + c*x^2])) / (f*(16*a*f \\ & ^2 + 8*b*f*(-e + \text{Sqrt}[e^2 - 4d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4d*f])^2)) / (16 \\ & *c*f^2)) / ((e^2 - 4d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) - (f*(a + x*(b + c* \\ & x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + \text{Sqrt}[e^2 - 4d*f])) + 2*(b*f - c*(e + \text{S} \\ & \text{qrt}[e^2 - 4d*f])) * (-b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4d*f])) - 4*c*f*(b*f - c* \\ & (e + \text{Sqrt}[e^2 - 4d*f])) * x) * \text{Sqrt}[a + b*x + c*x^2]) / (8*c*f^2) - ((-2*(b*f - \\ & c*(e + \text{Sqrt}[e^2 - 4d*f])) * (b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4d \\ & *f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4d*f]))) * \text{ArcTanh}[(b + 2*c*x) / (2*\text{S} \\ & \text{qrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (\text{Sqrt}[c]*f) - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d* \\ & f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4d*f] - b*f*\text{Sqrt}[e^2 - 4d*f]]*(4*(e \\ & + \text{Sqrt}[e^2 - 4d*f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4d*f])) * (b^2*f^2 - 4*c^2*(e^ \\ & 2 - 2*d*f + e*\text{Sqrt}[e^2 - 4d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4d*f] \\ &))) + 4*f*(2*c*f*(4*a*f - b*(e + \text{Sqrt}[e^2 - 4d*f]))^2 - (e + \text{Sqrt}[e^2 - 4d \\ & *f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4d*f])) * (b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e \\ & ^2 - 4d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4d*f]) - (-2*b*f + 2*c* \\ & (e + \text{Sqrt}[e^2 - 4d*f])) * x] / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f \\ & ^2 + c*e*\text{Sqrt}[e^2 - 4d*f] - b*f*\text{Sqrt}[e^2 - 4d*f]]*\text{Sqrt}[a + b*x + c*x^2])) \\ &) / (f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4d*f] \\ &)^2)) / (16*c*f^2)) / ((e^2 - 4d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f*(a \\ & + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4d*f] \\ &))) + 4*c^2*f*x) * \text{Sqrt}[a + b*x + c*x^2]) / (8*c*f^2) - ((-2*\text{Sqrt}[c]*(b^2*f^2 + \\ & 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - \\ & 4d*f]))) * \text{ArcTanh}[(b + 2*c*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / f - (2*\text{S} \\ & \text{qrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4d*f] - b*f* \\ & \text{Sqrt}[e^2 - 4d*f]]*(4*c*(e + \text{Sqrt}[e^2 - 4d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d \\ & \end{aligned}$$

```
*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*
f*(3*b^2*f*(e + Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*
(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))) *ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 -
4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/((e^2 - 4*d*f)*(a +
b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.369, size = 72576, normalized size = 103.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.108 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3} dx$$

Optimal. Leaf size=671

$$\frac{3 \left(2 \left(e - \sqrt{e^2 - 4df} \right) (ce - bf)(2af - be + 2cd) - f \left(4be(3af + cd) - 4a(4af^2 + ce^2) + b^2(-4df + e^2) \right) \right) \tanh^{-1} \left(\frac{\dots}{2\sqrt{\dots}} \right)}{4\sqrt{2} \left(e^2 - 4df \right)^{5/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2)
+ (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) - (3*(2*(2
*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3
*a*f) - b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a
+ b*x + c*x^2])]/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*
e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (3*(2*(2*c*d - b*e + 2*a*
f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3*a*f) - b^2*(e^2
+ 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])
)/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 11.597, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {971, 1013, 1032, 724, 206}

$$\frac{3 \left(-2 \left(e - \sqrt{e^2 - 4df} \right) (ce - bf)(2af - be + 2cd) + 4bef(3af + cd) - 4af(4af^2 + ce^2) + b^2(-f)(4df + e^2) \right) \tanh^{-1} \left(\frac{\dots}{2\sqrt{\dots}} \right)}{4\sqrt{2} \left(e^2 - 4df \right)^{5/2} \sqrt{2af^2 - \sqrt{e^2 - 4df}(ce - bf) - bef - 2cdf + ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2)
+ (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
```

$$\begin{aligned} & *x) * \text{Sqrt}[a + b*x + c*x^2] / (4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b* \\ & e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c* \\ & d - b*e + 2*a*f)*(c*e - b*f)*(e - \text{Sqrt}[e^2 - 4*d*f])) * \text{ArcTanh}[(4*a*f - b*(e \\ & - \text{Sqrt}[e^2 - 4*d*f]) + 2*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f])) * x] / (2*\text{Sqrt}[2]*\text{S} \\ & \text{qrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]] * \text{Sqrt} \\ & [a + b*x + c*x^2])) / (4*\text{Sqrt}[2]*(e^2 - 4*d*f)^{(5/2)} * \text{Sqrt}[c*e^2 - 2*c*d*f - \\ & b*e*f + 2*a*f^2 - (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a* \\ & f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f \\ &)*(c*e - b*f)*(e + \text{Sqrt}[e^2 - 4*d*f])) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4 \\ & *d*f]) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f])) * x] / (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c \\ & *d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]] * \text{Sqrt}[a + b*x + c*x^ \\ & 2])) / (4*\text{Sqrt}[2]*(e^2 - 4*d*f)^{(5/2)} * \text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 \\ & + (c*e - b*f)*\text{Sqrt}[e^2 - 4*d*f]]) \end{aligned}$$

Rule 971

$$\begin{aligned} & \text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_.)} * ((d_.) + (e_.)*(x_) + (f_.)*(x \\ & _)^2)^{(q_.)}, x_Symbol] := \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^{(p + 1)}*(d + e \\ & *x + f*x^2)^q] / ((b^2 - 4*a*c)*(p + 1)), x] - \text{Dist}[1/((b^2 - 4*a*c)*(p + 1)) \\ & , \text{Int}[(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^{(q - 1)} * \text{Simp}[2*c*d*(2*p + \\ & 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2 \\ & , x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[\\ & e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 0] \&\& !\text{IGtQ}[q, 0] \end{aligned}$$

Rule 1013

$$\begin{aligned} & \text{Int}[(g_.) + (h_.)*(x_)] * ((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)} * ((d_.) + (e \\ & _)*(x_) + (f_.)*(x_)^2)^{(q_.)}, x_Symbol] := \text{Simp}[(g*b - 2*a*h - (b*h - 2*g \\ & *c)*x)*(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^q] / ((b^2 - 4*a*c)*(p + 1 \\ &)), x] - \text{Dist}[1/((b^2 - 4*a*c)*(p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)}*(d + \\ & e*x + f*x^2)^{(q - 1)} * \text{Simp}[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) + \\ & (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(\\ & 2*p + 2*q + 3)*x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ} \\ & [b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 0] \end{aligned}$$

Rule 1032

$$\begin{aligned} & \text{Int}[(g_.) + (h_.)*(x_)] / (((a_.) + (b_.)*(x_) + (c_.)*(x_)^2) * \text{Sqrt}[(d_.) + (\\ & e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dis} \\ & \text{t}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], \\ & x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x \\ & ^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \\ & \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[b^2 - 4*a*c] \end{aligned}$$

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx = -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{\int \frac{\left(\frac{3}{2}(be - 4af) + 3(ce - bf)x\right)\sqrt{a + bx + cx^2}}{(d + ex + fx^2)^2} dx}{2(e^2 - 4df)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2)x)\sqrt{a + bx + cx^2}}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2)x)\sqrt{a + bx + cx^2}}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2)x)\sqrt{a + bx + cx^2}}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

$$= -\frac{(e + 2fx)(a + bx + cx^2)^{3/2}}{2(e^2 - 4df)(d + ex + fx^2)^2} + \frac{3(4cde + 4aef - b(e^2 + 4df) + 2(ce^2 - 2bef + 4af^2)x)\sqrt{a + bx + cx^2}}{4(e^2 - 4df)^2(d + ex + fx^2)}$$

Mathematica [B] time = 7.29595, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]

[Out]
$$\begin{aligned} & (-2f^2(a + x(b + cx))^{\frac{3}{2}})/((e^2 - 4df)^{\frac{3}{2}}(e - \sqrt{e^2 - 4df} \\ & + 2fx)^2) + (6f^2(a + x(b + cx))^{\frac{3}{2}})/((e^2 - 4df)^2(e - \sqrt{e^2 - 4df} \\ & + 2fx)) + (2f^2(a + x(b + cx))^{\frac{3}{2}})/((e^2 - 4df)^{\frac{3}{2}}(e + \sqrt{e^2 - 4df} \\ & + 2fx)^2) + (6f^2(a + x(b + cx))^{\frac{3}{2}})/((e^2 - 4df)^2(e + \sqrt{e^2 - 4df} \\ & + 2fx)) + (9f^2(a + x(b + cx))^{\frac{3}{2}})((-4b^2cf - 2c^2(bf + 2c(-e + \sqrt{e^2 - 4df})) - 4c^2fx) \sqrt{a + bx + cx^2})/(8c^2f^2) - ((2\sqrt{c}(b^2f^2 + 4c^2(e^2 - 2df - e\sqrt{e^2 - 4df})) + 4c^2f(a^2 - b(e - \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(b + 2cx)/(2\sqrt{c}\sqrt{a + bx + cx^2})])/f + (2\sqrt{2}\sqrt{c}e^2 - 2c^2df - b^2ef + 2a^2f^2 - c^2e\sqrt{e^2 - 4df} + b^2f\sqrt{e^2 - 4df}) * (4c^2f(8a^2bf^2 - 3b^2f(e - \sqrt{e^2 - 4df})) - 4a^2c^2f(e - \sqrt{e^2 - 4df})) + 4b^2c(e^2 - 2df - e\sqrt{e^2 - 4df})) + 4c^2(-e + \sqrt{e^2 - 4df}) * (b^2f^2 + 4c^2(e^2 - 2df - e\sqrt{e^2 - 4df})) + 4c^2f(a^2 - b(e - \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(4a^2f - b(-e + \sqrt{e^2 - 4df})) - (2b^2f + 2c^2(-e + \sqrt{e^2 - 4df}))x]/(2\sqrt{2}\sqrt{c}e^2 - 2c^2df - b^2ef + 2a^2f^2 - c^2e\sqrt{e^2 - 4df} + b^2f\sqrt{e^2 - 4df}) \sqrt{a + bx + cx^2}))/ (f(16a^2f^2 + 8b^2f(-e + \sqrt{e^2 - 4df})) + 4c^2(-e + \sqrt{e^2 - 4df})^2))/ (16c^2f^2))/ ((e^2 - 4df)^2(a + bx + cx^2)^{\frac{3}{2}}) - (3f^2(a + x(b + cx))^{\frac{3}{2}})((-4c^2f(4a^2f - b(e - \sqrt{e^2 - 4df})) - 2(b^2f - c(e - \sqrt{e^2 - 4df})))(b^2f + 2c^2(-e + \sqrt{e^2 - 4df})) - 4c^2f(b^2f - c(e - \sqrt{e^2 - 4df}))x) \sqrt{a + bx + cx^2})/(8c^2f^2) - ((-2(b^2f - c(e - \sqrt{e^2 - 4df})))(b^2f^2 - 4c^2(e^2 - 2df - e\sqrt{e^2 - 4df})) - 4c^2f(3a^2f - b(e - \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(b + 2cx)/(2\sqrt{c}\sqrt{a + bx + cx^2})])/(\sqrt{c}f) + (2\sqrt{2}\sqrt{c}e^2 - 2c^2df - b^2ef + 2a^2f^2 - c^2e\sqrt{e^2 - 4df} + b^2f\sqrt{e^2 - 4df}) * (-4(-e + \sqrt{e^2 - 4df}))(b^2f - c(e - \sqrt{e^2 - 4df})) * (b^2f^2 - 4c^2(e^2 - 2df - e\sqrt{e^2 - 4df})) - 4c^2f(3a^2f - b(e - \sqrt{e^2 - 4df}))) + 4f^2(2c^2f(4a^2f - b(e - \sqrt{e^2 - 4df}))^2 - (e - \sqrt{e^2 - 4df})(b^2f - c(e - \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(4a^2f - b(-e + \sqrt{e^2 - 4df})) - (2b^2f + 2c^2(-e + \sqrt{e^2 - 4df}))x]/(2\sqrt{2}\sqrt{c}e^2 - 2c^2df - b^2ef + 2a^2f^2 - c^2e\sqrt{e^2 - 4df} + b^2f\sqrt{e^2 - 4df}) \sqrt{a + bx + cx^2}))/ (f(16a^2f^2 + 8b^2f(-e + \sqrt{e^2 - 4df})) + 4c^2(-e + \sqrt{e^2 - 4df})^2))/ (16c^2f^2))/ ((e^2 - 4df)^{\frac{5}{2}}(a + bx + cx^2)^{\frac{3}{2}}) + (3f^2(a + x(b + cx))^{\frac{3}{2}})((-2b^2f - 2c^2(-e + \sqrt{e^2 - 4df})) * (a + bx + cx^2)^{\frac{3}{2}})/((-4a^2f^2 - 2b^2f(-e + \sqrt{e^2 - 4df})) - c^2(-e + \sqrt{e^2 - 4df})^2) * (-e + \sqrt{e^2 - 4df}) - 2fx)) + (((-4c^2f(b^2f + 4a^2cf - 2b^2c(e - \sqrt{e^2 - 4df})) - 4c^2(b^2f - c(e - \sqrt{e^2 - 4df}))) * (b^2f + 2c^2(-e + \sqrt{e^2 - 4df})) - 8c^2f(b^2f - c(e - \sqrt{e^2 - 4df}))x) \sqrt{a + bx + cx^2})/(8c^2f^2) - ((16c^{\frac{3}{2}}(b^2f - c(e - \sqrt{e^2 - 4df}))) * (c^2(e^2 - 2df - e\sqrt{e^2 - 4df})) + f(2a^2f - b(e - \sqrt{e^2 - 4df}))) \operatorname{ArcTanh}[(b + 2$$

$$\begin{aligned}
& c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]))/f + (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f \\
& f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f*\text{Sqrt}[e^2 - 4*d*f]]*(32*c^ \\
& 2*(-e + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d* \\
& f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))) + 16*c*f* \\
& (b^2*f + 4*a*c*f - 2*b*c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*\text{Sqrt}[\\
& e^2 - 4*d*f]) + f*(2*a*f - b*(e - \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(-4*a*f - b \\
& *(-e + \text{Sqrt}[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sq} \\
& \text{rt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*\text{Sqrt}[e^2 - 4*d*f] + b*f* \\
& \text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2))]/(f*(16*a*f^2 + 8*b*f*(-e + \text{Sqrt} \\
& [e^2 - 4*d*f]) + 4*c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2)/(-4*a*f^2 - \\
& 2*b*f*(-e + \text{Sqrt}[e^2 - 4*d*f]) - c*(-e + \text{Sqrt}[e^2 - 4*d*f])^2))/((e^2 - 4* \\
& d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((4*c \\
& *f*(-4*a*f + b*(e + \text{Sqrt}[e^2 - 4*d*f])) + 2*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f] \\
&))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + \text{Sqrt}[e^2 - \\
& 4*d*f]))*x)*\text{Sqrt}[a + b*x + c*x^2)]/(8*c*f^2) - ((-2*(b*f - c*(e + \text{Sqrt}[e^2 \\
& - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) - 4*c*f*(3* \\
& a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b \\
& *x + c*x^2))]/(\text{Sqrt}[c]*f) - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a* \\
& f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*(4*(e + \text{Sqrt}[e^2 - 4*d \\
& *f])*(b*f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*\text{Sq} \\
& \text{rt}[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) + 4*f*(2*c*f* \\
& (4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))^2 - (e + \text{Sqrt}[e^2 - 4*d*f])*(b*f - c*(e \\
& + \text{Sqrt}[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \\
& \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - \\
& 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^ \\
& 2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*\text{Sqrt}[a + b*x + c*x^2))]/(f*(16*a*f^2 - \\
& 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2 \\
&)))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x) \\
&)^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*c^2*f*x \\
&)*\text{Sqrt}[a + b*x + c*x^2)]/(8*c*f^2) - ((-2*\text{Sqrt}[c]*(b^2*f^2 + 4*c^2*(e^2 - 2 \\
& *d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcT} \\
& \text{anh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2))]/f - (2*\text{Sqrt}[2]*\text{Sqrt}[c*e \\
& ^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d \\
& *f]]*(4*c*(e + \text{Sqrt}[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*\text{Sqrt}[e^ \\
& 2 - 4*d*f]) + 4*c*f*(a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))) + 4*c*f*(3*b^2*f*(e \\
& + \text{Sqrt}[e^2 - 4*d*f]) + 4*a*c*f*(e + \text{Sqrt}[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*(\\
& e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]))) * \text{ArcTanh}[(4*a*f - b*(e + \text{Sqrt}[e^2 - 4* \\
& d*f]) - (-2*b*f + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f]))*x)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*e^2 - 2 \\
& *c*d*f - b*e*f + 2*a*f^2 + c*e*\text{Sqrt}[e^2 - 4*d*f] - b*f*\text{Sqrt}[e^2 - 4*d*f]]*S \\
& \text{qrt}[a + b*x + c*x^2))]/(f*(16*a*f^2 - 8*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + 4*c* \\
& (e + \text{Sqrt}[e^2 - 4*d*f])^2)))/(16*c*f^2))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2 \\
&)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + \text{Sqrt}[e^2 - 4* \\
& d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) \\
& - c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*(e + \text{Sqrt}[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b \\
& *f - c*(e + \text{Sqrt}[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])) + 4*
\end{aligned}$$

```

c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + Sqrt[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c
*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2]]/(8*c*f^2) - ((16*c^(3/2)
)*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2]))]/f - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-32*c^2*(e + Sqrt[e^2 -
4*d*f]))*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))) + 16*c*f*(b^2*f + 4*a*c*f -
2*b*c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2]))]/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(
e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4
*d*f]) - c*(e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x
^2)^(3/2))

```

Maple [B] time = 0.36, size = 178044, normalized size = 265.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.109 \quad \int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=717

$$\frac{\sqrt{a+bx+cx^2}(-504bc^2f(22a^2f^2+70abef+25b^2(df+e^2))+96c^3(128a^2ef^2+275abf(df+e^2)+50b^2(6def+e^3)))}{7680c^6}$$

[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 + d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))

Rubi [A] time = 2.70964, antiderivative size = 717, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}(-504bc^2f(22a^2f^2+70abef+25b^2(df+e^2))+96c^3(128a^2ef^2+275abf(df+e^2)+50b^2(6def+e^3)))}{7680c^6}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2], x]

[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d


```

*f) + 50*b^2*(e^3 + 6*d*e*f))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))

```

Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

Rule 640

```

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

```

Rule 621

```

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^3}{\sqrt{a + bx + cx^2}} dx &= \frac{f^3 x^5 \sqrt{a + bx + cx^2}}{6c} + \frac{\int \frac{6cd^3 + 18cd^2 ex + 18cd(e^2 + df)x^2 + 6ce(e^2 + 6df)x^3 - f(5af^2 - 18c(e^2 + df))x^4 + \frac{1}{2}f^2(36ce - 11bf)x^5}{\sqrt{a + bx + cx^2}} dx}{6c} \\
&= \frac{f^2(36ce - 11bf)x^4 \sqrt{a + bx + cx^2}}{60c^2} + \frac{f^3 x^5 \sqrt{a + bx + cx^2}}{6c} + \frac{\int \frac{30c^2 d^3 + 90c^2 d^2 ex + 90c^2 d(e^2 + df)x^2 - 2(36acef^2 - 11bf^2 c^2)x^3 + f^2(36ce - 11bf)x^4}{\sqrt{a + bx + cx^2}} dx}{60c^2} \\
&= \frac{f(99b^2 f^2 - 4cf(81be + 25af) + 360c^2(e^2 + df))x^3 \sqrt{a + bx + cx^2}}{480c^3} + \frac{f^2(36ce - 11bf)x^4 \sqrt{a + bx + cx^2}}{60c^2} \\
&= -\frac{(231b^3 f^3 - 36bcf^2(21be + 13af) - 320c^3(e^3 + 6def) + 24c^2 f(32aef + 35b(e^2 + df)))x^2 \sqrt{a + bx + cx^2}}{960c^4} \\
&= \frac{(1155b^4 f^3 - 252b^2 c f^2(15be + 14af) + 5760c^4 d(e^2 + df) + 24c^2 f(322abef + 50a^2 f^2 + 175b^2(e^2 + df)))x \sqrt{a + bx + cx^2}}{3840c^5} \\
&= \frac{(23040c^5 d^2 e - 3465b^5 f^3 + 420b^3 c f^2(27be + 34af) - 504bc^2 f(70abef + 22a^2 f^2 + 25b^2(e^2 + df)))\sqrt{a + bx + cx^2}}{3840c^5} \\
&= \frac{(23040c^5 d^2 e - 3465b^5 f^3 + 420b^3 c f^2(27be + 34af) - 504bc^2 f(70abef + 22a^2 f^2 + 25b^2(e^2 + df)))\sqrt{a + bx + cx^2}}{3840c^5} \\
&= \frac{(23040c^5 d^2 e - 3465b^5 f^3 + 420b^3 c f^2(27be + 34af) - 504bc^2 f(70abef + 22a^2 f^2 + 25b^2(e^2 + df)))\sqrt{a + bx + cx^2}}{3840c^5}
\end{aligned}$$

Mathematica [A] time = 1.40472, size = 615, normalized size = 0.86

$$\frac{\sqrt{a + x(b + cx)} \left(-168bc^2 f(66a^2 f^2 + 42abf(5e + fx) + b^2(75df + 75e^2 + 45efx + 11f^2 x^2)) + 48c^3(2a^2 f^2(128e + 25fx) + 128c^2 d^2(2e + fx) + 15d^2 x(6e^2 + 8efx + 3f^2 x^2)) + x^2(20e^3 + 45e^2 f x + 36ef^2 x^2 + 10f^3 x^3) + 48c^3(2a^2 f^2(128e + 25fx) + b^2(100e^3 + 175e^2 f x + 6ef(100d + 21fx^2) + f^2 x(175d + 33fx^2)) + 2abf(275e^2 + 161efx + f(275d + 39fx^2))) - 64c^4(a(80e^3 + 135e^2 f x + 96e^2 f^2 x^2) + b^2(100e^3 + 175e^2 f x + 6ef(100d + 21fx^2) + f^2 x(175d + 33fx^2))) \right)}{3840c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2], x]

[Out] (Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68*a*f + 11*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(75*e^2 + 75*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 15*d*x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 175*e^2*f*x + 6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*f*(275*e^2 + 161*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*e^2*f*x + 96*e^2*f^2*x^2) + b^2*(100*e^3 + 175*e^2*f*x + 6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)))

$$f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d*(18*e^2 + 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 + 22*f^3*x^3)))/((7680*c^6) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])))/(1024*c^(13/2))$$

Maple [B] time = 0.073, size = 1930, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]
$$\begin{aligned} & -45/16*b^2/c^{(7/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^{2*f}+55/16*b/c^3*a*(c*x^2+b*x+a)^{(1/2)}*d*f^2+55/16*b/c^3*a*(c*x^2+b*x+a)^{(1/2)}*e^{2*f} \\ & -9/8*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}*d*f^2-45/16*e*f^2*b/c^{(7/2)}*a^2*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}-7/8*b/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}*e^{2*f}-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^{(1/2)}+63/80*e*f^2*b^2/c^3*x^2*(c*x^2+b*x+a)^{(1/2)}-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^{(1/2)}+105/32*e*f^2*b^3/c^{(9/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+161/80*e*f^2*b/c^3*a*x*(c*x^2+b*x+a)^{(1/2)}-5/2*b/c^2*x*(c*x^2+b*x+a)^{(1/2)}*d*e*f+9/2*b/c^{(5/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*e*f+35/32*b^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}*e^{2*f}+2*x^2/c*(c*x^2+b*x+a)^{(1/2)}*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}*d*e*f-15/8*b^3/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*e*f-4*a/c^2*(c*x^2+b*x+a)^{(1/2)}*d*e*f-45/16*b^2/c^{(7/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^{(1/2)}+3/4*x^3/c*(c*x^2+b*x+a)^{(1/2)}*d*f^2-9/8*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}*e^{2*f}-147/160*f^3*b^2/c^4*a*x*(c*x^2+b*x+a)^{(1/2)}+39/80*f^3*b/c^3*a*x^2*(c*x^2+b*x+a)^{(1/2)}+35/32*b^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}*d*f^2+d^3*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-147/32*e*f^2*b^2/c^4*a*(c*x^2+b*x+a)^{(1/2)}+1/3*x^2/c*(c*x^2+b*x+a)^{(1/2)}*e^3+5/8*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}*e^3-5/16*b^3/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^3-2/3*a/c^2*(c*x^2+b*x+a)^{(1/2)}*e^3-231/512*f^3*b^5/c^6*(c*x^2+b*x+a)^{(1/2)}+231/1024*f^3*b^6/c^{(13/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-5/16*f^3*a^3/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+3*d^2*e/c*(c*x^2+b*x+a)^{(1/2)}+9/8*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*d^2+9/8*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*e^2+3/2*x/c*(c*x^2+b*x+a)$$

$$\begin{aligned} & \frac{1}{2} * f * d^2 + \frac{33}{160} * f^3 * b^2 / c^3 * x^3 * (c * x^2 + b * x + a)^{(1/2)} + \frac{8}{5} * e * f^2 * a^2 / c^3 * (c * x^2 + b * x + a)^{(1/2)} \\ & + \frac{189}{128} * e * f^2 * b^4 / c^5 * (c * x^2 + b * x + a)^{(1/2)} - \frac{189}{256} * e * f^2 * b^5 / c^{(11/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) \\ & - \frac{3}{2} * a / c^{(3/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * d * e^2 - \frac{3}{2} * d^2 * e * b / c^{(3/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) \\ & - \frac{3}{2} * a / c^{(3/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * f * d^2 + \frac{3}{2} * x / c * (c * x^2 + b * x + a)^{(1/2)} * d * e^2 - \frac{9}{4} * b / c^2 * (c * x^2 + b * x + a)^{(1/2)} * f * d^2 - \frac{9}{4} * b / c^2 * (c * x^2 + b * x + a)^{(1/2)} * d * e^2 + \frac{3}{4} * x^3 / c * (c * x^2 + b * x + a)^{(1/2)} * e^2 * f - \frac{105}{64} * b^3 / c^4 * (c * x^2 + b * x + a)^{(1/2)} * d * f^2 - \frac{105}{64} * b^3 / c^4 * (c * x^2 + b * x + a)^{(1/2)} * e^2 * f + \frac{105}{128} * b^4 / c^{(9/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * d * f^2 + \frac{105}{128} * b^4 / c^{(9/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * e^2 * f + \frac{9}{8} * a^2 / c^{(5/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * d * f^2 + \frac{9}{8} * a^2 / c^{(5/2)} * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * e^2 * f - \frac{5}{12} * b / c^2 * x * (c * x^2 + b * x + a)^{(1/2)} * e^3 + \frac{3}{4} * b / c^{(5/2)} * a * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) * e^3 - \frac{77}{320} * f^3 * b^3 / c^4 * x^2 * (c * x^2 + b * x + a)^{(1/2)} + \frac{77}{256} * f^3 * b^4 / c^5 * x * (c * x^2 + b * x + a)^{(1/2)} - \frac{315}{256} * f^3 * b^4 / c^{(11/2)} * a * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) + \frac{119}{64} * f^3 * b^3 / c^5 * a * (c * x^2 + b * x + a)^{(1/2)} + \frac{105}{64} * f^3 * b^2 / c^{(9/2)} * a^2 * \ln\left(\frac{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}{(1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}}\right) - \frac{231}{160} * f^3 * b / c^4 * a^2 * (c * x^2 + b * x + a)^{(1/2)} - \frac{5}{24} * f^3 * a / c^2 * x^3 * (c * x^2 + b * x + a)^{(1/2)} + \frac{5}{16} * f^3 * a^2 / c^3 * x * (c * x^2 + b * x + a)^{(1/2)} - \frac{11}{60} * f^3 * b / c^2 * x^4 * (c * x^2 + b * x + a)^{(1/2)} + \frac{1}{6} * f^3 * x^5 * (c * x^2 + b * x + a)^{(1/2)} / c \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 7.39034, size = 3661, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

```
[Out] [-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)*
d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*a^
2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c
^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b^2*c
^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 120*a*b
^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^5 + 230
40*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*b
*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*d
- 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
- 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(1
6*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16*
a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600*
b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(35
*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*d
^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a
))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*
a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c +
1680*a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 4
8*a^2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(
3*b^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 -
120*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f^3*x^5 + 23
040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*
b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*d
- 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
- 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(
16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16
*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600
*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(3
5*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*
d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x +
a))/c^7]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex + fx^2)^3}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((d + e*x + f*x**2)**3/sqrt(a + b*x + c*x**2), x)

Giac [A] time = 1.36933, size = 1112, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out]
$$\frac{1}{7680} \sqrt{c x^2 + b x + a} (2 (4 (2 (8 (10 f^3 x / c - (11 b c^4 f^3 - 36 c^5 f^2 e) / c^6) x + (360 c^5 d f^2 + 99 b^2 c^3 f^3 - 100 a c^4 f^3 - 324 b c^4 f^2 e + 360 c^5 f e^2) / c^6) x - (840 b c^4 d f^2 + 231 b^3 c^2 f^3 - 468 a b c^3 f^3 - 1920 c^5 d f e - 756 b^2 c^3 f^2 e + 768 a c^4 f^2 e + 840 b c^4 f e^2 - 320 c^5 e^3) / c^6) x + (5760 c^5 d^2 f + 4200 b^2 c^3 d f^2 - 4320 a c^4 d f^2 + 1155 b^4 c f^3 - 3528 a b^2 c^2 f^3 + 1200 a^2 c^3 f^3 - 9600 b c^4 d f e - 3780 b^3 c^2 f^2 e + 7728 a b c^3 f^2 e + 5760 c^5 d e^2 + 4200 b^2 c^3 f e^2 - 4320 a c^4 f e^2 - 1600 b c^4 e^3) / c^6) x - (17280 b c^4 d^2 f + 12600 b^3 c^2 d f^2 - 26400 a b c^3 d f^2 + 3465 b^5 f^3 - 14280 a b^3 c f^3 + 11088 a^2 b c^2 f^3 - 23040 c^5 d^2 e - 28800 b^2 c^3 d f e + 30720 a c^4 d f e - 11340 b^4 c f^2 e + 35280 a b^2 c^2 f^2 e - 12288 a^2 c^3 f^2 e + 17280 b c^4 d e^2 + 12600 b^3 c^2 f e^2 - 26400 a b c^3 f e^2 - 4800 b^2 c^3 e^3 + 5120 a c^4 e^3) / c^6) - \frac{1}{1024} (1024 c^6 d^3 + 1152 b^2 c^4 d^2 f - 1536 a c^5 d^2 f + 840 b^4 c^2 d f^2 - 2880 a b^2 c^3 d f^2 + 1152 a^2 c^4 d f^2 + 231 b^6 f^3 - 1260 a b^4 c f^3 + 1680 a^2 b^2 c^2 f^3 - 320 a^3 c^3 f^3 - 1536 b c^5 d^2 e - 1920 b^3 c^3 d f e + 4608 a b c^4 d f e - 756 b^5 c f^2 e + 3360 a b^3 c^2 f^2 e - 2880 a^2 b c^3 f^2 e + 1152 b^2 c^4 d e^2 - 1536 a c^5 d e^2 + 840 b^4 c^2 f e^2 - 2880 a b^2 c^3 f e^2 + 1152 a^2 c^4 f e^2 - 320 b^3 c^3 e^3 + 768 a b c^4 e^3) \log(\text{abs}(-2(\sqrt{c} x - \sqrt{c x^2 + b x + a}) \sqrt{c} - b)) / c^{13/2})$$

$$3.110 \quad \int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bde)+35b^4f\right)}{128c^{9/2}}$$

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rubi [A] time = 0.62621, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bde)+35b^4f\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2], x]

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx &= \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{4cd^2 + 8cdex - (3af^2 - 4c(e^2 + 2df))x^2 + \frac{1}{2}f(16ce - 7bf)x^3}{\sqrt{a + bx + cx^2}} dx}{4c} \\
&= \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} + \frac{f^2 x^3 \sqrt{a + bx + cx^2}}{4c} + \frac{\int \frac{12c^2 d^2 + (24c^2 de - 16acef + 7abf^2)x + \frac{1}{4}(35b^2 f^2 - 4c^2 d^2)}{\sqrt{a + bx + cx^2}} dx}{12c^2} \\
&= \frac{(35b^2 f^2 - 4cf(20be + 9af) + 48c^2(e^2 + 2df))x \sqrt{a + bx + cx^2}}{96c^3} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2} \\
&= \frac{(384c^3 de - 105b^3 f^2 + 20bcf(12be + 11af) - 16c^2(16aef + 9b(e^2 + 2df))) \sqrt{a + bx + cx^2}}{192c^4} + \frac{f(16ce - 7bf)x^2 \sqrt{a + bx + cx^2}}{24c^2}
\end{aligned}$$

Mathematica [A] time = 0.536985, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2(a^2f^2+4abef+b^2(2df+e^2))-40b^2cf(3af+2be)-64c^3(a(2df+e^2)+2bde)+35b^4\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2], x]

[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f + 7*b*f*x) + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) - 8*c^2*(a*f*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2)))/(192*c^4) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))

Maple [B] time = 0.062, size = 706, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]
$$-5/6*e*f*b/c^2*x*(c*x^2+b*x+a)^{(1/2)}+3/2*e*f*b/c^{(5/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+3/4*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f+2*d*e/c*(c*x^2+b*x+a)^{(1/2)}+1/2*x/c*(c*x^2+b*x+a)^{(1/2)}*e^2-3/4*b/c^2*(c*x^2+b*x+a)^{(1/2)}*e^2+3/8*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2-35/64*f^2*b^3/c^4*(c*x^2+b*x+a)^{(1/2)}+35/128*f^2*b^4/c^{(9/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+3/8*f^2*a^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/2*a/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^2+x/c*(c*x^2+b*x+a)^{(1/2)}*d*f-3/2*b/c^2*(c*x^2+b*x+a)^{(1/2)}*d*f+d^2*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-a/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f-d*e*b/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-7/24*f^2*b/c^2*x^2*(c*x^2+b*x+a)^{(1/2)}+35/96*f^2*b^2/c^3*x*(c*x^2+b*x+a)^{(1/2)}-15/16*f^2*b^2/c^{(7/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+55/48*f^2*b/c^3*a*(c*x^2+b*x+a)^{(1/2)}-3/8*f^2*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}+2/3*e*f*x^2/c*(c*x^2+b*x+a)^{(1/2)}+5/4*e*f*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}-5/8*e*f*b^3/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/3*e*f*a/c^2*(c*x^2+b*x+a)^{(1/2)}+1/4*f^2*x^3*(c*x^2+b*x+a)^{(1/2)}/c$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 2.84519, size = 1457, normalized size = 4.61

$$\left[\frac{3(128c^4d^2 - 128bc^3de + 16(3b^2c^2 - 4ac^3)e^2 + (35b^4 - 120ab^2c + 48a^2c^2)f^2 + 16(2(3b^2c^2 - 4ac^3)d - (5b^3c - 12ab^2c^2)))}{(c^2x^2 + b^2x + a^2)^{3/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(1/2)}, x, \text{algorithm}="fricas")$

```
[Out] [1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.39602, size = 410, normalized size = 1.3

$$\frac{1}{192} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(\frac{6f^2x}{c} - \frac{7bc^2f^2 - 16c^3fe}{c^4} \right) x + \frac{96c^3df + 35b^2cf^2 - 36ac^2f^2 - 80bc^2fe + 48c^3e^2}{c^4} \right) x - \frac{288bc^2d^2 + 288bc^2de + 288c^3e^2}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e
```

$$\begin{aligned}
&^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 2 \\
&40*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 9 \\
&6*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f \\
&^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a \\
&*c^3*e^2)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(9 \\
&/2)
\end{aligned}$$

$$3.111 \quad \int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

[Out] $((4*c*e - 3*b*f)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2) + (f*x*\text{Sqrt}[a + b*x + c*x^2])/ (2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*c^{(5/2)})$

Rubi [A] time = 0.110769, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2], x]

[Out] $((4*c*e - 3*b*f)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2) + (f*x*\text{Sqrt}[a + b*x + c*x^2])/ (2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*c^{(5/2)})$

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q-1)*(a + b*x + c*x^2)^(p+1))/(c*(q+2*p+1)), x] + Dist[1/(c*(q+2*p+1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q+2*p+1)*Pq - a*e*(q-1)*x^(q-2) - b*e*(q+p)*x^(q-1) - c*e*(q+2*p+1)*x^q, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p+1))/(2*c*(p+1)), x] + Dist[(2*c*d - b

*e)/(2*c), Int[(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}} dx &= \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2cd - af + \frac{1}{2}(4ce - 3bf)x}{\sqrt{a + bx + cx^2}} dx}{2c} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{4c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \text{Subst}\left(\int \frac{1}{4c - x^2} dx\right)}{2c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{(8c^2d + 3b^2f - 4c(be + af)) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.150978, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right) (-4c(af + be) + 3b^2f + 8c^2d)}{8c^{5/2}} + \frac{\sqrt{a + x(b + cx)}(-3bf + 4ce + 2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2], x]

[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)]/(4*c^2) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])

]])/(8*c^(5/2))

Maple [A] time = 0.062, size = 185, normalized size = 1.6

$$\frac{fx}{2c}\sqrt{cx^2+bx+a}-\frac{3bf}{4c^2}\sqrt{cx^2+bx+a}+\frac{3b^2f}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}}-\frac{af}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)

[Out] $\frac{1}{2}f*x*(c*x^2+b*x+a)^{(1/2)}/c-3/4*f*b/c^2*(c*x^2+b*x+a)^{(1/2)}+3/8*f*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/2*f*a/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+e/c*(c*x^2+b*x+a)^{(1/2)}-1/2*e*b/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+d*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.92911, size = 549, normalized size = 4.73

$$\left[\frac{(8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{c} \log\left(-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac\right) - 4(2c^2fx + 4c^2e)}{16c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A] time = 1.29089, size = 132, normalized size = 1.14

$$\frac{1}{4} \sqrt{cx^2 + bx + a} \left(\frac{2fx}{c} - \frac{3bf - 4ce}{c^2} \right) - \frac{(8c^2d + 3b^2f - 4acf - 4bce) \log \left(\left| -2 \left(\sqrt{cx} - \sqrt{cx^2 + bx + a} \right) \sqrt{c} - b \right| \right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```


$$3.112 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=374

$$\frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))
```

Rubi [A] time = 0.57883, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {983, 724, 206}

$$\frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af+2x(bf-c(\sqrt{e^2-4df}+e))-b(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af+2x(bf-c(e-\sqrt{e^2-4df}))-b(e-\sqrt{e^2-4df})}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f])*Sqrt[a + b*x + c*x^2]))/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))
```

Rule 983

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx = \frac{(2f) \int \frac{1}{(e-\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}} - \frac{(2f) \int \frac{1}{(e+\sqrt{e^2-4df}+2fx)\sqrt{a+bx+cx^2}} dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f) \text{Subst}\left(\int \frac{1}{16af^2-8bf(e-\sqrt{e^2-4df})+4c(e-\sqrt{e^2-4df})^2-x^2} dx, x, \frac{4af-b(e-\sqrt{e^2-4df})-(-2bf+2c)}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af-b(e-\sqrt{e^2-4df})+2(bf-c(e-\sqrt{e^2-4df}))x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f \tanh^{-1}\left(\frac{4af-b(e+\sqrt{e^2-4df})-(-2bf+2c)}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [A] time = 1.50838, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}f \left(\frac{\tanh^{-1} \left(\frac{4af-b(\sqrt{e^2-4df}+e-2fx)-2cx(\sqrt{e^2-4df}+e)}{2\sqrt{2}\sqrt{a+cx}\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-4df}-2df+e^2)}} \right) - \frac{\tanh^{-1} \left(\frac{4af+b(\sqrt{e^2-4df}-e+2fx)+2cx(\sqrt{e^2-4df}-e)}{2\sqrt{2}\sqrt{a+cx}\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{f(2af+b(\sqrt{e^2-4df}+b(-e))+c(-e\sqrt{e^2-4df}-2df+e^2)}} \right)}{\sqrt{e^2-4df}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]

[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] - ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f]

Maple [B] time = 0.359, size = 761, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)

[Out] $-1/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/(-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c$

$$2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-(-4*d*f+e^2)^{(1/2)}*b*f +(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*2^{(1/2)}*((-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 45.0179, size = 22873, normalized size = 61.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] $\frac{1}{4}\sqrt{2}\sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)}}{(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b$

$$\begin{aligned}
&^2 - 6*a*c)*d*e^2)*f)) * \log((2*(b^2*d - a*b*e)*f^2 + \sqrt{2}*(c^2*d*e^3 - 4* \\
&a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^ \\
&2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^ \\
&2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3* \\
&a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)* \\
&d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^ \\
&3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2) \\
&*d^2*e^3)*f) * \sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^ \\
&3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d \\
&^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b \\
&*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a \\
&^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b* \\
&c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c \\
&)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2 \\
&*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a \\
&*b^2*c - 2*a^2*c^2)*d*e^4)*f)) * \sqrt{c*x^2 + b*x + a} * \sqrt{(c*e^2 + 2*a*f^2 \\
&- (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4* \\
&a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a \\
&*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f) * \sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d \\
&^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (\\
&b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c) \\
&*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - \\
&a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d \\
&^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + \\
&2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - \\
&4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b \\
&*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^ \\
&3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 \\
&- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d \\
&*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8* \\
&a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4* \\
&a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a \\
&*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + \\
&a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a* \\
&b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c \\
&*e^4)*f)*x) * \sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3 \\
&*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^ \\
&2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b* \\
&e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^ \\
&2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c \\
&^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c) \\
&*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2* \\
&b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a* \\
&b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*\sqrt{2}*\sqrt{(c*e^2 + 2*a*f^2 - (2* \\
&c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
\end{aligned}$$

$$\begin{aligned}
& e + a^2e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 \\
& - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*\log((2*(b^2*d - a*b*e)*f^2 - \sqrt{2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f))*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*\sqrt{(c*x^2 + b*x + a)*\sqrt{(c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*\sqrt{(c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d^2*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/((c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
\end{aligned}$$

$$\begin{aligned}
& + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - \\
& (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*\sqrt{(c^2*e^2 - 2*b*c*e*f + \\
& b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - \\
& 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2 \\
& *b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d \\
& ^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c \\
& ^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2* \\
& c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - \\
& 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + \\
& (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4 \\
& *sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 \\
& + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 \\
& - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 \\
& 2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + \\
& a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a \\
& ^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c \\
& + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)* \\
& f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a* \\
& b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^ \\
& 3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a \\
& *c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4 \\
&)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2* \\
& e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 \\
& - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b \\
& *d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + \\
& a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - \\
& a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b \\
& ^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3 \\
& *e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d \\
& ^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^ \\
& 2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e \\
& ^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2* \\
& e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^ \\
& 3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2* \\
& b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2 \\
&)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d \\
& *e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b* \\
& c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^ \\
& 2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - \\
& (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b \\
& *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* \\
& e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4 \\
& *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 \\
& *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ \\
& 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
\end{aligned}$$

$$\begin{aligned}
& 2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 \\
& - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(\\
& a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4* \\
& b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^ \\
& 2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + \\
& a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - \\
& (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f) - 2*(b*c*d*e \\
& - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3 \\
& *d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c \\
& ^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^ \\
& 2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^ \\
& 2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2 \\
& *e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^ \\
& 4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^ \\
& 3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e \\
& ^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 \\
& + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b \\
& ^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2) \\
& *d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d* \\
& e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c \\
& *e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2 \\
& *c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d \\
& + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + \\
& a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - \\
& (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - \\
& 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + \\
& 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 \\
& - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c) \\
& *d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b \\
& ^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 \\
& - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3* \\
& d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2 \\
& *e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e \\
& ^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2 \\
& *d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b \\
& *e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e \\
& ^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2* \\
& e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^ \\
& 3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - \\
& a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a* \\
& b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 \\
& - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b \\
& ^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4* \\
& a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b \\
& ^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
\end{aligned}$$

$$\begin{aligned}
& - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\
& - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2) \\
& *d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2 \\
& *(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (\\
& b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 \\
& + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d* \\
& e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f \\
& ^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2 \\
& *e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 \\
& + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e \\
& + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c \\
& + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2) \\
& *f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20 \\
& *a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2 \\
& *a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + \\
& 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d* \\
& e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a \\
& ^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b \\
& ^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - \\
& (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(\\
& a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a \\
& *b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f \\
& + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + \\
& (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b* \\
& c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2 \\
& *f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a \\
& ^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 \\
& - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 \\
& - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 \\
& - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2) \\
& *d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2* \\
& (2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b \\
& ^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d), x)

[Out] $\text{Integral}(1/(\text{sqrt}(a + b*x + c*x**2))*(d + e*x + f*x**2)), x)$

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(c*x^2+b*x+a)^{(1/2)}/(f*x^2+e*x+d), x, \text{algorithm}="giac")$

[Out] Timed out

$$3.113 \quad \int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2} dx$$

Optimal. Leaf size=789

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(-4a^2f^2 + 3abef + b^2(e^2 - 6df)) - c(4af(e^2 - 3df) + b(e^3 - 5def))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - \sqrt{e^2 - 4df}}}$$

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(3*a*b*e*f - 4*a^2*f^2 + b^2*(e^2 - 6*d*f)) - c*(4*a*f*(e^2 - 3*d*f) + b*(e^3 - 5*d*e*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*(e^2 - 4*d*f)^(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(3*a*b*e*f - 4*a^2*f^2 + b^2*(e^2 - 6*d*f)) - c*(4*a*f*(e^2 - 3*d*f) + b*(e^3 - 5*d*e*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*(e^2 - 4*d*f)^(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 8.20973, antiderivative size = 787, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 1032, 724, 206}

$$\frac{(f(e - \sqrt{e^2 - 4df})(ce - bf)(2af - be + 2cd) - 2f(-4a^2f^3 + 3abef^2 - 4acf(e^2 - 3df) + b^2f(e^2 - 6df)) - bc(e^3 - 5def))}{2\sqrt{2}(e^2 - 4df)^{3/2}((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{2af^2 - \sqrt{e^2 - 4df}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2), x]

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*(e^2 - 4*d*f)^(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f])))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))/(2*Sqrt[2]*(e^2 - 4*d*f)^(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2} dx = \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)\sqrt{a+bx+cx^2}}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)\sqrt{a+bx+cx^2}}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)\sqrt{a+bx+cx^2}}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

$$= \frac{(f(be^2 - 2bdf - aef) - c(e^3 - 3def) + f(f(be - 2af) - c(e^2 - 2df))x)\sqrt{a+bx+cx^2}}{(e^2 - 4df)((cd - af)^2 - (bd - ae)(ce - bf))(d + ex + fx^2)}$$

Mathematica [A] time = 6.73391, size = 1377, normalized size = 1.75

$$\frac{8(cx^2 + bx + a)f^3}{(e^2 - 4df)\left(4af^2 - 2b(e - \sqrt{e^2 - 4df})f + c(e - \sqrt{e^2 - 4df})^2\right)(e + 2fx - \sqrt{e^2 - 4df})\sqrt{a + x(b + cx)}} - \frac{1}{(e^2 - 4df)(4af^2 - 2b(e - \sqrt{e^2 - 4df})f + c(e - \sqrt{e^2 - 4df})^2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2), x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2]))]/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))]/((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2]))]/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]))]/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.329, size = 3858, normalized size = 4.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] -2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f
```

$$\begin{aligned}
& f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*2^{(1/2)}*(((-4 \\
& *d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2 \\
&)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b* \\
& f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)))/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d* \\
& f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2))/(x-1/2*(-e+(-4*d* \\
& f+e^2)^{(1/2)))/f))+2/(4*d*f-e^2)/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)} \\
& *c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x+1/2*(-4*d*f+e^2)^{(1/2)}/f+1/2*e/f)* \\
& ((x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(\\
& x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(\\
& 1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}+1/(4*d*f-e^2)*f/((-4*d*f+ \\
& e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^{(1/2)}/ \\
& (((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^ \\
& 2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b \\
& *e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d \\
& *f+e^2)^{(1/2)))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}* \\
& c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2) \\
&)/f)^2*c+4/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2) \\
&)/f)+2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+ \\
& c*e^2)/f^2)^{(1/2))/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f))*c*(-4*d*f+e^2)^{(1/2)}-1 \\
& /(4*d*f-e^2)*f^2/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b* \\
& e*f-2*c*d*f+c*e^2)*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e \\
& +2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d \\
& *f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^{(1/ \\
& 2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1 \\
& /2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(\\
& x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+ \\
& 1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2) \\
& }*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2))/(x+1/2*(e+(-4*d*f+e^2)^{(1/2) \\
&)/f))*b+1/(4*d*f-e^2)*f/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a* \\
& f^2-b*e*f-2*c*d*f+c*e^2)*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/ \\
& 2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f \\
& +(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^ \\
& 2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*2^{(1/2)}*(((-4*d*f+e \\
& ^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2) \\
& }*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c* \\
& e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)))/f)+2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2) \\
& ^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2))/(x+1/2*(e+(-4*d*f+e^2)^{(\\
& 1/2)))/f))*c*e+2/(4*d*f-e^2)/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e \\
& +2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x-1/2*(-4*d*f+e^2)^{(1/2)}/f+1/2*e/f)*((x- \\
& 1/2*(-e+(-4*d*f+e^2)^{(1/2)))/f)^2*c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2* \\
& (-e+(-4*d*f+e^2)^{(1/2)))/f)+1/2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c \\
& *e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}-1/(4*d*f-e^2)*f/((-4*d*f+e^2)^{(1 \\
& /2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^{(1/2)}/(((-4*d \\
& *f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(\\
& 1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*
\end{aligned}$$

$$\begin{aligned}
& d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f))*c*(-4*d*f+e^2)^{(1/2)}-1/(4*d*f-e^2)*f^2/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f))*b+1/(4*d*f-e^2)*f/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^{(1/2)}/(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c+4*(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})))/f))*c*e+2*f/(4*d*f-e^2)/((-4*d*f+e^2)^{(1/2)}*2^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})))/f)+1/2*2^{(1/2)}*(((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*(4*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})))/f)+2*((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)})/(x+1/2*(e+(-4*d*f+e^2)^{(1/2)})))/f))
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(fx^2 + ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)`

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")`

[Out] Timed out

$$3.114 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=649

$$2(-x(-2acf + b^2f - bce + 2c^2d)(a^2c^2f^2 - 4ab^2cf^2 + 7abc^2ef - 2ac^3df - 3ac^3e^2 + b^2c^2df + b^2c^2e^2 - 2b^3cef + b^4f^2 - b$$

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/(c^5*(b^2 -
4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
+ b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/((128*c^(11/2))
```

Rubi [A] time = 2.10557, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 621, 206}

$$2(-x(-2acf + b^2f - bce + 2c^2d)(a^2c^2f^2 - 4ab^2cf^2 + 7abc^2ef - 2ac^3df - 3ac^3e^2 + b^2c^2df + b^2c^2e^2 - 2b^3cef + b^4f^2 - b$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x]

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
```

$$\begin{aligned} &^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b* \\ &c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e \\ &*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/(c^5*(b^2 - \\ &4*a*c)*\text{Sqrt}[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a* \\ &f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*\text{Sqrt}[\\ &a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c \\ &^2*(e^2 + d*f))*x*\text{Sqrt}[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^ \\ &2*\text{Sqrt}[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2) \\ &+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80* \\ &c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) \\ &+ b*(e^3 + 6*d*e*f)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2] \\ &)]/(128*c^(11/2)) \end{aligned}$$

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{3/2}} dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2e - 9a^2df^2)}{(a + bx + cx^2)^{3/2}}$$

Mathematica [A] time = 1.6847, size = 745, normalized size = 1.15

$$\frac{-16b^2c^2(-a^2f^2(230e + 169fx) + ac(2ef(36d - 43fx^2) + f^2x(186d - 13fx^2)) + 186e^2fx + 12e^3) + c^2x(-24d^2f + 6d(-$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]

[Out] (315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c*f*(105*a*f*(4*e + 9*f*x) + c*x*(-360*e^2 - 360*d*f + 140*e*f*x + 21*f^2*x^2)) - 8*b^3*c*(210*a^2*f^3 - c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2*e*f*(-72*d + 7*f*x^2)) + a*c*f*(-90*e^2 - 530*e*f*x + f*(-90*d + 77*f*x^2)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x + 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x) + a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2*x^2 + 3*f^3*x^3))))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/(128*c^(11/2))

Maple [B] time = 0.069, size = 2827, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)

[Out] $\frac{3/2*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}*e^3-3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^3+115/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-45/4*e*f^2*b/c^3*a*x/(c*x^2+b*x+a)^{(1/2)}+113/8*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+9*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}*d*e*f-9/2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*e*f-8*e*f^2*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f^2+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2*f-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*f^2-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^2*f+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*e^2+4*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^3+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*f*d^2+24*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*e*f+6*x^2/c/(c*x^2+b*x+a)^{(1/2)}*d*e*f-9/2*b^2/c^3/(c$

$$\begin{aligned}
& x^2+bx+a)^{(1/2)}*d*e*f-3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x*e^3-9* \\
& b/c^{(5/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*d*e*f+12*a/c^2/(c*x^2 \\
& +bx+a)^{(1/2)}*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x*d*f^2- \\
& 16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x+115/4*e*f^2*b^3/c^3*a/ \\
& (4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)} \\
& *x*e^2*f+12*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*d*e*f-9*b^3/c^2/ \\
& (4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x*d*e*f-105/32*e*f^2*b^4/c^5/(c*x^2+bx+a)^{(1/2)} \\
& -105/16*e*f^2*b^3/c^{(9/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})- \\
& 8*e*f^2*a^2/c^3/(c*x^2+bx+a)^{(1/2)}-5/8*f^3*a/c^2*x^3/(c*x^2+bx+a)^{(1/2)}-3 \\
& /8*f^3*b/c^2*x^4/(c*x^2+bx+a)^{(1/2)}+21/32*f^3*b^2/c^3*x^3/(c*x^2+bx+a)^{(1/2)} \\
& -105/64*f^3*b^3/c^4*x^2/(c*x^2+bx+a)^{(1/2)}-315/128*f^3*b^4/c^5*x/(c*x^2 \\
& +bx+a)^{(1/2)}+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}-105/16*f^ \\
& 3*b^3/c^5*a/(c*x^2+bx+a)^{(1/2)}+113/16*f^3*b/c^4*a^2/(c*x^2+bx+a)^{(1/2)}-10 \\
& 5/16*f^3*b^2/c^{(9/2)}*a*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})-15/8*f^3 \\
& *a^2/c^3*x/(c*x^2+bx+a)^{(1/2)}-3*x/c/(c*x^2+bx+a)^{(1/2)}*f*d^2-3*x/c/(c*x^2 \\
& +bx+a)^{(1/2)}*d*e^2+3/2*b/c^2/(c*x^2+bx+a)^{(1/2)}*f*d^2+3/2*b/c^2/(c*x^2+bx \\
& +a)^{(1/2)}*d*e^2+3/2*x^3/c/(c*x^2+bx+a)^{(1/2)}*d*f^2+3/2*x^3/c/(c*x^2+bx+a \\
&)^{(1/2)}*e^2*f+45/16*b^3/c^4/(c*x^2+bx+a)^{(1/2)}*d*f^2+45/16*b^3/c^4/(c*x^2+ \\
& bx+a)^{(1/2)}*e^2*f+45/8*b^2/c^{(7/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)}) \\
& *d*f^2+45/8*b^2/c^{(7/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*e^2 \\
& *f-9/2*a/c^{(5/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*d*f^2-9/2*a/c^{(5/2)} \\
& *\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*e^2*f+e*f^2*x^4/c/(c*x^2+ \\
& bx+a)^{(1/2)}-39/4*b/c^3*a/(c*x^2+bx+a)^{(1/2)}*d*f^2-39/4*b/c^3*a/(c*x^2+bx \\
& +a)^{(1/2)}*e^2*f+9/2*a/c^2*x/(c*x^2+bx+a)^{(1/2)}*d*f^2+9/2*a/c^2*x/(c*x^2+bx \\
& +a)^{(1/2)}*e^2*f+115/8*e*f^2*b^2/c^4*a/(c*x^2+bx+a)^{(1/2)}+45/4*e*f^2*b/c^{(7/2)} \\
& *a*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})-4*e*f^2*a/c^2*x^2/(c*x^2 \\
& +bx+a)^{(1/2)}+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*f*d^2+3/2*b^3/c^2 \\
& / (4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*d*e^2+45/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+bx \\
& +a)^{(1/2)}*d*f^2+45/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*e^2*f+x^2/c/(\\
& c*x^2+bx+a)^{(1/2)}*e^3-3/4*b^2/c^3/(c*x^2+bx+a)^{(1/2)}*e^3-3/2*b/c^{(5/2)}*\ln \\
& ((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*e^3+2*a/c^2/(c*x^2+bx+a)^{(1/2)}*e \\
& ^3+315/256*f^3*b^5/c^6/(c*x^2+bx+a)^{(1/2)}+315/128*f^3*b^4/c^{(11/2)}*\ln((1/2 \\
& *b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})+1/4*f^3*x^5/c/(c*x^2+bx+a)^{(1/2)}+2*a/ \\
& c^2*b^2/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*e^3+15/8*f^3*a^2/c^{(7/2)}*\ln((1/2*b+ \\
& cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})+3/c^{(3/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+b \\
& x+a)^{(1/2)})*f*d^2+3/c^{(3/2)}*\ln((1/2*b+cx)/c^{(1/2)}+(c*x^2+bx+a)^{(1/2)})*d \\
& *e^2-3*d^2*e/c/(c*x^2+bx+a)^{(1/2)}+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+bx+a) \\
& ^{(1/2)}+105/16*f^3*b^2/c^4*a*x/(c*x^2+bx+a)^{(1/2)}+49/16*f^3*b/c^3*a*x^2/(c* \\
& x^2+bx+a)^{(1/2)}+113/16*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}-15/ \\
& 4*b/c^2*x^2/(c*x^2+bx+a)^{(1/2)}*d*f^2-15/4*b/c^2*x^2/(c*x^2+bx+a)^{(1/2)}*e \\
& ^2*f-45/8*b^2/c^3*x/(c*x^2+bx+a)^{(1/2)}*d*f^2-45/8*b^2/c^3*x/(c*x^2+bx+a)^{(1/2)} \\
& *e^2*f-105/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x-105/8*f^3 \\
& *b^4/c^4*a/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}*x-7/4*e*f^2*b/c^2*x^3/(c*x^2+bx \\
& +a)^{(1/2)}+35/8*e*f^2*b^2/c^3*x^2/(c*x^2+bx+a)^{(1/2)}+105/16*e*f^2*b^3/c^4*x \\
& / (c*x^2+bx+a)^{(1/2)}-105/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+bx+a)^{(1/2)}+3
\end{aligned}$$

$$\frac{15}{128}f^3b^6/c^5/(4ac-b^2)/(cx^2+bx+a)^{1/2}x - \frac{105}{16}f^3b^5/c^5a/(4ac-b^2)/(cx^2+bx+a)^{1/2} - 6d^2eb/(4ac-b^2)/(cx^2+bx+a)^{1/2}x - 3d^2eb^2/c/(4ac-b^2)/(cx^2+bx+a)^{1/2}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 15.911, size = 6743, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/256*(3*(128*(ab^2c^4 - 4a^2c^5)*d*e^2 - 64*(ab^3c^3 - 4a^2b*c^4) \\ & *e^3 + 5*(21*ab^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8* \\ & (6*(5*ab^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*ab^5*c - 40*a^2*b^ \\ & 3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4 \\ & - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3* \\ & c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 - \\ & 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b \\ & ^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f) \\ & *x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e \\ & + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4 \\ & *a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c \\ & + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16 \\ & *a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8* \\ & (b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 - \\ & 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - \\ & b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6* \\ & d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5 \\ & - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3 \end{aligned}$$

$$\begin{aligned}
& *a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3 - 104*a*b^2*c^4 + 80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4 - 4*a*b*c^5)*e)*f^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b^4*c^2 - 460*a^2*b^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^3 - 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*a*b*c^5)*d - (35*b^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2*c^5 - 4*a*c^6)*d*e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d^2 - 8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f + (256*c^7*d^3 - 384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b^3*c^4 - 10*a*b*c^5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3 + 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x), -1/128*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f)*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8*(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(128*b*c^6*d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5 - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3 - 104*a*b^2*c^4 + 80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4 - 4*a*b*c^5)*e)*f^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b^4*c^2 - 460*a^2*b^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^3 - 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*a*b*c^5)*d - (35*b^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2*c^5 - 4*a*c^6)*d*e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d^2 - 8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f + (256*c^7*d^3 - 384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b^3*c^4 - 10*a*b*c^5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3 + 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*
\end{aligned}$$

$$b^5c^2 - 530ab^3c^3 + 488a^2b^2c^4)e) * f^2 + 48(8(b^2c^5 - 2ac^6) * d^2 - 8(3b^3c^4 - 10ab^2c^5) * d * e + (15b^4c^3 - 62a^2b^2c^4 + 24a^2c^5) * e^2) * f) * x) * \sqrt{c * x^2 + b * x + a} / (a * b^2 * c^6 - 4 * a^2 * c^7 + (b^2 * c^7 - 4 * a * c^8) * x^2 + (b^3 * c^6 - 4 * a * b * c^7) * x)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.3817, size = 1484, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")

[Out]
$$\frac{1}{64} \left(\left(\left(2 \left(4 \left(2 \left(b^2 c^4 f^3 - 4 a c^5 f^3 \right) x / \left(b^2 c^5 - 4 a c^6 \right) - \left(3 b^3 c^3 f^3 - 12 a b^2 c^4 f^3 - 8 b^2 c^4 f^2 e + 32 a c^5 f^2 e \right) / \left(b^2 c^5 - 4 a c^6 \right) \right) x + \left(48 b^2 c^4 d f^2 - 192 a c^5 d f^2 + 21 b^4 c^2 f^3 - 104 a b^2 c^3 f^3 + 80 a^2 c^4 f^3 - 56 b^3 c^3 f^2 e + 224 a b^2 c^4 f^2 e + 48 b^2 c^4 f e^2 - 192 a c^5 f e^2 \right) / \left(b^2 c^5 - 4 a c^6 \right) \right) x - \left(240 b^3 c^3 d f^2 - 960 a b^2 c^4 d f^2 + 105 b^5 c f^3 - 616 a b^3 c^2 f^3 + 784 a^2 b^2 c^3 f^3 - 384 b^2 c^4 d f e + 1536 a c^5 d f e - 280 b^4 c^2 f^2 e + 1376 a b^2 c^3 f^2 e - 1024 a^2 c^4 f^2 e + 240 b^3 c^3 f e^2 - 960 a b^2 c^4 f e^2 - 64 b^2 c^4 e^3 + 256 a c^5 e^3 \right) / \left(b^2 c^5 - 4 a c^6 \right) \right) x - \left(256 c^6 d^3 + 384 b^2 c^4 d^2 f - 768 a c^5 d^2 f + 720 b^4 c^2 d f^2 - 2976 a b^2 c^3 d f^2 + 1152 a^2 c^4 d f^2 + 315 b^6 f^3 - 1890 a b^4 c f^3 + 2704 a^2 b^2 c^2 f^3 - 480 a^3 c^3 f^3 - 384 b^2 c^5 d^2 e - 1152 b^3 c^3 d f e + 3840 a b^2 c^4 d f e - 840 b^5 c f^2 e + 4240 a b^3 c^2 f^2 e - 3904 a^2 b^2 c^3 f^2 e + 384 b^2 c^4 d e^2 - 768 a c^5 d e^2 + 720 b^4 c^2 f e^2 - 2976 a b^2 c^3 f e^2 + 1152 a^2 c^4 f e^2 - 192 b^3 c^3 e^3 + 640 a b^2 c^4 e^3 \right) / \left(b^2 c^5 - 4 a c^6 \right) \right) x - \left(128 b^2 c^5 d^3 + 384 a b^2 c^4 d^2 f + 720 a b^3 c^2 d f^2 - 2496 a^2 b^2 c^3 \right)$$

$$\begin{aligned}
& *d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3 \\
& 680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^ \\
& 2*c^5 - 4*a*c^6))/\sqrt{c*x^2 + b*x + a} - 3/128*(128*c^4*d^2*f + 240*b^2*c^ \\
& 2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3 \\
& - 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 2 \\
& 40*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*\log(\text{abs}(-2*(\sqrt{c}*x - \\
& \sqrt{c*x^2 + b*x + a})*\sqrt{c} - b))/c^{(11/2)}
\end{aligned}$$

$$3.115 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=309

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3a^2f^2 - 2af^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(c^3*(b^2 - 4ac)*\sqrt{a + bx + cx^2}) + (f*(8*c*e - 7*b*f)*\sqrt{a + b*x + c*x^2})/(4*c^3) + (f^2*x*\sqrt{a + b*x + c*x^2})/(2*c^2) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}]])/(8*c^(7/2))$$

[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))

Rubi [A] time = 0.446654, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2(-x(c^2(2a^2f^2 + 6abef + b^2(2df + e^2)) - 2b^2cf(2af + be) - 2c^3(a(2df + e^2) + bde) + b^4f^2 + 2c^4d^2) - bc(-3a^2f^2 - 2af^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(c^3*(b^2 - 4ac)*\sqrt{a + bx + cx^2}) + (f*(8*c*e - 7*b*f)*\sqrt{a + b*x + c*x^2})/(4*c^3) + (f^2*x*\sqrt{a + b*x + c*x^2})/(2*c^2) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*\text{ArcTanh}[(b + 2*c*x)/(2*\sqrt{c}*\sqrt{a + b*x + c*x^2}]])/(8*c^(7/2))$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x]

[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

Rule 640

```

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

```

Rule 621

```

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{c^3(b^2 - 4ac)\sqrt{a +}} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{c^3(b^2 - 4ac)\sqrt{a +}} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{c^3(b^2 - 4ac)\sqrt{a +}} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{c^3(b^2 - 4ac)\sqrt{a +}} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{c^3(b^2 - 4ac)\sqrt{a +}}
\end{aligned}$$

Mathematica [A] time = 0.756993, size = 288, normalized size = 0.93

$$\frac{4bc(-13a^2f^2 + ac(4df + 2e^2 + 20efx - 5f^2x^2) + 2c^2d(d - 2ex)) + 8c^2(a^2f(8e + 3fx) + ac(x(-2e^2 + 4efx + f^2x^2) - 4c^3(4ac - b^2))}{4c^3(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x]

[Out] (15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(-13*a^2*f^2 + 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x^2)) - 2*b^2*c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x^2)) + 8*c^2*(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(-2*e^2 + 4*e*f*x + f^2*x^2)))/(4*c^3*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*c^(7/2))

Maple [B] time = 0.067, size = 1011, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out] $8*e*f*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*f-3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+4*e*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+1/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e^{-2}-13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+3*e*f*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}-3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-2*d*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e^2+b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*f+15/8*f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x-2*x/c/(c*x^2+b*x+a)^{(1/2)}*d*f+2*e*f*x^2/c/(c*x^2+b*x+a)^{(1/2)}-5/4*f^2*b/c^2*x^2/(c*x^2+b*x+a)^{(1/2)}-15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^{(1/2)}+15/16*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^{(1/2)}-3*e*f*b/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+4*e*f*a/c^2/(c*x^2+b*x+a)^{(1/2)}+b/c^2/(c*x^2+b*x+a)^{(1/2)}*d*f+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*e^2+3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^{(1/2)}-13/4*f^2*b/c^3*a/(c*x^2+b*x+a)^{(1/2)}+2/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*f-x/c/(c*x^2+b*x+a)^{(1/2)}*e^2+1/2*b/c^2/(c*x^2+b*x+a)^{(1/2)}*e^2+15/8*f^2*b^2/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-3/2*f^2*a/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/2*f^2*x^3/c/(c*x^2+b*x+a)^{(1/2)}+15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^{(1/2)}-2*d*e/c/(c*x^2+b*x+a)^{(1/2)}+2*d^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 9.59512, size = 2750, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")

[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)

[Out] Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)

Giac [A] time = 1.65767, size = 549, normalized size = 1.78

$$\left(\frac{2(b^2c^2f^2 - 4ac^3f^2)x}{b^2c^3 - 4ac^4} - \frac{5b^3cf^2 - 20abc^2f^2 - 8b^2c^2fe + 32ac^3fe}{b^2c^3 - 4ac^4} \right) x - \frac{16c^4d^2 + 16b^2c^2df - 32ac^3df + 15b^4f^2 - 62ab^2cf^2 + 24a^2c^2f^2 - 16bc^3de - 24b^3cfe + 80abc^2d^2 - 16a^2c^3e^2}{b^2c^3 - 4ac^4} \sqrt{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")

[Out] 1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 - 20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 24*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*f*e + 8*b^2*c^2*e^2 - 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f + 15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2*f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/8*(16*c^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)

$$3.116 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(3/2)

Rubi [A] time = 0.0797038, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2), x]

[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(3/2)

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} - \frac{2 \int -\frac{(b^2 - 4ac)f}{2c\sqrt{a + bx + cx^2}} dx}{b^2 - 4ac} \\ &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{c} \\ &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{(2f) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}} \right)}{c} \\ &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}} \right)}{c^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.328344, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}(abf - 2ac(e + fx) + b^2fx + bc(d - ex) + 2c^2dx)}{\sqrt{a + x(b + cx)}} - f(b^2 - 4ac) \log(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx)}{c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2), x]

[Out] ((2*sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*sqrt[c]*sqrt[a + x*(b + c*x)])/(c^(3/2)*(-b^2 + 4*a*c))

Maple [B] time = 0.052, size = 249, normalized size = 2.2

$$-\frac{fx}{c} \frac{1}{\sqrt{cx^2 + bx + a}} + \frac{bf}{2c^2} \frac{1}{\sqrt{cx^2 + bx + a}} + \frac{b^2fx}{c(4ac - b^2)} \frac{1}{\sqrt{cx^2 + bx + a}} + \frac{b^3f}{2c^2(4ac - b^2)} \frac{1}{\sqrt{cx^2 + bx + a}} + f \ln\left(\frac{b}{2} + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2), x)

[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 4.44962, size = 941, normalized size = 8.48

$$\left[\frac{((b^2c - 4ac^2)fx^2 + (b^3 - 4abc)fx + (ab^2 - 4a^2c)f)\sqrt{c} \log\left(-8c^2x^2 - 8bcx - b^2 - 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac\right)}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^2 + (b^3c^2 - 4ac^3)x - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")

[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)

[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)

Giac [A] time = 1.50197, size = 165, normalized size = 1.49

$$\frac{2 \left(\frac{(2c^2d + b^2f - 2acf - bce)x}{b^2c - 4ac^2} + \frac{bcd + abf - 2ace}{b^2c - 4ac^2} \right)}{\sqrt{cx^2 + bx + a}} - \frac{f \log \left(\left| -2 \left(\sqrt{cx} - \sqrt{cx^2 + bx + a} \right) \sqrt{c} - b \right| \right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")

[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(c

$$c)x - \sqrt{c^2x^2 + bx + a})\sqrt{c - b})/c^{3/2}$$

$$3.117 \quad \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx$$

Optimal. Leaf size=666

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^2ce + b^3(-f))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} - \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]
)]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])]
```

Rubi [A] time = 1.82916, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 1032, 724, 206}

$$\frac{2(-cx(-2acf + b^2f - bce + 2c^2d) - bc(cd - 3af) - 2ac^2e + b^2ce + b^3(-f))}{(b^2 - 4ac)\sqrt{a + bx + cx^2}((cd - af)^2 - (bd - ae)(ce - bf))} - \frac{f(f(2af - b(\sqrt{e^2 - 4df} + e)) + c(e\sqrt{e^2 - 4df} - b(\sqrt{e^2 - 4df} + e)))}{\sqrt{2}\sqrt{e^2 - 4df}((cd - af)^2 - (bd - ae)(ce - bf))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2]
)]/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x]/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]])*Sqrt[a + b*x + c*x^2])]/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])]
```

$$2*af - b*(e + \sqrt{e^2 - 4*df})) * \text{ArcTanh}[(4*af - b*(e - \sqrt{e^2 - 4*df})) / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*\sqrt{e^2 - 4*df}}*\sqrt{a + b*x + c*x^2})] / (\sqrt{2}*\sqrt{e^2 - 4*df}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e - \sqrt{e^2 - 4*df}))}) + (f*(c*(e^2 - 2*d*f - e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e - \sqrt{e^2 - 4*df})))) * \text{ArcTanh}[(4*af - b*(e + \sqrt{e^2 - 4*df})) + 2*(b*f - c*(e + \sqrt{e^2 - 4*df})) * x] / (2*\sqrt{2}*\sqrt{c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*\sqrt{e^2 - 4*df}}*\sqrt{a + b*x + c*x^2})) / (\sqrt{2}*\sqrt{e^2 - 4*df}*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*\sqrt{c*(e^2 - 2*d*f + e*\sqrt{e^2 - 4*df}) + f*(2*af - b*(e + \sqrt{e^2 - 4*df}))})]$$

Rule 974

$$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := \text{Simp}[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x]*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)) / ((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] - \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q * \text{Simp}[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(2*p + 2*q + 5)*x^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, q\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \&\& !(!\text{IntegerQ}[p] \&\& \text{ILtQ}[q, -1]) \&\& !\text{IGtQ}[q, 0]$$

Rule 1032

$$\text{Int}[(g_. + (h_.)*(x_)) / (((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*\sqrt{(d_.) + (e_.)*(x_) + (f_.)*(x_)^2})], x_Symbol] := \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c*g - h*(b - q))/q, \text{Int}[1/((b - q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\sqrt{d + e*x + f*x^2}), x], x]] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[e^2 - 4*d*f, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$$

Rule 724

$$\text{Int}[1/(((d_.) + (e_.)*(x_))*\sqrt{(a_.) + (b_.)*(x_) + (c_.)*(x_)^2})], x_Symbol] := \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\sqrt{a + b*x + c*x^2}], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)} dx &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} + \frac{2 \int \frac{1}{(b^2 - 4ac)\sqrt{a+bx+cx^2}} dx}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} + \frac{(f(2f^2 - 2efx - e^2))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} - \frac{(2f(2f^2 - 2efx - e^2))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} \\ &= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} - \frac{f(c(e^2 - 2efx - f^2))}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a+bx+cx^2}} \end{aligned}$$

Mathematica [A] time = 6.79196, size = 700, normalized size = 1.05

$$2f \left[\frac{2(-2c(2af+cx(\sqrt{e^2-4df+e}))+2b^2f-bc(\sqrt{e^2-4df+e}-2fx))}{(b^2-4ac)\sqrt{a+x(b+cx)}(4af^2-2bf(\sqrt{e^2-4df+e})+c(\sqrt{e^2-4df+e})^2)} + \frac{2c(cx(\sqrt{e^2-4df-e})-2af)+2b^2f+bc(\sqrt{e^2-4df-e}+2fx)}{(b^2-4ac)\sqrt{a+x(b+cx)}(f(2af+b(\sqrt{e^2-4df-e}))+c(-e\sqrt{e^2-4df}-2df+e^2))} + \frac{\sqrt{2}f^2 \operatorname{arctanh}\left(\frac{\sqrt{e^2-4df+e}}{\sqrt{e^2-4df-e}}\right)}{(b^2-4ac)\sqrt{a+x(b+cx)}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]

[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)))/(b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d

$$\begin{aligned}
& *f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[a + x*(b + c*x)] - (2* \\
& (2*b^2*f - b*c*(e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + \text{Sqrt}[e \\
& ^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + \text{Sqrt}[e^2 - 4*d*f]) + \\
& c*(e + \text{Sqrt}[e^2 - 4*d*f])^2)*\text{Sqrt}[a + x*(b + c*x)] + (\text{Sqrt}[2]*f^2*\text{ArcTanh} \\
& [(4*a*f - 2*c*(e + \text{Sqrt}[e^2 - 4*d*f])*x - b*(e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x) \\
&)/(2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f - b*(e + \\
& \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)])))/(c*(e^2 - 2*d*f + e*\text{Sqrt}[e^2 \\
& - 4*d*f]) + f*(2*a*f - b*(e + \text{Sqrt}[e^2 - 4*d*f]))^(3/2) - (\text{Sqrt}[2]*f^2*\text{Sq} \\
& \text{rt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4* \\
& d*f]))]*\text{ArcTanh}[(4*a*f + 2*c*(-e + \text{Sqrt}[e^2 - 4*d*f])*x + b*(-e + \text{Sqrt}[e^2 \\
& - 4*d*f] + 2*f*x))/(2*\text{Sqrt}[2]*\text{Sqrt}[c*(e^2 - 2*d*f - e*\text{Sqrt}[e^2 - 4*d*f]) + \\
& f*(2*a*f + b*(-e + \text{Sqrt}[e^2 - 4*d*f]))]*\text{Sqrt}[a + x*(b + c*x)])))/(c*(-e^2 + \\
& 2*d*f + e*\text{Sqrt}[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - \text{Sqrt}[e^2 - 4*d*f]))^(2)) \\
& / \text{Sqrt}[e^2 - 4*d*f]
\end{aligned}$$

Maple [B] time = 0.355, size = 4099, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(c*x^2+b*x+a)^{(3/2)}/(f*x^2+e*x+d), x)$

[Out]
$$\begin{aligned}
& 2/(-4*d*f+e^2)^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2 \\
& -b*e*f-2*c*d*f+c*e^2)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(c*(-4*d*f \\
& +e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)} \\
& *b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}-4* \\
& f/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2) \\
& /((4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4 \\
& *d*f+e^2)^{(1/2)})/f)^2*c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f \\
& +e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2- \\
& b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*x*c^2-4/(-4*d*f+e^2)^{(1/2)}*f^2/((-4*d*f+e^2) \\
&)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2 \\
& /f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)}) \\
&)/f)^2*c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f \\
&)+1/2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+ \\
& c*e^2)/f^2)^{(1/2)}*x*b*c+4/(-4*d*f+e^2)^{(1/2)}*f/((-4*d*f+e^2)^{(1/2)}*b*f-(-4* \\
& d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2 \\
& -1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(c*(-4 \\
& *d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e \\
& ^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)} \\
&)*x*c^2*e-2*f/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-
\end{aligned}$$

$$\begin{aligned}
& 2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x \\
& -1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2 \\
& *(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}* \\
& c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*b*c-2/(-4*d*f+e^2)^{(1/2)}*f^2/((\\
& -4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(\\
& 4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f \\
& +e^2)^{(1/2)})/f)^2*c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2 \\
&)^{(1/2)})/f)+1/2*((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e* \\
& f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*b^2+2/(-4*d*f+e^2)^{(1/2)}*f/((-4*d*f+e^2)^{(1/2)}* \\
& b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^ \\
& 2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2* \\
& c+(c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*((\\
& -4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f \\
& ^2)^{(1/2)}*b*c*e-2/(-4*d*f+e^2)^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(\\
& 1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}*b*f- \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*\ln(((4*d*f \\
& +e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c* \\
& (-4*d*f+e^2)^{(1/2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+1/2*2^{(1/2)} \\
& *(((4*d*f+e^2)^{(1/2)}*b*f-(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^ \\
& 2)/f^2)^{(1/2)}*(4*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)^2*c+4*(c*(-4*d*f+e^2)^{(1 \\
& /2)}+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^{(1/2)})/f)+2*((4*d*f+e^2)^{(1/2)}*b*f- \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}/(x-1/2*(-e+ \\
& (-4*d*f+e^2)^{(1/2)})/f))-2/(-4*d*f+e^2)^{(1/2)}/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d \\
& *f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^ \\
& (1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1 \\
& /2))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2 \\
& *c*d*f+c*e^2)/f^2)^{(1/2)}-4*f/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c* \\
& e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e \\
& ^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^{(1/ \\
& 2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)}*b*f+ \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*x*c^2+4/(-4*d \\
& *f+e^2)^{(1/2)}*f^2/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b \\
& *e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2 \\
&)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e) \\
& *(x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2) \\
& ^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*x*b*c-4/(-4*d*f+e^2)^{(1/ \\
& 2)}*f/((-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+ \\
& c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+ \\
& (-4*d*f+e^2)^{(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(- \\
& 4*d*f+e^2)^{(1/2))/f)+1/2*(-(-4*d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2* \\
& a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^{(1/2)}*x*c^2*e-2*f/((-4*d*f+e^2)^{(1/2)}*b*f+ \\
& (-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^ \\
& 2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)^2*c+1/f \\
& *(-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^{(1/2))/f)+1/2*(-(-4 \\
& *d*f+e^2)^{(1/2)}*b*f+(-4*d*f+e^2)^{(1/2)}*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
\end{aligned}$$

$$\begin{aligned} &)^{(1/2)} * b * c + 2 / (-4 * d * f + e^2)^{(1/2)} * f^2 / (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f)^2 * c + 1 / f * (-c * (-4 * d * f + e^2)^{(1/2)} + b * f - c * e) * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f + 1/2 * (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * b^2 - 2 / (-4 * d * f + e^2)^{(1/2)} * f / (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / (4 * a * c - 4 * c^2 / f * d + c^2 / f^2 * e^2 - 1 / f^2 * (-4 * d * f + e^2) * c^2 - b^2) / ((x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f)^2 * c + 1 / f * (-c * (-4 * d * f + e^2)^{(1/2)} + b * f - c * e) * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f + 1/2 * (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * b * c * e + 2 / (-4 * d * f + e^2)^{(1/2)} / (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) * f^2 * 2^{(1/2)} / ((-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * \ln(((-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2 + 1 / f * (-c * (-4 * d * f + e^2)^{(1/2)} + b * f - c * e) * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f + 1/2 * 2^{(1/2)} * ((-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} * (4 * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f)^2 * c + 4 / f * (-c * (-4 * d * f + e^2)^{(1/2)} + b * f - c * e) * (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f + 2 * (-(-4 * d * f + e^2)^{(1/2)} * b * f + (-4 * d * f + e^2)^{(1/2)} * c * e + 2 * a * f^2 - b * e * f - 2 * c * d * f + c * e^2) / f^2)^{(1/2)} / (x + 1/2 * (e + (-4 * d * f + e^2)^{(1/2)})) / f)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral(1/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")

[Out] Timed out

$$3.118 \quad \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}} dx$$

Optimal. Leaf size=891

$$\frac{x\sqrt{cx^2+bx+af^3}}{2c^3} + \frac{(12ce-11bf)\sqrt{cx^2+bx+af^2}}{4c^4} + \frac{(24(e^2+df)c^2-20f(3be+af)c+35b^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+af^2}}\right)}{8c^{9/2}}$$

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/(3*c^5*(b^2
- 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c
*f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2
*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x)/(3*c^5*(b^2 - 4*a*c)^2*sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])])
)/(8*c^(9/2))
```

Rubi [A] time = 1.76833, antiderivative size = 891, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{x\sqrt{cx^2+bx+af^3}}{2c^3} + \frac{(12ce-11bf)\sqrt{cx^2+bx+af^2}}{4c^4} + \frac{(24(e^2+df)c^2-20f(3be+af)c+35b^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+af^2}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]

[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x)/(3*c^5*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 10*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) - b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e + 26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x)/(3*c^5*(b^2 - 4*a*c)^2*sqrt[a + b*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3*x*sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])])/(8*c^(9/2))

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1661

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a + b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
]:> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{5/2}} dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}$$

$$= \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2 + df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 + df)) - 9a^2d^2f^2 + 6a^2df^3 - 9a^2e^2f^2)}$$

Mathematica [A] time = 2.37583, size = 872, normalized size = 0.98

$$-105f^3x^2b^7 - 10f^2x(21af + 2cx(7fx - 9e))b^6 - 3f(35a^2f^2 - 10acx(12e + 23fx))f + c^2x^2(24e^2 - 80fxe + 7f^2x^2 + 24df$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2), x]

[Out] $(-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c*f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^2) + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2 - 10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^2*x^2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2*f + 4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^3 - e^3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e^2 + 39*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e^3*x^3 + 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*e^2 - 74*e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3*d + 29*f*x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*c^3*d*x*(d^2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*d - 10*f*x^2) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*e + 2*f*x) + x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^2*(a^3*f^2*(25*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) + a^2*c*f*x*(-21*e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*f*x) - 2*d*x*(3*e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f^2*x^2 + f^3*x^3))) / (12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x + 2*sqrt[c]*sqrt[a + x*(b + c*x)]) / (8*c^(9/2))$

Maple [B] time = 0.071, size = 4635, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2), x)

[Out] $6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2*f+3/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2+3/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f+1/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f-6*b/c*a/(4*a*c-b^2)/(c*$

$$\begin{aligned}
& x^2+bx+a)^{(3/2)} * x * d * e * f + 8 * e * f^2 * a^2 / c^3 / (c * x^2 + b * x + a)^{(3/2)} + 3/2 * b^2 / c^2 * a / \\
& (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * d * f^2 + 3/2 * b^2 / c^2 * a / (4 * a * c - b^2) / (c * x^2 + b * \\
& x + a)^{(3/2)} * x * e^2 * f + 96 * e * f^2 * a^2 / c * b / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x - 19 / \\
& 4 * e * f^2 * b^3 / c^3 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x + 12 * e * f^2 * a^2 / c^2 * b / (4 * a \\
& * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x + 4 * b^3 / c / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * d \\
& * e * f + 12 * b^2 / c * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * d * f^2 + 12 * b^2 / c * a / (4 * a * c \\
& - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * e^2 * f - 38 * e * f^2 * b^3 / c^2 * a / (4 * a * c - b^2)^2 / (c * x^2 \\
& + b * x + a)^{(1/2)} * x - 48 * b * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x * d * e * f - 24 * b^2 / c * a \\
& / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * d * e * f - 3 * b^2 / c^2 * a / (4 * a * c - b^2) / (c * x^2 + b * x \\
& + a)^{(3/2)} * d * e * f + 1/2 * b^3 / c^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * d * e * f + 3/4 * b^3 \\
& / c^3 * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f + 6 * b^3 / c^2 * a / (4 * a * c - b^2)^2 / (c * x \\
& ^2 + b * x + a)^{(1/2)} * d * f^2 - 8 * d^2 * e * b^2 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} + 3 * e * f^2 \\
& * x^4 / c / (c * x^2 + b * x + a)^{(3/2)} - 15/2 * e * f^2 * b / c^{(7/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x \\
& ^2 + b * x + a)^{(1/2)}) - 15/4 * e * f^2 * b^2 / c^4 / (c * x^2 + b * x + a)^{(1/2)} + 5/32 * e * f^2 * b^4 / c^5 / \\
& (c * x^2 + b * x + a)^{(3/2)} - 3/2 * b / c^2 * x / (c * x^2 + b * x + a)^{(3/2)} * d * e * f - 16 * d^2 * e * b * c / (4 * a \\
& * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x - b / c * a / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * e^3 \\
& + 5/16 * e * f^2 * b^5 / c^4 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x + 5/2 * e * f^2 * b^5 / c^3 / (4 * \\
& a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x + 3 * e * f^2 * a / c^3 * b * x / (c * x^2 + b * x + a)^{(3/2)} + 23 * f \\
& ^3 * b^4 / c^3 * a / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * x + 3 / c^{(5/2)} * \ln((1/2 * b + c * x) / c \\
& ^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * d * f^2 + 3 / c^{(5/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b \\
& * x + a)^{(1/2)}) * e^2 * f - x^2 / c / (c * x^2 + b * x + a)^{(3/2)} * e^3 + 1/24 * b^2 / c^3 / (c * x^2 + b * x + a) \\
& ^{(3/2)} * e^3 - 2/3 * a / c^2 / (c * x^2 + b * x + a)^{(3/2)} * e^3 - d^2 * e / c / (c * x^2 + b * x + a)^{(3/2)} + 2 / \\
& 3 * d^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * b + 35/8 * f^3 * b^2 / c^{(9/2)} * \ln((1/2 * b + c * x) \\
& / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + 1/2 * f^3 * x^5 / c / (c * x^2 + b * x + a)^{(3/2)} - 5/2 * f^3 * a / c \\
& ^{(7/2)} * \ln((1/2 * b + c * x) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + 35/16 * f^3 * b^3 / c^5 / (c * x^2 \\
& + b * x + a)^{(1/2)} - 35/384 * f^3 * b^5 / c^6 / (c * x^2 + b * x + a)^{(3/2)} + 16/3 * d^3 * c / (4 * a * c - b^2) \\
& ^2 / (c * x^2 + b * x + a)^{(1/2)} * b - 3/2 * x / c / (c * x^2 + b * x + a)^{(3/2)} * f * d^2 - 3/2 * x / c / (c * x^2 + b \\
& * x + a)^{(3/2)} * d * e^2 + 1/4 * b / c^2 / (c * x^2 + b * x + a)^{(3/2)} * f * d^2 + 1/4 * b / c^2 / (c * x^2 + b * x + \\
& a)^{(3/2)} * d * e^2 + 173/96 * f^3 * b^3 / c^5 * a / (c * x^2 + b * x + a)^{(3/2)} - 35/8 * f^3 * b^2 / c^4 * x / \\
& (c * x^2 + b * x + a)^{(1/2)} + 5/2 * f^3 * a / c^3 * x / (c * x^2 + b * x + a)^{(1/2)} - 5/4 * f^3 * a / c^4 * b / (c * \\
& x^2 + b * x + a)^{(1/2)} + 5/6 * f^3 * a / c^2 * x^3 / (c * x^2 + b * x + a)^{(3/2)} - 11/2 * f^3 * b / c^4 * a^2 / (\\
& c * x^2 + b * x + a)^{(3/2)} + 3/2 / c^3 * b / (c * x^2 + b * x + a)^{(1/2)} * e^2 * f - x^3 / c / (c * x^2 + b * x + a)^{(\\
& 3/2)} * d * f^2 - x^3 / c / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f - 1/16 * b^3 / c^4 / (c * x^2 + b * x + a)^{(3/2)} \\
&) * d * f^2 - 1/16 * b^3 / c^4 / (c * x^2 + b * x + a)^{(3/2)} * e^2 * f - 3 / c^2 * x / (c * x^2 + b * x + a)^{(1/2)} * \\
& d * f^2 - 3 / c^2 * x / (c * x^2 + b * x + a)^{(1/2)} * e^2 * f + 3/2 / c^3 * b / (c * x^2 + b * x + a)^{(1/2)} * d * f^2 \\
& - 7/4 * f^3 * b / c^2 * x^4 / (c * x^2 + b * x + a)^{(3/2)} + 35/16 * f^3 * b^5 / c^5 / (4 * a * c - b^2) / (c * x^2 \\
& + b * x + a)^{(1/2)} - 35/24 * f^3 * b^2 / c^3 * x^3 / (c * x^2 + b * x + a)^{(3/2)} + 35/16 * f^3 * b^3 / c^4 * x \\
& ^2 / (c * x^2 + b * x + a)^{(3/2)} + 35/64 * f^3 * b^4 / c^5 * x / (c * x^2 + b * x + a)^{(3/2)} - 35/384 * f^3 * b \\
& ^7 / c^6 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} - 35/48 * f^3 * b^7 / c^5 / (4 * a * c - b^2)^2 / (c * x \\
& ^2 + b * x + a)^{(1/2)} - 1/4 * b / c^2 * x / (c * x^2 + b * x + a)^{(3/2)} * e^3 + 1/24 * b^4 / c^3 / (4 * a * c - b^2 \\
&) / (c * x^2 + b * x + a)^{(3/2)} * e^3 + 1/3 * b^4 / c^2 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a)^{(1/2)} * e^3 \\
& + 4/3 * d^3 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * c + 32/3 * d^3 * c^2 / (4 * a * c - b^2)^2 / (c * \\
& x^2 + b * x + a)^{(1/2)} * x + 1/2 * b^2 / c / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * x * d * e^2 + a / c / (4 \\
& * a * c - b^2) / (c * x^2 + b * x + a)^{(3/2)} * b * f * d^2 + 2 * b^4 / c^2 / (4 * a * c - b^2)^2 / (c * x^2 + b * x + a) \\
& ^{(1/2)} * d * e * f - 5/2 * f^3 * a / c^3 * b^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * x + 1/2 * b^2 / c /
\end{aligned}$$

$$\begin{aligned}
& (4ac-b^2)/(c^2+bx+a)^{3/2} * x * f * d^2 + a/c / (4ac-b^2) / (c^2+bx+a)^{3/2} \\
& * b * d * e^2 - 33/4 * f^3 * b^2 / c^3 * a^2 / (4ac-b^2) / (c^2+bx+a)^{3/2} * x - 66 * f^3 * b^2 / \\
& c^2 * a^2 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * x + 23/8 * f^3 * b^4 / c^4 * a / (4ac-b^2) / \\
& (c^2+bx+a)^{3/2} * x + 6 * e * f^2 * a^2 / c^3 * b^2 / (4ac-b^2) / (c^2+bx+a)^{3/2} + 4 \\
& 8 * e * f^2 * a^2 / c^2 * b^2 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} - 15/2 * e * f^2 * b^3 / c^3 / (4 \\
& ac-b^2) / (c^2+bx+a)^{1/2} * x - 1/8 * b^4 / c^3 / (4ac-b^2) / (c^2+bx+a)^{3/2} \\
& * x * d * f^2 - 1/8 * b^4 / c^3 / (4ac-b^2) / (c^2+bx+a)^{3/2} * x * e^2 * f - b^4 / c^2 / (4ac \\
& - b^2)^2 / (c^2+bx+a)^{1/2} * x * d * f^2 - b^4 / c^2 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} \\
& * x * e^2 * f + 3/4 * b^3 / c^3 * a / (4ac-b^2) / (c^2+bx+a)^{3/2} * d * f^2 - 19/8 * e * f^2 * b \\
& ^4 / c^4 * a / (4ac-b^2) / (c^2+bx+a)^{3/2} - 19 * e * f^2 * b^4 / c^3 * a / (4ac-b^2)^2 / (\\
& c^2+bx+a)^{1/2} + 16 * a * c / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * x * f * d^2 + 16 * a * c / \\
& (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * x * d * e^2 + 3/2 * b / c^2 * x^2 / (c^2+bx+a)^{3/2} \\
& * d * f^2 + 3/2 * b / c^2 * x^2 / (c^2+bx+a)^{3/2} * e^2 * f + 3/8 * b^2 / c^3 * x / (c^2+bx+a)^{3/2} \\
& * d * f^2 + 3/8 * b^2 / c^3 * x / (c^2+bx+a)^{3/2} * e^2 * f + 5/4 * e * f^2 * b^6 / c^4 / (4ac \\
& - b^2)^2 / (c^2+bx+a)^{1/2} - 3 * e * f^2 * b^2 / c^4 * a / (c^2+bx+a)^{3/2} + 15/2 * e * f \\
& ^2 * b / c^3 * x / (c^2+bx+a)^{1/2} + 35/8 * f^3 * b^4 / c^4 / (4ac-b^2) / (c^2+bx+a)^{1/2} \\
& * x - 1/2 * b^5 / c^3 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * d * f^2 - 1/2 * b^5 / c^3 / (4ac \\
& - b^2)^2 / (c^2+bx+a)^{1/2} * e^2 * f + b / c^3 * a / (c^2+bx+a)^{3/2} * d * f^2 + b / c^3 \\
& * a / (c^2+bx+a)^{3/2} * e^2 * f + 3/2 / c^3 * b^3 / (4ac-b^2) / (c^2+bx+a)^{1/2} * d * \\
& f^2 + 3/2 / c^3 * b^3 / (4ac-b^2) / (c^2+bx+a)^{1/2} * e^2 * f - 6 * x^2 / c / (c^2+bx+a) \\
& ^{3/2} * d * e * f + 1/4 * b^2 / c^3 / (c^2+bx+a)^{3/2} * d * e * f + 1/12 * b^3 / c^2 / (4ac-b^2) \\
& / (c^2+bx+a)^{3/2} * x * e^3 + 2/3 * b^3 / c / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * x * e^ \\
& 3 - 1/2 * b^2 / c^2 * a / (4ac-b^2) / (c^2+bx+a)^{3/2} * e^3 - 8 * b * a / (4ac-b^2)^2 / (c \\
& ^2+bx+a)^{1/2} * x * e^3 - 4 * b^2 / c * a / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * e^3 - 4 * a / \\
& c^2 / (c^2+bx+a)^{3/2} * d * e * f - 15/4 * e * f^2 * b^4 / c^4 / (4ac-b^2) / (c^2+bx+a)^{1/2} \\
& + 12 * e * f^2 * a / c^2 * x^2 / (c^2+bx+a)^{3/2} + 5/2 * e * f^2 * b / c^2 * x^3 / (c^2+bx \\
& + a)^{3/2} - 15/4 * e * f^2 * b^2 / c^3 * x^2 / (c^2+bx+a)^{3/2} - 15/16 * e * f^2 * b^3 / c^4 * x / \\
& (c^2+bx+a)^{3/2} + 5/32 * e * f^2 * b^6 / c^5 / (4ac-b^2) / (c^2+bx+a)^{3/2} - d^2 * \\
& e * b^2 / c / (4ac-b^2) / (c^2+bx+a)^{3/2} + 23/2 * f^3 * b^5 / c^4 * a / (4ac-b^2)^2 / (c \\
& ^2+bx+a)^{1/2} - 33/8 * f^3 * b^3 / c^4 * a^2 / (4ac-b^2) / (c^2+bx+a)^{3/2} - 33 * f \\
& ^3 * b^3 / c^3 * a^2 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} - 5/4 * f^3 * a / c^4 * b^3 / (4ac-b \\
& ^2) / (c^2+bx+a)^{1/2} - 33/16 * f^3 * b^2 / c^4 * a * x / (c^2+bx+a)^{3/2} - 1/16 * b^5 / \\
& c^4 / (4ac-b^2) / (c^2+bx+a)^{3/2} * d * f^2 - 1/16 * b^5 / c^4 / (4ac-b^2) / (c^2+bx \\
& + a)^{3/2} * e^2 * f - 33/4 * f^3 * b / c^3 * a * x^2 / (c^2+bx+a)^{3/2} - 35/192 * f^3 * b^6 / c \\
& ^5 / (4ac-b^2) / (c^2+bx+a)^{3/2} * x - 35/24 * f^3 * b^6 / c^4 / (4ac-b^2)^2 / (c^2 \\
& + bx+a)^{1/2} * x + 23/16 * f^3 * b^5 / c^5 * a / (4ac-b^2) / (c^2+bx+a)^{3/2} + 1/4 * b^3 \\
& / c^2 / (4ac-b^2) / (c^2+bx+a)^{3/2} * f * d^2 + 1/4 * b^3 / c^2 / (4ac-b^2) / (c^2+bx \\
& + a)^{3/2} * d * e^2 + 4 * b^2 / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * x * f * d^2 + 4 * b^2 / (4 \\
& ac-b^2)^2 / (c^2+bx+a)^{1/2} * x * d * e^2 + 2 * b^3 / c / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} \\
& * f * d^2 + 2 * b^3 / c / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * d * e^2 + 2 * a / (4ac-b^2) \\
& / (c^2+bx+a)^{3/2} * x * f * d^2 + 2 * a / (4ac-b^2) / (c^2+bx+a)^{3/2} * x * d * e^2 + 8 * \\
& a / (4ac-b^2)^2 / (c^2+bx+a)^{1/2} * b * f * d^2 + 8 * a / (4ac-b^2)^2 / (c^2+bx+a) \\
& ^{1/2} * b * d * e^2 - 2 * d^2 * e * b / (4ac-b^2) / (c^2+bx+a)^{3/2} * x
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 48.3546, size = 8338, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 - \\ & 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2 \\ & *c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4 \\ & + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60* \\ & a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^4 \\ & + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64* \\ & a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^2 \\ & - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3* \\ & b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f \\ & ^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c \\ & + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6 \\ & *a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2) \\ & *x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7 \\ & - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8 \\ & *a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^ \\ & 3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + \\ & a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3 \\ & + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^ \\ & 2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x \\ & ^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105 \\ & *a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c \\ & ^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (3 \end{aligned}$$

$$\begin{aligned}
& 5b^6c^2 - 279a^2b^4c^3 + 588a^2b^2c^4 - 160a^3c^5) f^3 - 12(2(b^4 \\
& c^4 - 7a^2b^2c^5 + 8a^2c^6) d - (5b^5c^3 - 37a^2b^3c^4 + 64a^2b^2c^5) e) f^2 + 12((b^2c^6 + 4a^2c^7) d^2 + (b^3c^5 - 12a^2b^2c^6) d e - 2(b \\
& ^4c^4 - 7a^2b^2c^5 + 8a^2c^6) e^2) f) x^3 - 12(2(3a^2b^3c^3 - 20a^3b^2c^4) d - (15a^2b^4c^2 - 100a^3b^2c^3 + 128a^4c^4) e) f^2 + 3(\\
& 64b^2c^7d^3 - 96b^2c^6d^2e + 24(b^3c^5 + 4a^2b^2c^6) d e^2 - 16(a^2b^2c^5 + 4a^2c^6) e^3 - (35b^7c - 230a^2b^5c^2 + 232a^2b^3c^3 + 448a^3b^2c^4) f^3 - 12(2(b^5c^3 - 6a^2b^3c^4) d - (5b^6c^2 - 30a^2b^4c^3 + 16a^2b^2c^4 + 64a^3c^5) e) f^2 + 24((b^3c^5 + 4a^2b^2c^6) d^2 - 4(a^2b^2c^5 + 4a^2c^6) d e - (b^5c^3 - 6a^2b^3c^4) e^2) f) x^2 + 24(8a^2b^2c^5d^2 - 32a^3c^5d e - (3a^2b^3c^3 - 20a^3b^2c^4) e^2) f + 6(48a^2b^2c^5d e^2 - 32a^2b^2c^5e^3 + 8(b^2c^6 + 4a^2c^7) d^3 - 12(b^3c^5 + 4a^2b^2c^6) d^2 e - (35a^2b^6c - 265a^2b^4c^2 + 504a^3b^2c^3 - 80a^4c^4) f^3 - 12(2(a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5) d - (5a^2b^5c^2 - 35a^2b^3c^3 + 52a^3b^2c^4) e) f^2 + 24(2a^2b^2c^5d^2 - 8a^2b^2c^5d e - (a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5) e^2) f) x) \sqrt{cx^2 + bx + a}) / (a^2b^4c^5 - 8a^3b^2c^6 + 16a^4c^7 + (b^4c^7 - 8a^2b^2c^8 + 16a^2c^9) x^4 + 2(b^5c^6 - 8a^2b^3c^7 + 16a^2b^2c^8) x^3 + (b^6c^5 - 6a^2b^4c^6 + 32a^3c^8) x^2 + 2(a^2b^5c^5 - 8a^2b^3c^6 + 16a^3b^2c^7) x), -1/24(3((24(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) e^2 f + 5(7b^6c^2 - 60a^2b^4c^3 + 144a^2b^2c^4 - 64a^3c^5) f^3 + 12(2(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) d - 5(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5) e) f^2) x^4 + 24(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4) e^2 f + 5(7a^2b^6 - 60a^3b^4c + 144a^4b^2c^2 - 64a^5c^3) f^3 + 2(24(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5) e^2 f + 5(7b^7c - 60a^2b^5c^2 + 144a^2b^3c^3 - 64a^3b^2c^4) f^3 + 12(2(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5) d - 5(b^6c^2 - 8a^2b^4c^3 + 16a^2b^2c^4) e) f^2) x^3 + 12(2(a^2b^4c^2 - 8a^3b^2c^3 + 16a^4c^4) d - 5(a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3) e) f^2 + (24(b^6c^2 - 6a^2b^4c^3 + 32a^3c^5) e^2 f + 5(7b^8 - 46a^2b^6c + 24a^2b^4c^2 + 224a^3b^2c^3 - 128a^4c^4) f^3 + 12(2(b^6c^2 - 6a^2b^4c^3 + 32a^3c^5) d - 5(b^7c - 6a^2b^5c^2 + 32a^3b^2c^4) e) f^2) x^2 + 2(24(a^2b^5c^2 - 8a^2b^3c^3 + 16a^3b^2c^4) e^2 f + 5(7a^2b^7 - 60a^2b^5c + 144a^3b^3c^2 - 64a^4b^2c^3) f^3 + 12(2(a^2b^5c^2 - 8a^2b^3c^3 + 16a^3b^2c^4) d - 5(a^2b^6c - 8a^2b^4c^2 + 16a^3b^2c^3) e) f^2) x) \sqrt{-c} \arctan(1/2 \sqrt{cx^2 + bx + a}) (2cx + b) \sqrt{-c} / (c^2x^2 + bcx + ac)) - 2(192a^2b^2c^5d e^2 - 128a^3c^5e^3 + 6(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) f^3 x^5 + 3(12(b^4c^4 - 8a^2b^2c^5 + 16a^2c^6) e f^2 - 7(b^5c^3 - 8a^2b^3c^4 + 16a^2b^2c^5) f^3) x^4 - 8(b^3c^5 - 12a^2b^2c^6) d^3 - 48(a^2b^2c^5 + 4a^2c^6) d^2 e - (105a^2b^5c - 760a^3b^3c^2 + 1296a^4b^2c^3) f^3 + 4(32c^8d^3 - 48b^2c^7d^2e + 12(b^2c^6 + 4a^2c^7) d e^2 + 2(b^3c^5 - 12a^2b^2c^6) e^3 - (35b^6c^2 - 279a^2b^4c^3 + 588a^2b^2c^4 - 160a^3c^5) f^3 - 12(2(b^4c^4 - 7a^2b^2c^5 + 8a^2c^6) d - (5b^5c^3 - 37a^2b^3c^4 + 64a^2b^2c^5) e) f^2 + 12((b^2c^6 + 4a^2c^7) d^2 + (b^3c^5 - 12a^2b^2c^6) d e - 2(b^4c^4 - 7a^2b^2c^5 + 8a^2c^6) e^2) f) x^3 - 12(2(3a^2b^3c^3
\end{aligned}$$

$$\begin{aligned}
& 3 - 20a^3bc^4)d - (15a^2b^4c^2 - 100a^3b^2c^3 + 128a^4c^4)e) * f \\
& ^2 + 3(64b^7c^3d^3 - 96b^2c^6d^2e + 24(b^3c^5 + 4a^2bc^6)d^2e^2 - \\
& 16(a^2b^2c^5 + 4a^2c^6)e^3 - (35b^7c - 230a^2b^5c^2 + 232a^2b^3c^3 \\
& + 448a^3bc^4) * f^3 - 12(2(b^5c^3 - 6a^2b^3c^4)d - (5b^6c^2 - 30a \\
& a^2b^4c^3 + 16a^2b^2c^4 + 64a^3c^5)e) * f^2 + 24((b^3c^5 + 4a^2bc^6) \\
& * d^2 - 4(a^2b^2c^5 + 4a^2c^6)d^2e - (b^5c^3 - 6a^2b^3c^4)e^2) * f) * x^2 \\
& + 24(8a^2b^2c^5d^2 - 32a^3c^5d^2e - (3a^2b^3c^3 - 20a^3bc^4)e^2) \\
&) * f + 6(48a^2b^2c^5d^2e^2 - 32a^2b^2c^5e^3 + 8(b^2c^6 + 4a^2c^7)d^3 \\
& - 12(b^3c^5 + 4a^2bc^6)d^2e - (35a^2b^6c - 265a^2b^4c^2 + 504a^3b^2c^3 \\
& - 80a^4c^4) * f^3 - 12(2(a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5)d \\
& - (5a^2b^5c^2 - 35a^2b^3c^3 + 52a^3bc^4)e) * f^2 + 24(2a^2b^2c^5d^2 \\
& ^2 - 8a^2b^2c^5d^2e - (a^2b^4c^3 - 7a^2b^2c^4 + 4a^3c^5)e^2) * f) * x) * s \\
& \text{sqrt}(c*x^2 + b*x + a) / (a^2b^4c^5 - 8a^3b^2c^6 + 16a^4c^7 + (b^4c^7 \\
& - 8a^2b^2c^8 + 16a^2c^9) * x^4 + 2(b^5c^6 - 8a^2b^3c^7 + 16a^2b^2c^8) * \\
& x^3 + (b^6c^5 - 6a^2b^4c^6 + 32a^3c^8) * x^2 + 2(a^2b^5c^5 - 8a^2b^3c^6 \\
& + 16a^3bc^7) * x)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)

[Out] Timed out

Giac [A] time = 1.54095, size = 1891, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")

[Out] $1/12 * (((3 * (2 * (b^4 * c^3 * f^3 - 8 * a * b^2 * c^4 * f^3 + 16 * a^2 * c^5 * f^3) * x / (b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6) - (7 * b^5 * c^2 * f^3 - 56 * a * b^3 * c^3 * f^3 + 112 * a^2 * b * c^4 * f^3 - 12 * b^4 * c^3 * f^2 * e + 96 * a * b^2 * c^4 * f^2 * e - 192 * a^2 * c^5 * f^2 * e) / (b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6)) * x + 4 * (32 * c^7 * d^3 + 12 * b^2 * c^5 * d^2 * f + 48 * a$

$$\begin{aligned}
& *c^6*d^2*f - 24*b^4*c^3*d*f^2 + 168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 - 3 \\
& 5*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*b^2*c^3*f^3 + 160*a^3*c^4*f^3 - 4 \\
& 8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4 \\
& 44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^2 \\
& - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4* \\
& e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6 \\
& *d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c \\
& ^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c \\
& ^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6 \\
& *c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e \\
& + 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f* \\
& e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6 \\
& 6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2 \\
& *d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2* \\
& b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a* \\
& b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2* \\
& e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2 \\
& *b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 \\
& + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f \\
& + 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^ \\
& 3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768 \\
& *a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^ \\
& 3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2 \\
& + 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a) \\
& ^{(3/2)} - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2 \\
& *f*e^2)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(9/2)} \\
&)
\end{aligned}$$

$$3.119 \quad \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}} dx$$

Optimal. Leaf size=444

$$\frac{2(-2cx(-c^2(16a^2f^2 + 12abef + b^2(-(2df + e^2)))) + b^2cf(14af + be) - c^3(8bde - 4a(2df + e^2)) - 2b^4f^2 + 8c^4d^2)}{3c^3(b^2 - 4ac)}$$

[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x)/(3*c^3*(b^2 - 4*a*c)^2*sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])])/c^(5/2)

Rubi [A] time = 0.450728, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1660, 12, 621, 206}

$$\frac{2(-2cx(-c^2(16a^2f^2 + 12abef + b^2(-(2df + e^2)))) + b^2cf(14af + be) - c^3(8bde - 4a(2df + e^2)) - 2b^4f^2 + 8c^4d^2)}{3c^3(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]

[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + 2*d*f)))*x)/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x)/(3*c^3*(b^2 - 4*a*c)^2*sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x + c*x^2])])/c^(5/2)

$$3c^3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2} + (f^2 \operatorname{ArcTanh}[(b + 2cx)/(2\sqrt{c} \sqrt{a + bx + cx^2})])/c^{5/2}$$

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex + fx^2)^2}{(a + bx + cx^2)^{5/2}} dx &= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{3c^3(b^2 - 4ac)(a + b)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{3c^3(b^2 - 4ac)(a + b)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{3c^3(b^2 - 4ac)(a + b)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{3c^3(b^2 - 4ac)(a + b)} \\
&= \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd - af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2 + 2df)) - (2c^4d^2 + b^4f^2 - 2)}{3c^3(b^2 - 4ac)(a + b)}
\end{aligned}$$

Mathematica [A] time = 1.31537, size = 387, normalized size = 0.87

$$\frac{2(2b^2c(21a^2f^2x - 2ac(d(e - 6fx) + x(-3e^2 + 3efx - 7f^2x^2))) + c^2x(3d^2 + 2dx(fx - 6e) + e^2x^2)) + b^3(-3a^2f^2 + 18ac)}{3c^3(b^2 - 4ac)(a + bx + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x]

[Out] (2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^3*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x)*(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6*d*x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*e + 3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*(3*e + 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(-6*e + f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f^2*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/2)

Maple [B] time = 0.061, size = 1786, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^{(5/2)}, x)$

[Out] $\frac{1}{2}f^2b/c^2x^2/(c*x^2+b*x+a)^{(3/2)} + \frac{1}{8}f^2b^2/c^3x/(c*x^2+b*x+a)^{(3/2)} - \frac{1}{48}f^2b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} - \frac{1}{6}f^2b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} + \frac{1}{3}f^2b/c^3a/(c*x^2+b*x+a)^{(3/2)} + \frac{1}{2}f^2/c^3b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} - 2*ef*x^2/c/(c*x^2+b*x+a)^{(3/2)} + \frac{1}{12}e*f*b^2/c^3/(c*x^2+b*x+a)^{(3/2)} - \frac{4}{3}e*f*a/c^2/(c*x^2+b*x+a)^{(3/2)} - 16e*f*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x - 8e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} - 2e*f*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x + \frac{32}{3}d^2*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x + \frac{16}{3}d^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * b + \frac{4}{3}d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x*c - \frac{16}{3}d*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} - \frac{32}{3}d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x - e*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} + \frac{1}{3}b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x*d*f + \frac{2}{3}a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * b*d*f + \frac{32}{3}a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x*d*f + \frac{1}{2}f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x - \frac{1}{48}f^2*b^3/c^4/(c*x^2+b*x+a)^{(3/2)} - f^2/c^2*x/(c*x^2+b*x+a)^{(1/2)} + \frac{1}{2}f^2/c^3*b/(c*x^2+b*x+a)^{(1/2)} - \frac{2}{3}d*e/c/(c*x^2+b*x+a)^{(3/2)} + \frac{2}{3}d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * b - \frac{1}{2}x/c/(c*x^2+b*x+a)^{(3/2)} * e^2 + \frac{1}{12}b/c^2/(c*x^2+b*x+a)^{(3/2)} * e^2 + \frac{4}{3}b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * d*f + \frac{4}{3}a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x*d*f + \frac{1}{3}a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * b*e^2 + \frac{16}{3}a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x*e^2 + \frac{16}{3}a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * b*d*f + f^2/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x - \frac{4}{3}d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x - \frac{2}{3}d*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} - x/c/(c*x^2+b*x+a)^{(3/2)} * d*f + f^2/c^(5/2) * ln((1/2*b+c*x)/c^(1/2) + (c*x^2+b*x+a)^(1/2)) + \frac{4}{3}e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x + \frac{4}{3}f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x + \frac{1}{6}b/c^2/(c*x^2+b*x+a)^{(3/2)} * d*f - \frac{1}{3}f^2*x^3/c/(c*x^2+b*x+a)^{(3/2)} - \frac{1}{2}e*f*b/c^2*x/(c*x^2+b*x+a)^{(3/2)} + \frac{1}{12}e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} + \frac{2}{3}e*f*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} - \frac{1}{24}f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x + \frac{1}{6}b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x*e^2 + \frac{1}{6}b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * d*f + \frac{8}{3}b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x*d*f + \frac{1}{12}b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * e^2 + \frac{4}{3}b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x*e^2 + \frac{2}{3}b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * e^2 + \frac{2}{3}a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x*e^2 + \frac{8}{3}a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * b*e^2 - \frac{1}{3}f^2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} * x + \frac{1}{4}f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} + \frac{2}{3}f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^{(1/2)} + \frac{1}{6}e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(3/2)} * x$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 27.422, size = 3252, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*f^2)*\sqrt{c}*\log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*\sqrt{c*x^2 + b*x + a}*(2*c*x + b)*\sqrt{c} - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 + (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 12*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*\sqrt{c*x^2 + b*x + a}]/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*f^2)*\sqrt{-c}*\arctan(1/2*\sqrt{c*x^2 + b*x + a}*(2*c*x + b)*\sqrt{-c}/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 + \end{aligned}$$

$$(2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 12*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)

[Out] Timed out

Giac [A] time = 1.38829, size = 792, normalized size = 1.78

$$\frac{f^2 \log\left(\left|-2\left(\sqrt{cx} - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{c^{\frac{5}{2}}} + \frac{2\left(\left(\frac{2(8c^5d^2+2b^2c^3df+8ac^4df-2b^4cf^2+14ab^2c^2f^2-16a^2c^3f^2-8bc^4de+b^3c^2fe-12abc^3fe+}{b^4c^2-8ab^2c^3+16a^2c^4}\right)}{c^{\frac{5}{2}}}\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")

[Out] -f^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 2/3*(((2*(8*c^5*d^2 + 2*b^2*c^3*d*f + 8*a*c^4*d*f - 2*b^4*c*f^2 + 14*a*b^2*c^2*f^2 - 16*a^2*c^3*f^2 - 8*b*c^4*d*e + b^3*c^2*f*e - 12*a*b*c^3*f*e + b^2*c^3*e^2 + 4*a*c^4*e^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^4*d^2 + 2*b^3*c^2*d*f + 8*a*b*c^3*d*f - b^5*f^2 + 6*a*b^3*c*f^2 - 8*b^2*c^3*d*e - 4*a*b^2*c^2*f*e - 16*a^2*c^3*f*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 + 4*a*b^

$$\begin{aligned}
& 2*c^2*d*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2 - b^3*c^2*d*e - 4*a \\
& *b*c^3*d*e - 8*a^2*b*c^2*f*e + 2*a*b^2*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16 \\
& *a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3 \\
& *f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e - \\
& 8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^{(\\
& 3/2)
\end{aligned}$$

$$3.120 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}} dx$$

Optimal. Leaf size=131

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])

Rubi [A] time = 0.0852556, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2(b + 2cx)\left(4af + \frac{b^2f}{c} - 4be + 8cd\right)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2), x]

[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{5/2}} dx &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{2 \int \frac{8cd - 4be + 4af + \frac{b^2f}{c}}{2(a + bx + cx^2)^{3/2}} dx}{3 (b^2 - 4ac)} \\ &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} - \frac{\left(8cd - 4be + 4af + \frac{b^2f}{c} \right) \int \frac{1}{(a + bx + cx^2)}}{3 (b^2 - 4ac)} \\ &= \frac{2 \left(c \left(2ae - b \left(d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{3c (b^2 - 4ac) (a + bx + cx^2)^{3/2}} + \frac{2 \left(8cd - 4be + 4af + \frac{b^2f}{c} \right) (b + 2cx)}{3 (b^2 - 4ac)^2 \sqrt{a + bx + cx^2}} \end{aligned}$$

Mathematica [A] time = 0.394814, size = 147, normalized size = 1.12

$$\frac{8b(2a^2f + 3ac(d - ex + fx^2) - 2c^2x^2(ex - 3d)) + 16c(-a^2e + acx(3d + fx^2) + 2c^2dx^3) - 4b^2(a(e - 6fx) - cx(3d - 6fx))}{3(b^2 - 4ac)^2(a + x(b + cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2), x]

[Out] (-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d + f*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f - 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2))

Maple [A] time = 0.052, size = 185, normalized size = 1.4

$$\frac{16ac^2fx^3 + 4b^2cfx^3 - 16bc^2ex^3 + 32c^3dx^3 + 24abcfx^2 + 6b^3fx^2 - 24b^2cex^2 + 48bc^2dx^2 + 24ab^2fx - 24abcex + 48a^2c^2d}{48a^2c^2 - 24acb^2 + 3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x)

[Out] $\frac{2/3/(c*x^2+b*x+a)^{(3/2)}*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d*x^3+12*a*b*c*f*x^2+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-12*a*b*c*e*x+24*a*c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2*e+12*a*b*c*d-b^3*d)}{(16*a^2*c^2-8*a*b^2*c+b^4)}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 23.2333, size = 614, normalized size = 4.69

$$\frac{2(8a^2bf + 2(8c^3d - 4bc^2e + (b^2c + 4ac^2)f)x^3 + 3(8bc^2d - 4b^2ce + (b^3 + 4abc)f)x^2 - (b^3 - 12abc)d - 2(ab^2 + 4a^2c^2d) + 2(a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)x^3 + (b^6 - 6ab^5c + 32a^3c^3)x^2 + 2(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2))}{3(a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)x^3 + (b^6 - 6ab^5c + 32a^3c^3)x^2 + 2(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out] $\frac{2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b^2 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e)*x}{(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 6*a*b^5*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)}$

*x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2), x)

[Out] Timed out

Giac [B] time = 1.36368, size = 356, normalized size = 2.72

$$\frac{\left(\left(\frac{2(8c^3d+b^2cf+4ac^2f-4bc^2e)x}{b^4c^2-8ab^2c^3+16a^2c^4} + \frac{3(8bc^2d+b^3f+4abcf-4b^2ce)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x + \frac{3(2b^2cd+8ac^2d+4ab^2f-b^3e-4abce)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x - \frac{b^3d-12abcd-8a^2bf+2ab^2e+8a^2ce}{b^4c^2-8ab^2c^3+16a^2c^4}}{3(cx^2+bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2), x, algorithm="giac")

[Out] 1/3*(((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*e - 4*a*b*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*d - 12*a*b*c*d - 8*a^2*b*f + 2*a*b^2*e + 8*a^2*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)

$$3.121 \quad \int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx$$

Optimal. Leaf size=51

$$\frac{1}{10} \tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right) + \frac{1}{5} \tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqrt[-7 + 2*x + 5*x^2]]/5

Rubi [A] time = 0.0656829, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {986, 1029, 203, 207}

$$\frac{1}{10} \tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right) + \frac{1}{5} \tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]

[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqrt[-7 + 2*x + 5*x^2]]/5

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
```

[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f), 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx &= -\left(\frac{1}{50} \int \frac{-100-50x}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx\right) + \frac{1}{50} \int \frac{-50-50x}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)} dx \\ &= 400 \operatorname{Subst}\left(\int \frac{1}{160000+100x^2} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right) + 1600 \operatorname{Subst}\left(\int \frac{-6}{\sqrt{-7+2x+5x^2}} dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right) \\ &= \frac{1}{10} \tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right) + \frac{1}{5} \tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right) \end{aligned}$$

Mathematica [C] time = 0.0402311, size = 81, normalized size = 1.59

$$\left(\frac{1}{10} - \frac{i}{20}\right) \tanh^{-1}\left(\frac{\left(\frac{1}{100} + \frac{i}{50}\right)((100-40i)x + (164-8i))}{\sqrt{5x^2+2x-7}}\right) - \left(\frac{1}{20} - \frac{i}{10}\right) \tanh^{-1}\left(\frac{\left(\frac{1}{50} + \frac{i}{100}\right)((-100-40i)x - (164+8i))}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]

[Out] (-1/20 + I/10)*ArcTan[((1/50 + I/100)*((-164 - 8*I) - (100 + 40*I)*x))/Sqrt[-7 + 2*x + 5*x^2]] + (1/10 - I/20)*ArcTanh[((1/100 + I/50)*((164 - 8*I) + (100 - 40*I)*x))/Sqrt[-7 + 2*x + 5*x^2]]

Maple [B] time = 0.108, size = 144, normalized size = 2.8

$$-\frac{1}{10} \sqrt{-4 \frac{(2+x)^2}{(-1-x)^2} + 9} \left(2 \operatorname{Arctanh} \left(\frac{1}{5} \sqrt{-4 \frac{(2+x)^2}{(-1-x)^2} + 9} \right) + \arctan \left(\frac{10+5x}{-2-2x} \sqrt{-4 \frac{(2+x)^2}{(-1-x)^2} + 9} \left(4 \frac{(2+x)^2}{(-1-x)^2} - 9 \right)^{-1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x)`

[Out] `-1/10*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(2+x)^2/(-1-x)^2+9)^(1/2))+arctan(5/2*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)/(4*(2+x)^2/(-1-x)^2-9)*(2+x)/(-1-x)))/(-4*(2+x)^2/(-1-x)^2-9)/(1+(2+x)/(-1-x))^2^(1/2)/(1+(2+x)/(-1-x))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2 + 12x + 8)\sqrt{5x^2 + 2x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)`

Fricas [B] time = 1.91039, size = 424, normalized size = 8.31

$$\frac{1}{20} \arctan \left(\frac{27x^2 + 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56} \right) + \frac{1}{20} \arctan \left(-\frac{27x^2 - 20\sqrt{5x^2 + 2x - 7}(x + 2) + 36x}{31x^2 + 16x - 56} \right) + \frac{1}{20} \log \left(\frac{15x^2 + 5\sqrt{5x^2 + 2x - 7}(x + 1)}{31x^2 + 16x - 56} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")`

[Out] `1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x + 1))/(31*x^2 + 16*x - 56))`

+ 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1) + 26*x + 9)/x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(5x+7)}(5x^2+12x+8)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)

[Out] Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)

Giac [B] time = 1.49367, size = 277, normalized size = 5.43

$$-\frac{1}{10} \arctan\left(-\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} + 5}{2(\sqrt{5} + 5)}\right) - \frac{1}{10} \arctan\left(\frac{5\sqrt{5}x + 6\sqrt{5} - 5\sqrt{5x^2 + 2x - 7} - 5}{2(\sqrt{5} - 5)}\right) + \frac{1}{10} \log\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")

[Out] -1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/(sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5) + 65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)

$$3.122 \quad \int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}} dx$$

Optimal. Leaf size=1432

result too large to display

```
[Out] -(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(d + e*x + f*x^2)))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2))*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))^2]*EllipticF[2*ArcTan[((2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x])]/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)])))/4)/(((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))]
```

Rubi [A] time = 6.21881, antiderivative size = 1432, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} =$

0.103, Rules used = {992, 935, 1103}

$$\sqrt[4]{db^2 + (\sqrt{b^2 - 4acd} - ae)b - a(2cd + \sqrt{b^2 - 4ace} - 2af)} \left(b + 2cx + \sqrt{b^2 - 4ac} \right)^{3/2} \sqrt{2a + (b + \sqrt{b^2 - 4ac})x} \sqrt{\frac{4fa^2}{(4fa^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]

[Out] -((((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^(2*(d + e*x + f*x^2)))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))^2)*EllipticF[2*ArcTan[((2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)))/4)/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (

$$\frac{b + \sqrt{b^2 - 4ac}x^2}{((b + \sqrt{b^2 - 4ac})^2d - 2a(b + \sqrt{b^2 - 4ac})e + 4a^2f)(b + \sqrt{b^2 - 4ac} + 2cx^2)}$$

Rule 992

`Int[1/(Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]`

Rule 935

`Int[1/(Sqrt[(d_) + (e_)*(x_)]*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)]/((e*f - d*g)*Sqrt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 - b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]`

Rule 1103

`Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]`

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}} dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\right) \int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+(b+\sqrt{b^2-4ac})x}}} \sqrt{a+bx+cx^2}}{\left(2(b+\sqrt{b^2-4ac}+2cx)^{3/2}\sqrt{2a+(b+\sqrt{b^2-4ac})x}\sqrt{\frac{(4ac-(b+\sqrt{b^2-4ac})^2)}{\left((b+\sqrt{b^2-4ac})^2 d-2a(b+\sqrt{b^2-4ac})\right)}}\right)}$$

$$= \frac{\sqrt[4]{b^2d+b(\sqrt{b^2-4ac}d-ae)-a(2cd+\sqrt{b^2-4ac}e-2af)}(b+\sqrt{b^2-4ac}+2cx)}{\dots}$$

Mathematica [A] time = 2.487, size = 670, normalized size = 0.47

$$\frac{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(-\sqrt{e^2-4df}+e+2fx\right)\sqrt{\frac{c\sqrt{b^2-4ac}\left(\sqrt{e^2-4df}+e+2fx\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)\right)}}}{\sqrt{a+x(b+cx)}\sqrt{d+x(e+fx)}\left(f\left(\sqrt{b^2-4ac}-\dots\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]

[Out] -((((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-(c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqrt[-((c*(4*a*f + Sqrt[b^2 - 4*a*c])*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*c]*f*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4

$$*d*f)))*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[\text{ArcSin}[\text{Sqrt}[((-b + \text{Sqrt}[b^2 - 4*a*c])*f + c*(e - \text{Sqrt}[e^2 - 4*d*f]))*(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)))/(((b + \text{Sqrt}[b^2 - 4*a*c])*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - \text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[e^2 - 4*d*f])/((2*c*d - b*e + 2*a*f + \text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[e^2 - 4*d*f]))]/(((-b + \text{Sqrt}[b^2 - 4*a*c])*f + c*(e - \text{Sqrt}[e^2 - 4*d*f]))*\text{Sqrt}[(c*\text{Sqrt}[b^2 - 4*a*c]*(-e + \text{Sqrt}[e^2 - 4*d*f] - 2*f*x)))/(((b + \text{Sqrt}[b^2 - 4*a*c])*f + c*(-e + \text{Sqrt}[e^2 - 4*d*f]))*(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x)))]*\text{Sqrt}[a + x*(b + c*x)]*\text{Sqrt}[d + x*(e + f*x))]]$$

Maple [A] time = 0.836, size = 929, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(c*x^2+b*x+a)^{(1/2)}/(f*x^2+e*x+d)^{(1/2)}, x)$

[Out] $8*(2*b*f^2*x^2-2*c*e*f*x^2+2*x^2*c*f*(-4*d*f+e^2)^{(1/2)}+2*(-4*a*c+b^2)^{(1/2)}*f^2*x^2+2*b*e*f*x+2*x*b*f*(-4*d*f+e^2)^{(1/2)}-8*c*x*f*d+2*(-4*a*c+b^2)^{(1/2)}*e*f*x+2*x*f*(-4*a*c+b^2)^{(1/2)}*(-4*d*f+e^2)^{(1/2)}-2*b*d*f+b*e^2+b*e*(-4*d*f+e^2)^{(1/2)}-2*d*e*c-2*c*d*(-4*d*f+e^2)^{(1/2)}-2*(-4*a*c+b^2)^{(1/2)}*d*f+(-4*a*c+b^2)^{(1/2)}*e^2+e*(-4*a*c+b^2)^{(1/2)}*(-4*d*f+e^2)^{(1/2)})*EllipticF(((f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^{(1/2)}-e)/(f*(-4*a*c+b^2)^{(1/2)}+c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^{(1/2)}+e))^{(1/2)}, ((f*(-4*a*c+b^2)^{(1/2)}+c*(-4*d*f+e^2)^{(1/2)}-b*f+c*e)*(f*(-4*a*c+b^2)^{(1/2)}+c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/(f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}-b*f+c*e)/(f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e))^{(1/2)})*((-4*d*f+e^2)^{(1/2)}*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})*f/(f*(-4*a*c+b^2)^{(1/2)}+c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^{(1/2)}+e))^{(1/2)}*((-4*d*f+e^2)^{(1/2)}*(-b-2*c*x+(-4*a*c+b^2)^{(1/2)})*f/(f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}-b*f+c*e)/(2*f*x+(-4*d*f+e^2)^{(1/2)}+e))^{(1/2)}*(-(f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^{(1/2)}-e)/(f*(-4*a*c+b^2)^{(1/2)}+c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^{(1/2)}+e))^{(1/2)}*(c*x^2+b*x+a)^{(1/2)}*(f*x^2+e*x+d)^{(1/2)}/(1/c/f*(-2*f*x+(-4*d*f+e^2)^{(1/2)}-e)*(2*f*x+(-4*d*f+e^2)^{(1/2)}+e)*(-b-2*c*x+(-4*a*c+b^2)^{(1/2)})*(b+2*c*x+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}/(-4*d*f+e^2)^{(1/2)}/(f*(-4*a*c+b^2)^{(1/2)}-c*(-4*d*f+e^2)^{(1/2)}+b*f-c*e)/(c*f*x^4+b*f*x^3+c*e*x^3+a*f*x^2+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}}{cfx^4 + (ce + bf)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)/(c*f*x^4 + (c*e + b*f)*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx + cx^2}\sqrt{d + ex + fx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)

[Out] Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

$$3.123 \quad \int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}} dx$$

Optimal. Leaf size=652

$$\sqrt{\frac{23}{11}}(-4x - i\sqrt{23} + 1) \sqrt{4x + i\sqrt{23} - 1} \sqrt{6 - (1 - i\sqrt{23})x} \sqrt{\frac{(-\sqrt{23}+11i)(5x^2+3x+2)}{(\sqrt{23}+7i)(-4x-i\sqrt{23}+1)^2}} \left(1 - \frac{\sqrt{\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}(6-(1-i\sqrt{23})x)}{-4x-i\sqrt{23}+1} \right) \sqrt{\frac{11(-\sqrt{23}+3i)}{(\sqrt{23}+7i)}} \\ (23 + i\sqrt{23})^4 \sqrt{\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}} \sqrt{2x^2 - x + 3} \sqrt{5x^2 + 3x + 2} \sqrt{\frac{11(-\sqrt{23}+3i)}{(\sqrt{23}+7i)}}$$

[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))])*(6 - (1 - I*Sqrt[23])*x)))/(1 - I*Sqrt[23] - 4*x)*Sqrt[(11 - (41*(I + Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x)))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2)]/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))])*(6 - (1 - I*Sqrt[23])*x)))/(1 - I*Sqrt[23] - 4*x)^2]*EllipticF[2*ArcTan[((-((3*I - Sqrt[23])/(7*I + Sqrt[23]))))^1/4]*Sqrt[6 - (1 - I*Sqrt[23])*x])/Sqrt[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/88)/((23 + I*Sqrt[23])*(-(3*I - Sqrt[23])/(7*I + Sqrt[23]))^1/4)*Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x)))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2)]

Rubi [A] time = 0.676932, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {992, 935, 1103}

$$\sqrt{\frac{23}{11}}(-4x - i\sqrt{23} + 1) \sqrt{4x + i\sqrt{23} - 1} \sqrt{6 - (1 - i\sqrt{23})x} \sqrt{\frac{(-\sqrt{23}+11i)(5x^2+3x+2)}{(\sqrt{23}+7i)(-4x-i\sqrt{23}+1)^2}} \left(1 - \frac{\sqrt{\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}(6-(1-i\sqrt{23})x)}{-4x-i\sqrt{23}+1} \right) \sqrt{\frac{11(-\sqrt{23}+3i)}{(\sqrt{23}+7i)}} \\ (23 + i\sqrt{23})^4 \sqrt{\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}} \sqrt{2x^2 - x + 3} \sqrt{5x^2 + 3x + 2} \sqrt{\frac{11(-\sqrt{23}+3i)}{(\sqrt{23}+7i)}(6 -$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]))*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x)*Sqrt[(11 - (41*(I + Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x))]/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2))/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]))*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x)^2)*EllipticF[2*ArcTan[(-(3*I - Sqrt[23])/(7*I + Sqrt[23]))^(1/4)*Sqrt[6 - (1 - I*Sqrt[23])*x]]/Sqrt[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/88)/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23]))^(1/4)*Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23]))*(6 - (1 - I*Sqrt[23])*x)]/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^2)))]
```

Rule 992

```
Int[1/(Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 935

```
Int[1/(Sqrt[(d_) + (e_)*(x_)]*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 - b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/((2*q*Sqrt[a + b*x^2 + c*x^4]
```

), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}} dx = \frac{\left(\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+(-1+i\sqrt{23})x}\right) \int \frac{1}{\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+(-1+i\sqrt{23})x}\sqrt{2+3x+5x^2}} dx}{\sqrt{3-x+2x^2}}$$

$$= \frac{\left(2(-1+i\sqrt{23}+4x)^{3/2}\sqrt{6+(-1+i\sqrt{23})x}\sqrt{\frac{(24-(-1+i\sqrt{23})^2)^2(2+3x+5x^2)}{(180-18(-1+i\sqrt{23})+2(-1+i\sqrt{23})^2)(-1+i\sqrt{23})}}\right)}{(24-(-1+i\sqrt{23}))}$$

$$= \frac{\sqrt{\frac{23}{11}}(-1+i\sqrt{23}+4x)^{3/2}\sqrt{6-(1-i\sqrt{23})x}\sqrt{\frac{(11i-\sqrt{23})(2+3x+5x^2)}{(7i+\sqrt{23})(1-i\sqrt{23}-4x)^2}}\left(1-\frac{\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}}{1-i\sqrt{23}}\right)}{(23+i\sqrt{23})^4\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{3-x+2x^2}}$$

Mathematica [A] time = 0.61975, size = 390, normalized size = 0.6

$$\frac{(-4x + i\sqrt{23} + 1)(10ix + \sqrt{31} + 3i)\sqrt{\frac{20ix - 2\sqrt{31} + 6i}{(11i + 5\sqrt{23} - 2\sqrt{31})(4ix + \sqrt{23} - i)}}\sqrt{\frac{(-22 - 10i\sqrt{23} + 4i\sqrt{31})x - \sqrt{713} - i\sqrt{31} - 3i\sqrt{23} + 63}{(11i + 5\sqrt{23} + 2\sqrt{31})(4ix + \sqrt{23} - i)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{(-11i + 5\sqrt{23} - 2\sqrt{31})\sqrt{\frac{10ix + \sqrt{31} + 3i}{(11i + 5\sqrt{23} + 2\sqrt{31})(4ix + \sqrt{23} - i)}}\sqrt{2x^2 - x + 3}}{\sqrt{2x^2 - x + 3}}\right)\right)}{(11i + 5\sqrt{23} + 2\sqrt{31})(4ix + \sqrt{23} - i)\sqrt{2x^2 - x + 3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]

[Out] ((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31] + (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[23] + (4*I)*Sqrt[31])*x]/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt[31])

+ Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))), (1197 + 41*Sqrt[713])/484)) /((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))])*Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2])

Maple [A] time = 3.535, size = 420, normalized size = 0.6

$$\frac{\frac{4i}{23} (2i\sqrt{31} + 5i\sqrt{23} - 11) (i\sqrt{23} - 4x + 1)^2 \sqrt{23}\sqrt{10}}{2i\sqrt{31} - 5i\sqrt{23} - 11} \sqrt{5x^2 + 3x + 2} \sqrt{2x^2 - x + 3}}{\sqrt{\frac{(2i\sqrt{31} - 5i\sqrt{23} - 11)(-1 + 4x + \dots)}{(2i\sqrt{31} + 5i\sqrt{23} - 11)(i\sqrt{23} - 4 \dots)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x)

[Out] 4/23*I*(5*x^2+3*x+2)^(1/2)*(2*x^2-x+3)^(1/2)*(2*I*31^(1/2)+5*I*23^(1/2)-11)*(-2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*23^(1/2)-4*x+1)^2*(I*23^(1/2)*(I*31^(1/2)+10*x+3)/(2*I*31^(1/2)-5*I*23^(1/2)+11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*23^(1/2)*10^(1/2)*EllipticF((-2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2), ((2*I*31^(1/2)+5*I*23^(1/2)+11)*(2*I*31^(1/2)+5*I*23^(1/2)-11)/(2*I*31^(1/2)-5*I*23^(1/2)+11)/(2*I*31^(1/2)-5*I*23^(1/2)-11))^(1/2))/(10*x^4+x^3+16*x^2+7*x+6)^(1/2)/(2*I*31^(1/2)-5*I*23^(1/2)-11)/((-1+4*x+I*23^(1/2))*(I*23^(1/2)-4*x+1)*(I*31^(1/2)+10*x+3)*(I*31^(1/2)-10*x-3))^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}}{10x^4 + x^3 + 16x^2 + 7x + 6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\sqrt{5x^2 + 3x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x**2+3*x+2)**(1/2)/(2*x**2-x+3)**(1/2),x)

[Out] Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2 + 3x + 2}\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,
```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```



```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```